Theory and practice of algorithmic self-assembly

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Inria
Paris

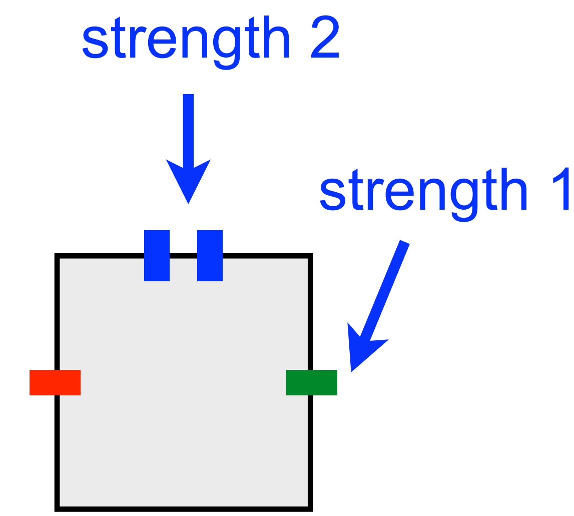
ER02 - ENS Lyon - Jan 2017 - Lectures 3 & 4

Structure

- Monday (lecture 1). Some high-level motivations, basic algorithmic self-assembly **models** (definitions) and very recent results on **implementing algorithmic DNA nanotube circuits, a self-assembly model,** in the wet-lab
- Tuesday (lecture 2): DNA sequence design and results on the DNA nanotube circuit model
- Wednesday (lecture 3). Complexity theory for self assembly.
  - **Theorem**: The (cooperative, temperature >= 2) abstract tile assembly model is intrinsic universal
- Thursday (lecture 4). Complexity theory for self assembly.
  - **Theorem**: The noncooperative (temperature 1) abstract tile assembly model does not simulate the cooperative model

Abstract tile assembly model

- Square **tiles**
  - finite set of tile types, unlimited supply of each type, non-rotatable
- Each side has a **glue** (colour) and **strength** (0,1,2,3,…)
- System has a **temperature** (e.g. 2)



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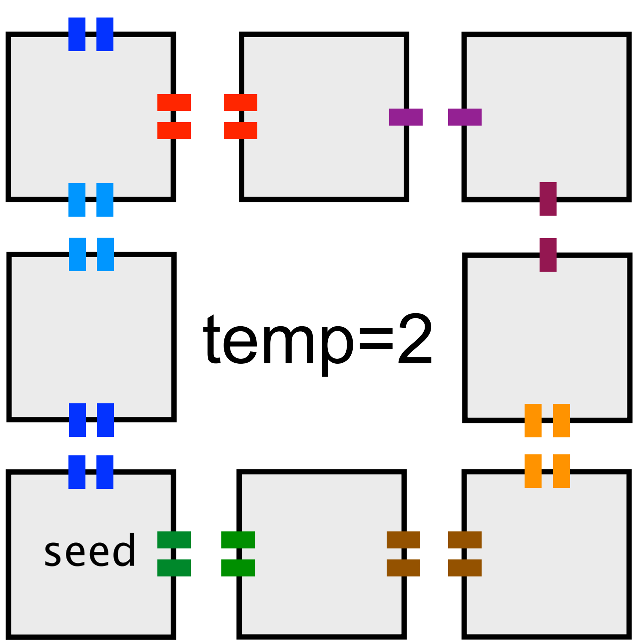
- **Tile assembly system:**
  tile set, **seed tile**, **temperature**



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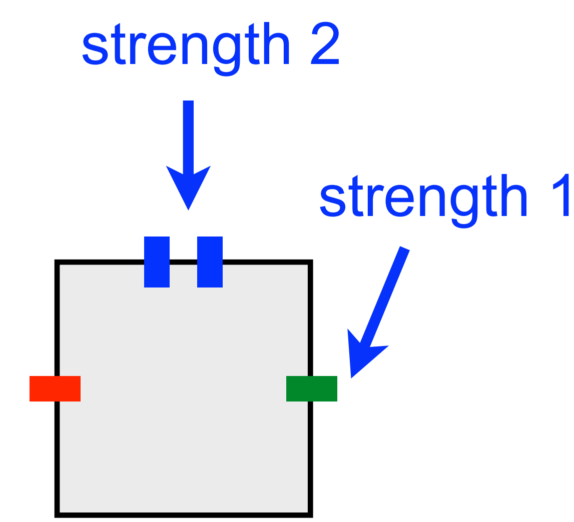

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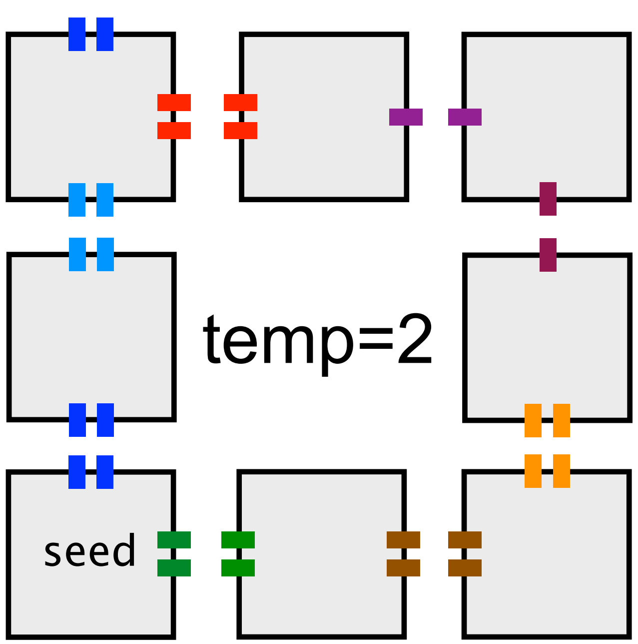

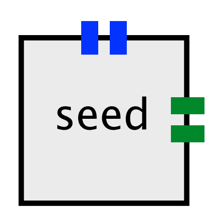

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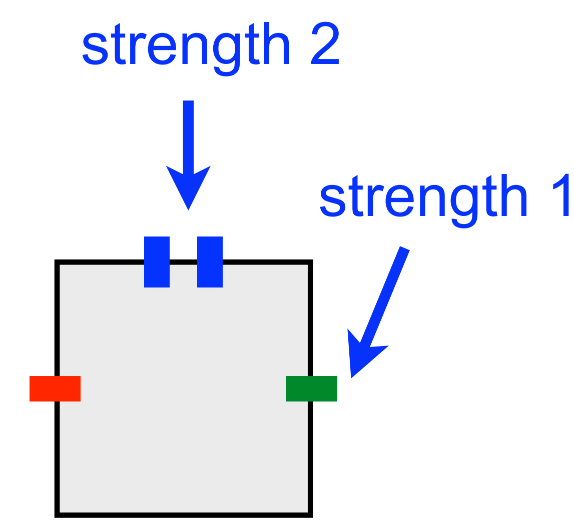

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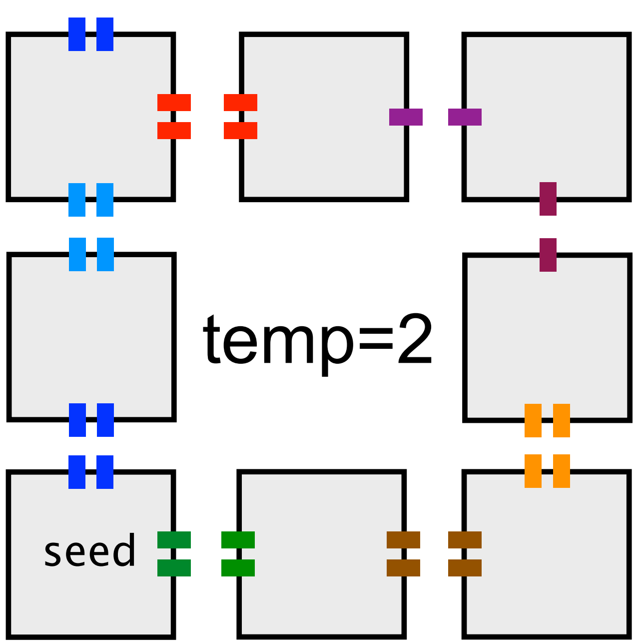

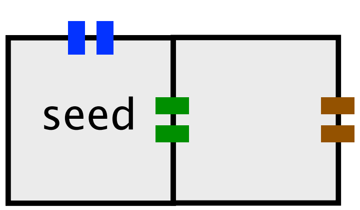

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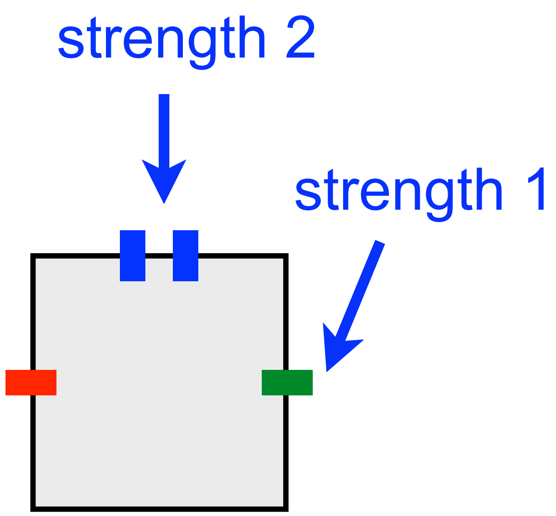

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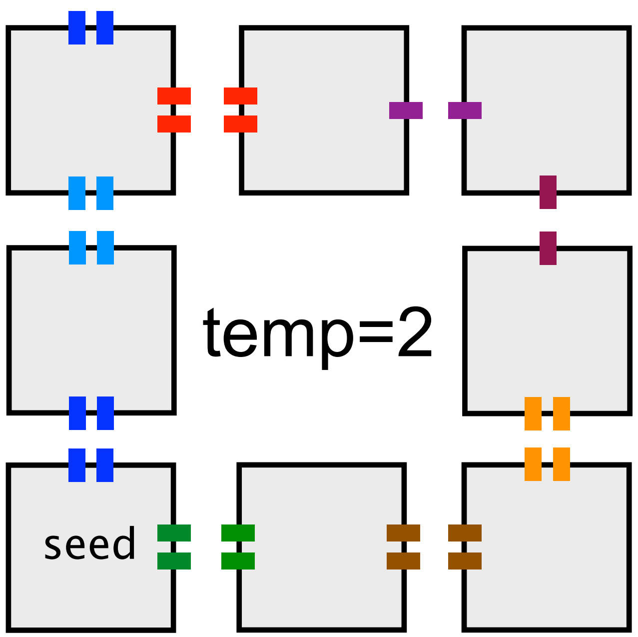

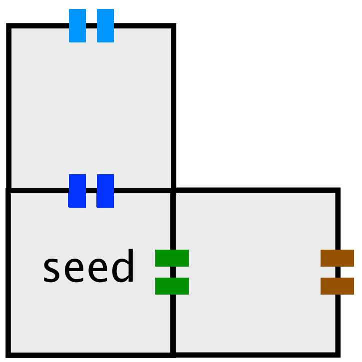

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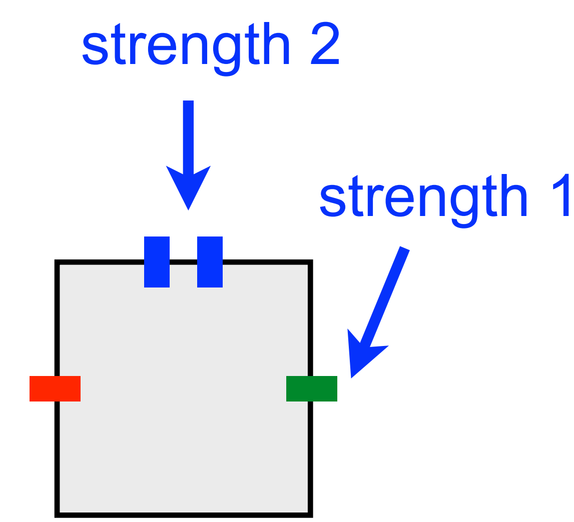

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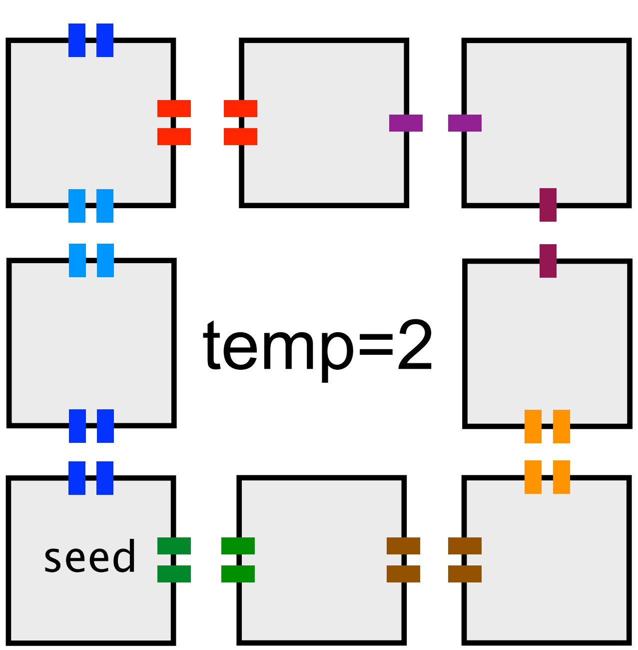

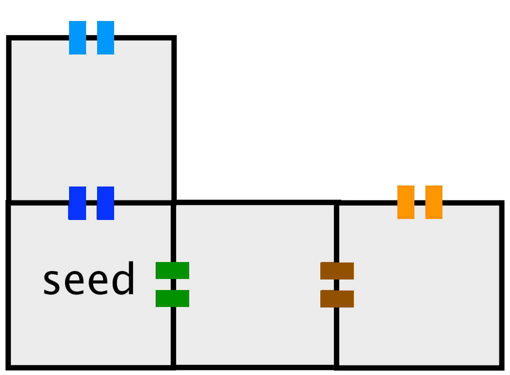

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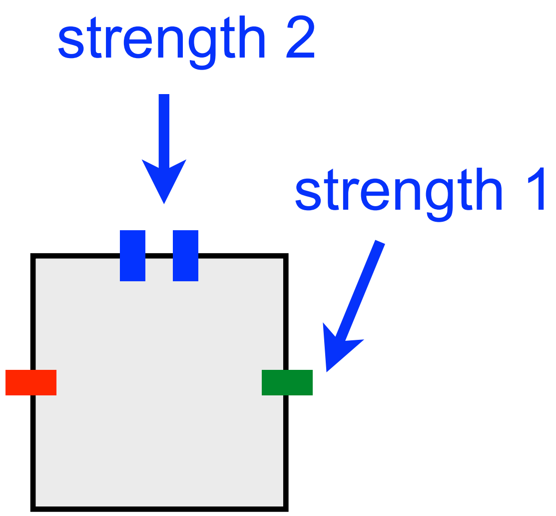

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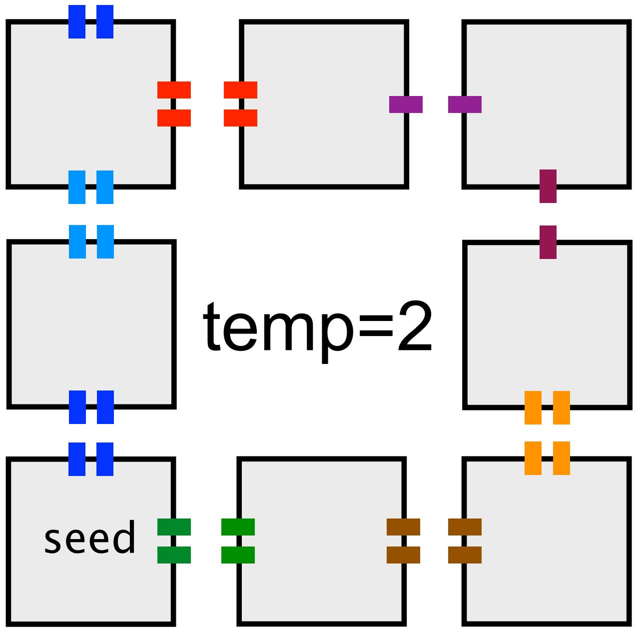

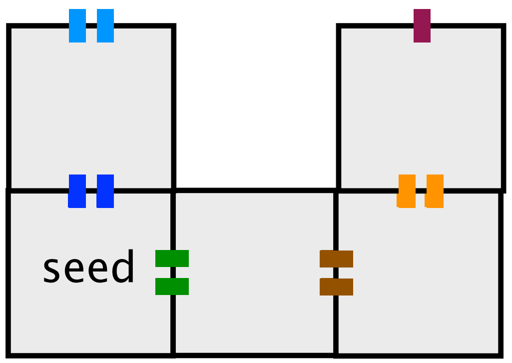

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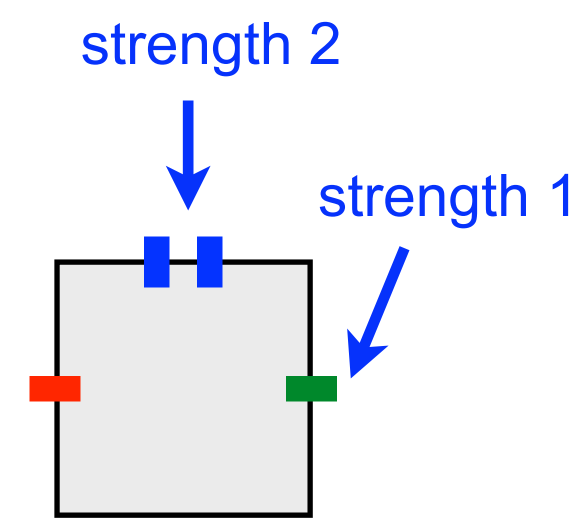

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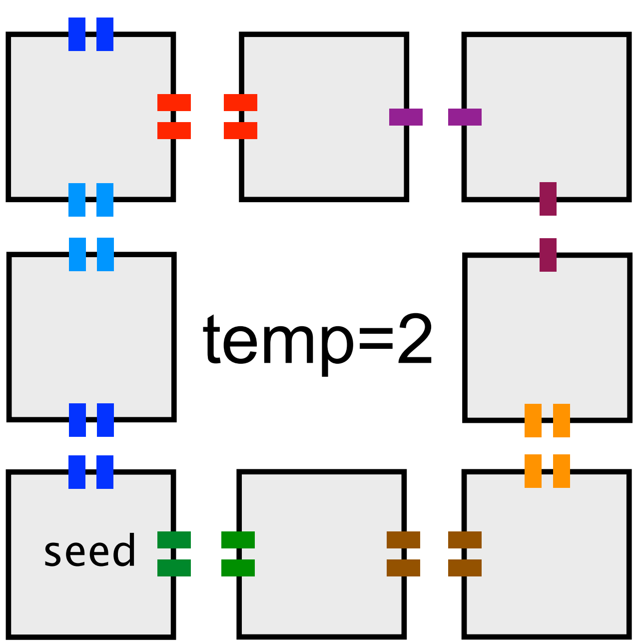

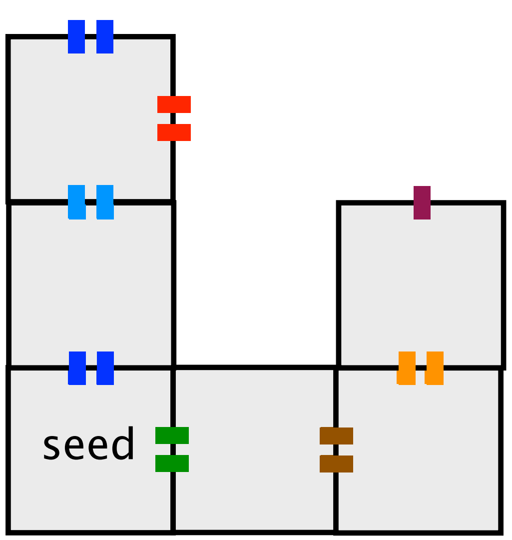

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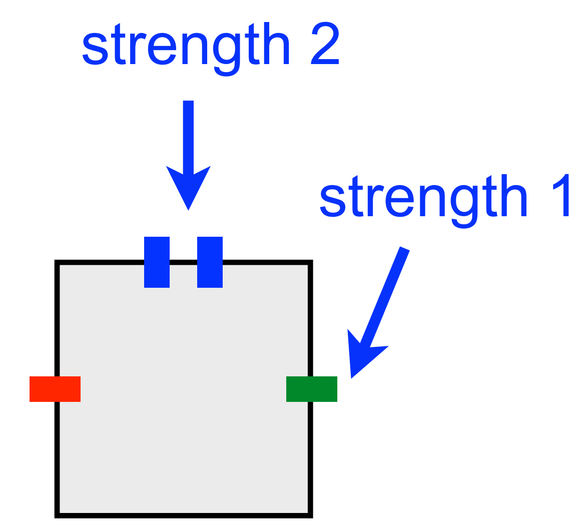

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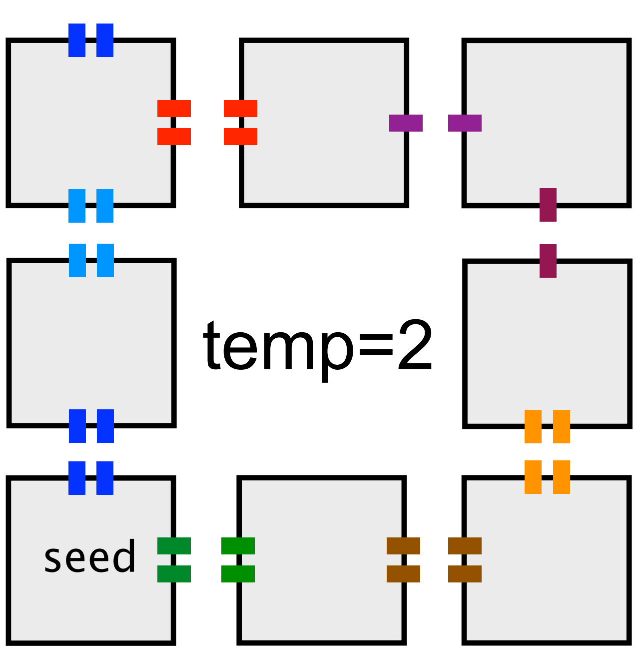

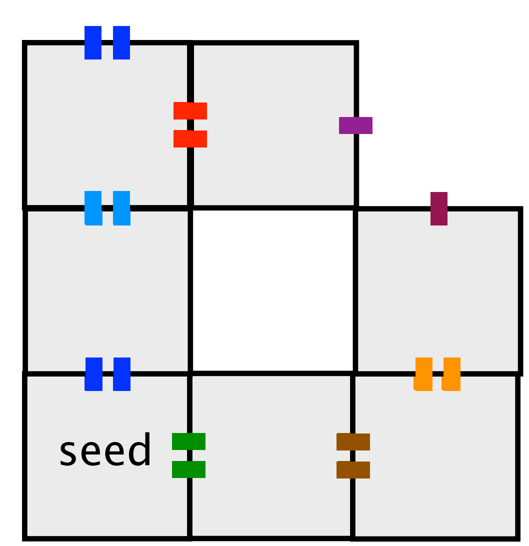

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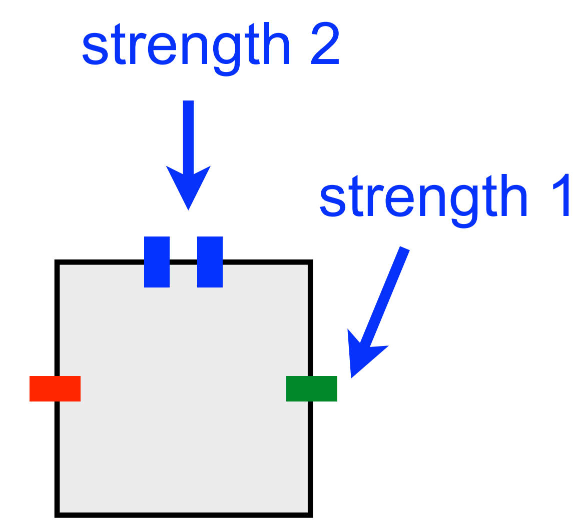

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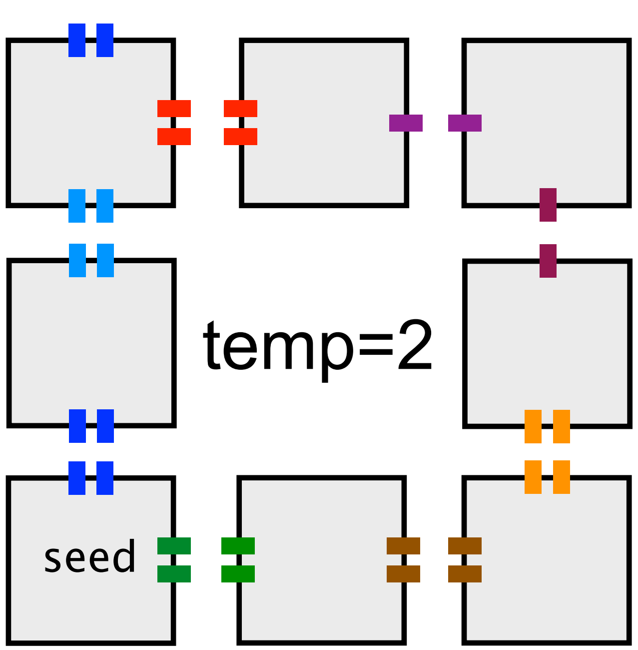

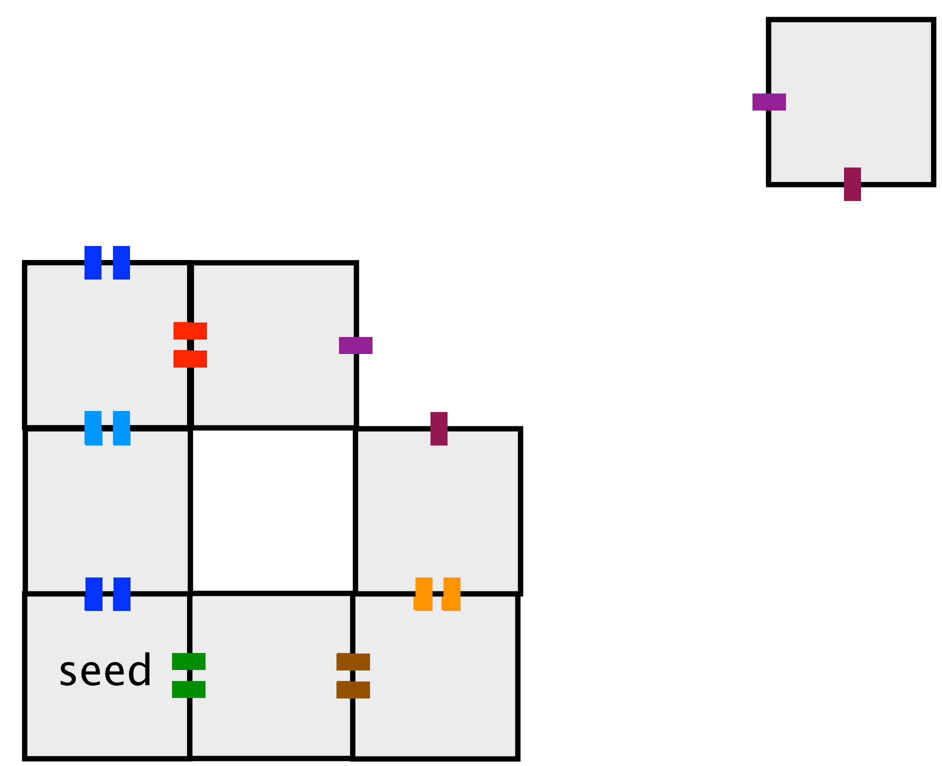

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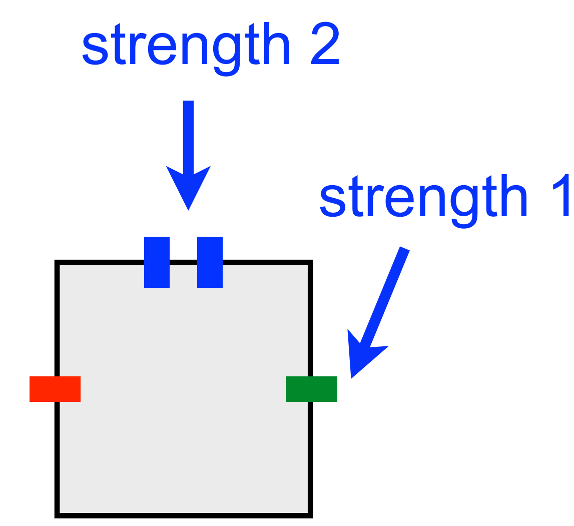

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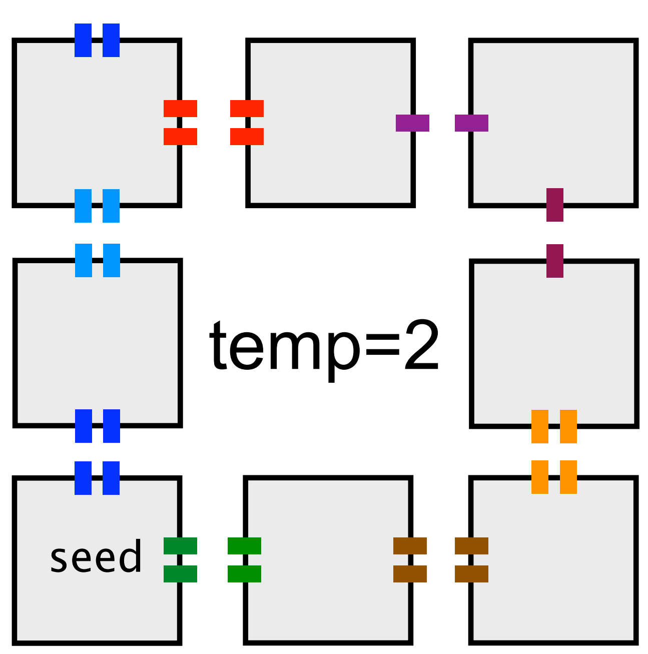

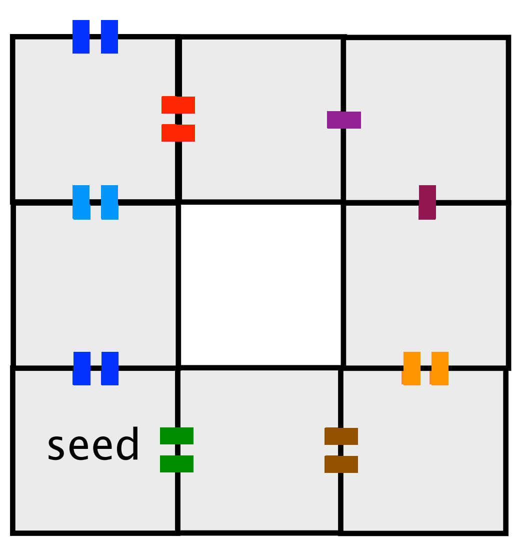

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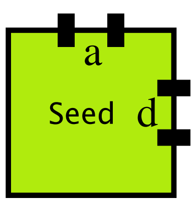

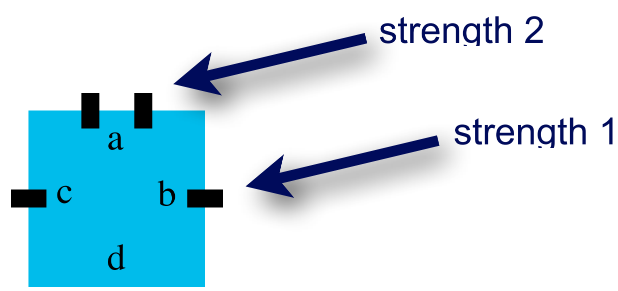

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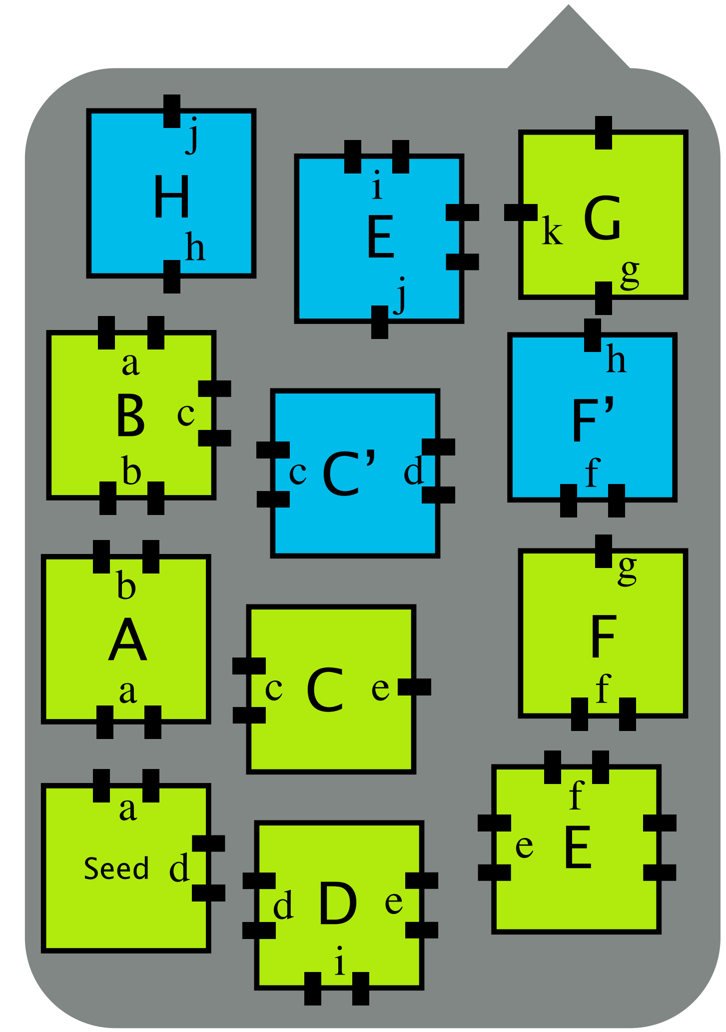

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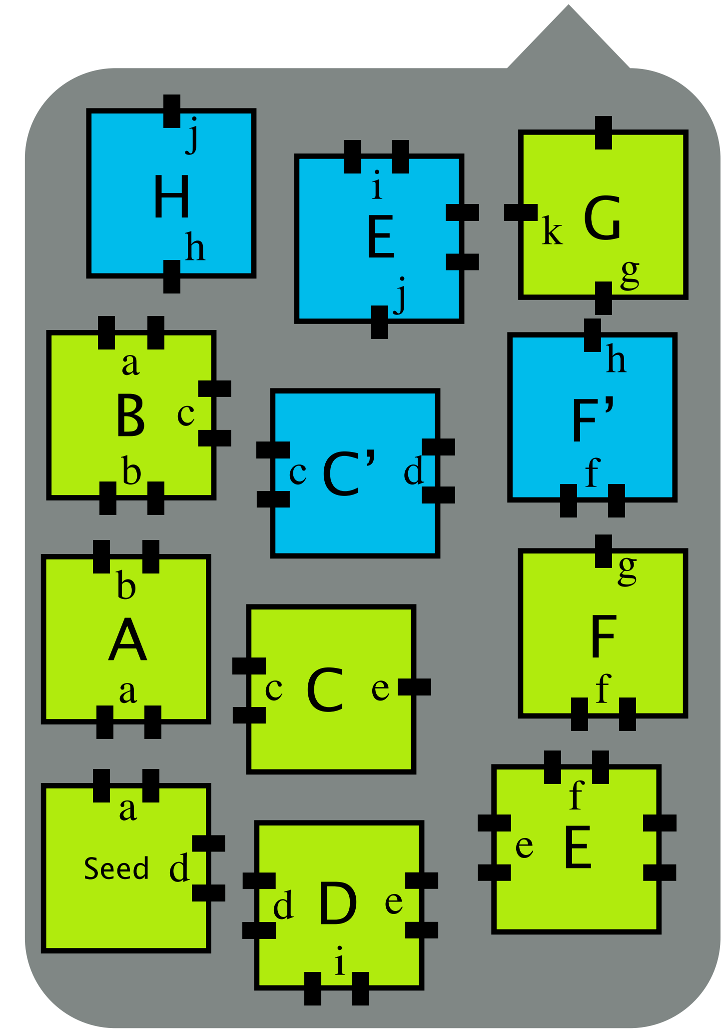

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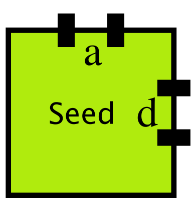

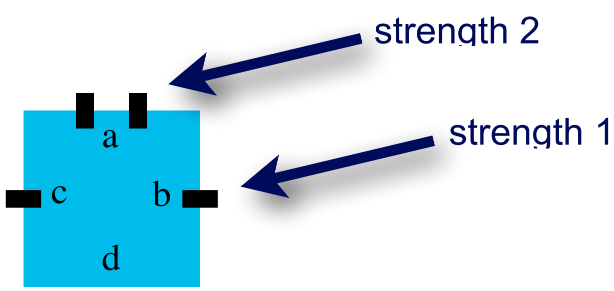

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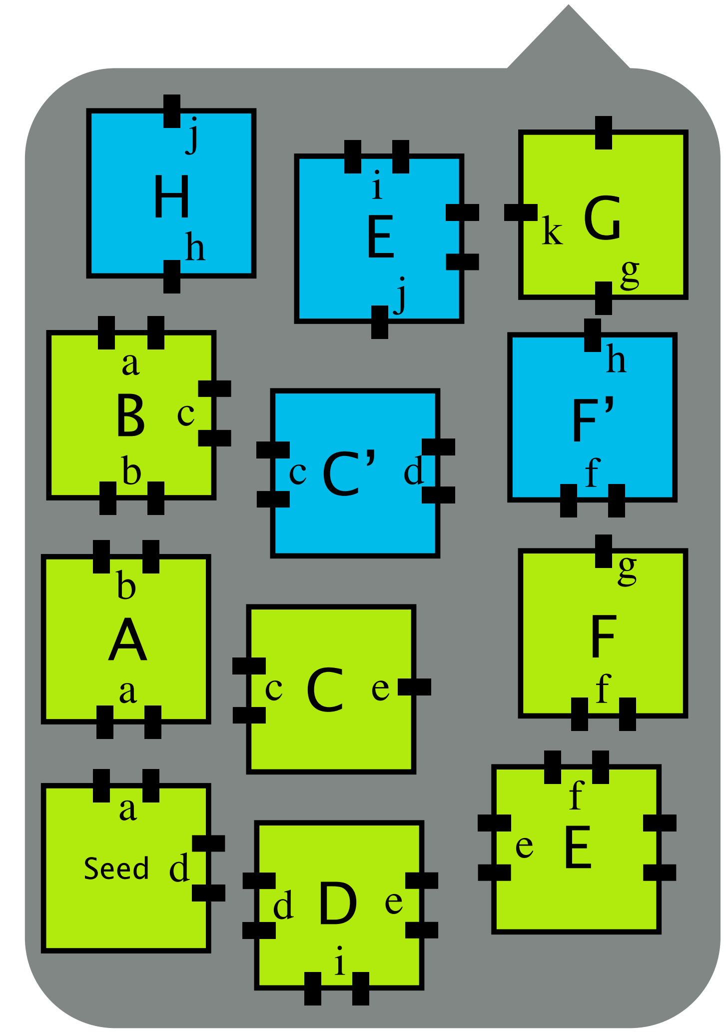

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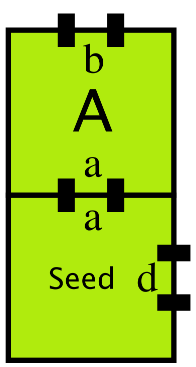

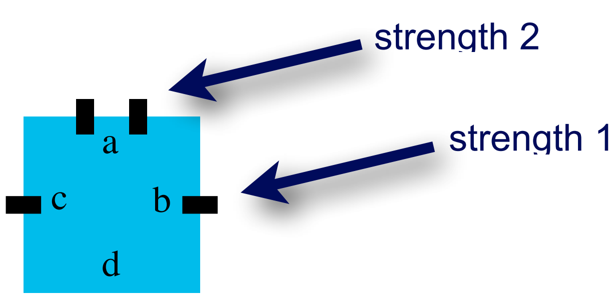

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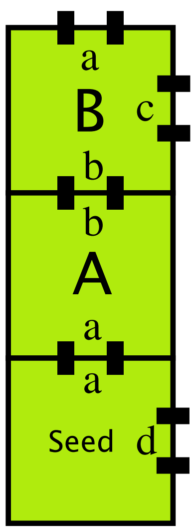

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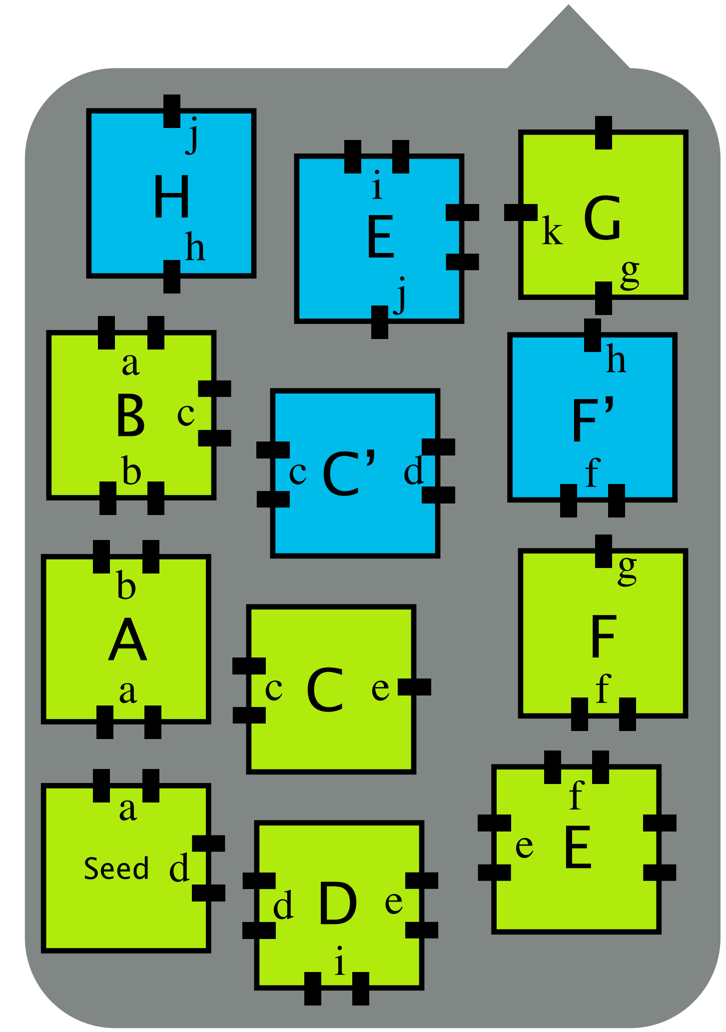

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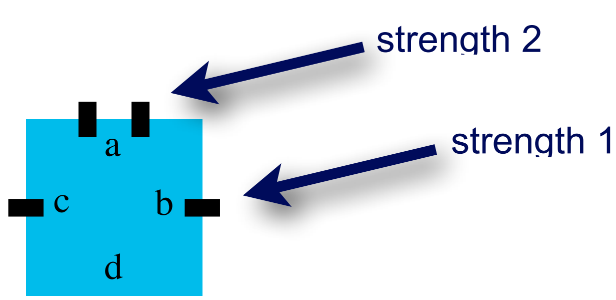

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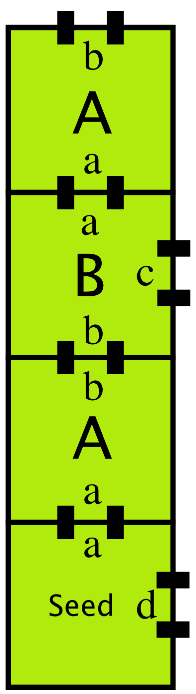

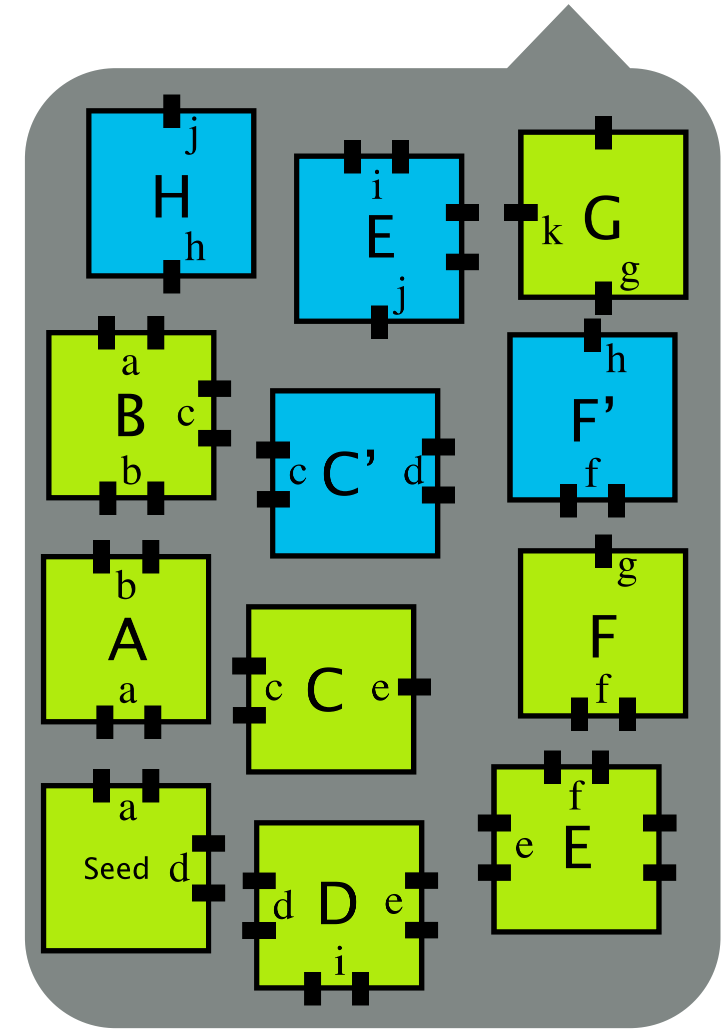

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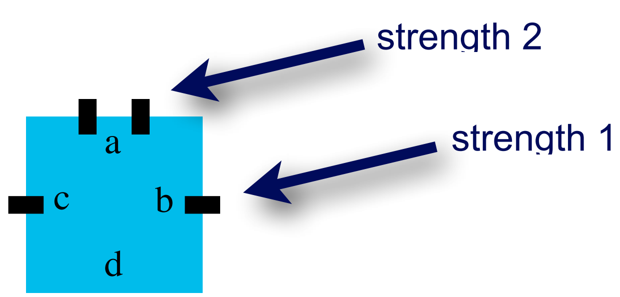

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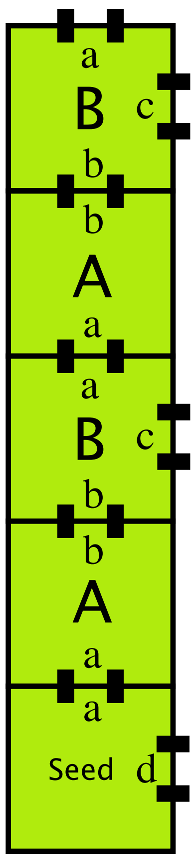

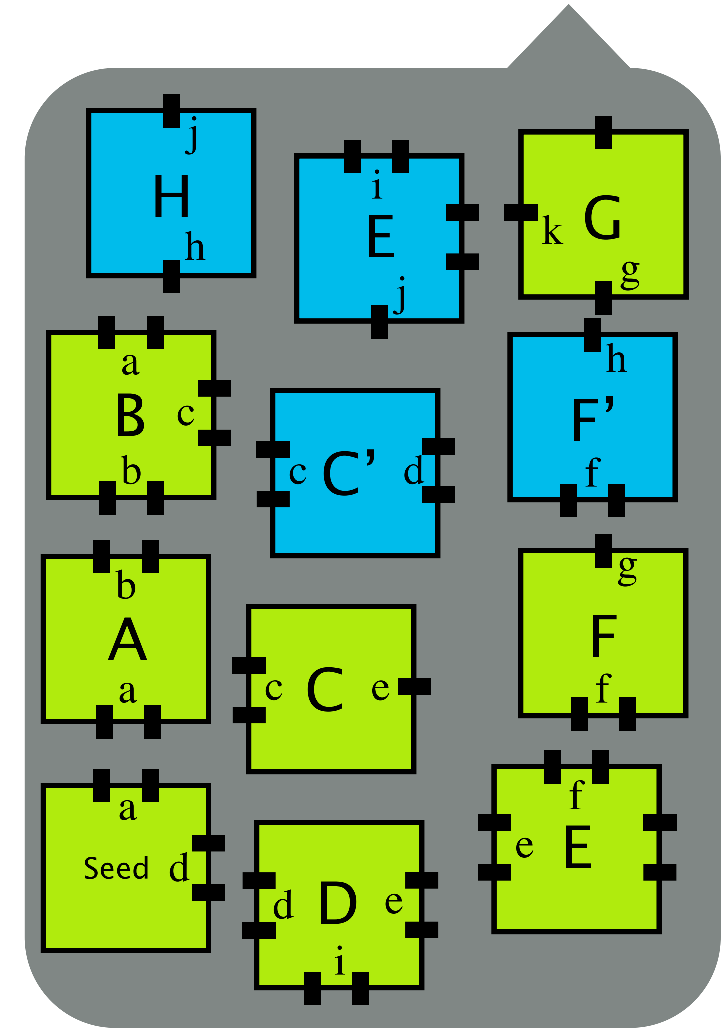

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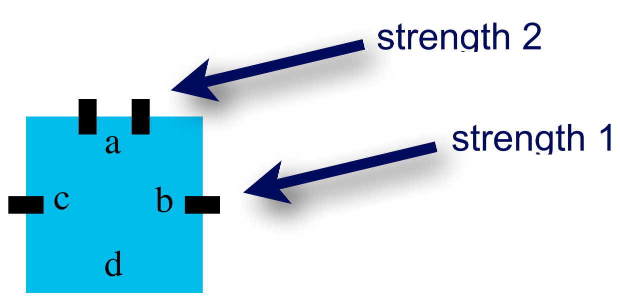

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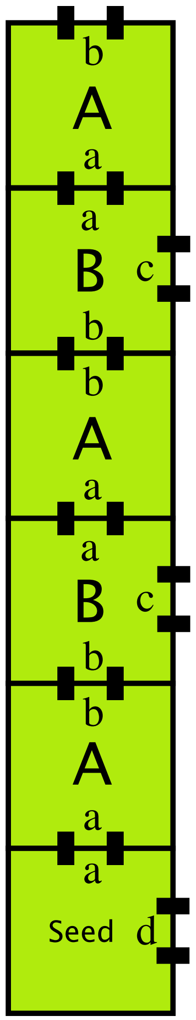

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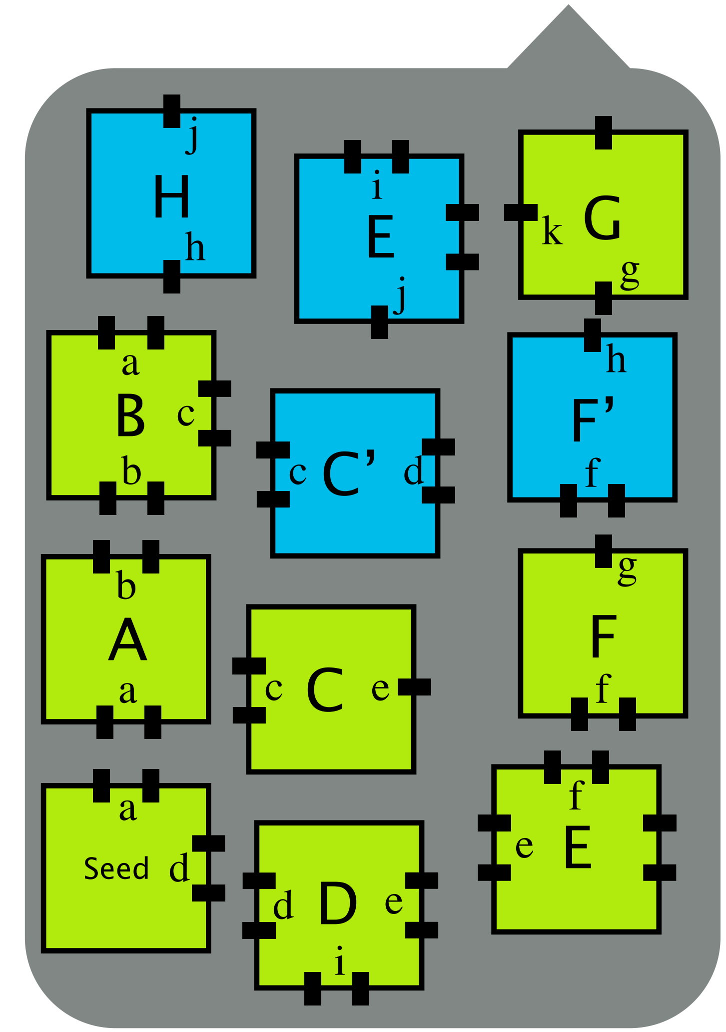

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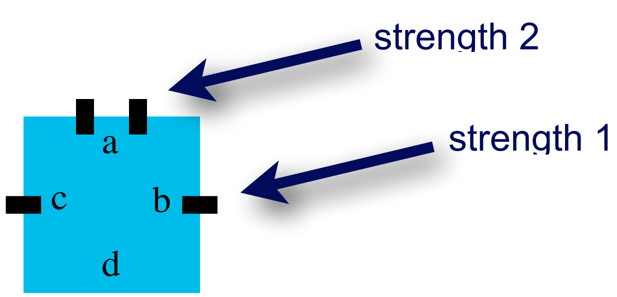

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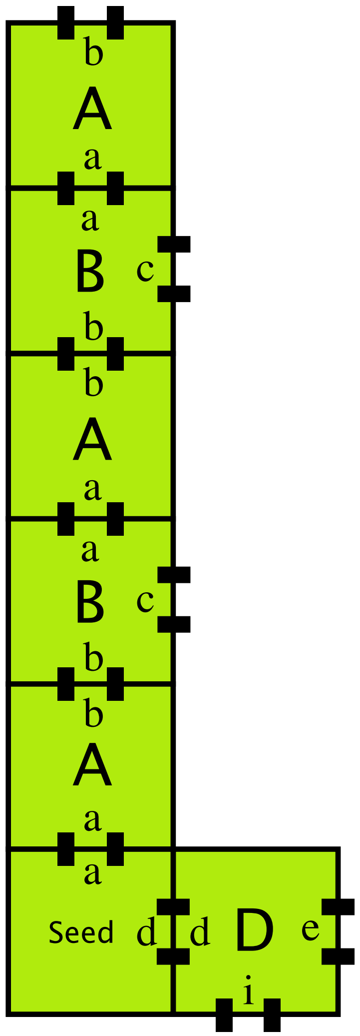

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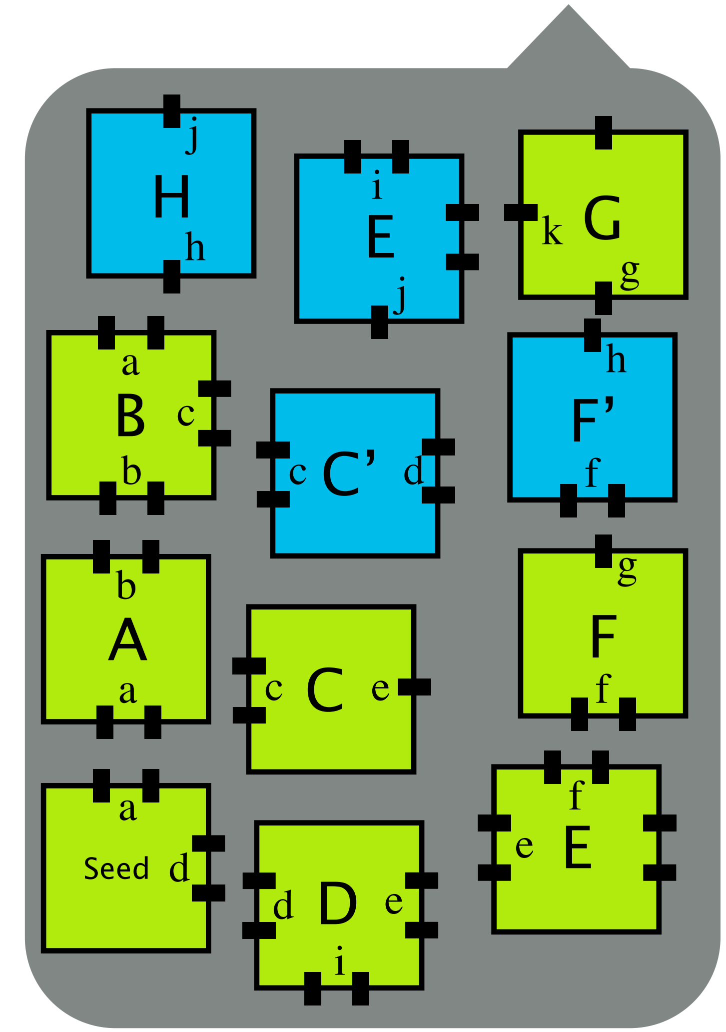

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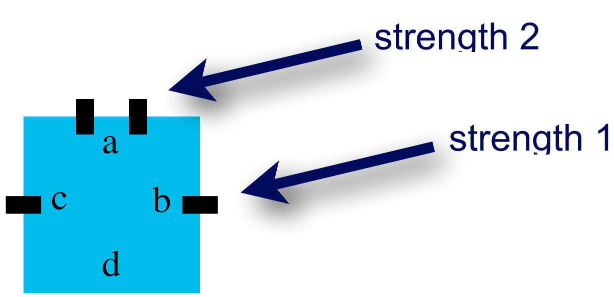

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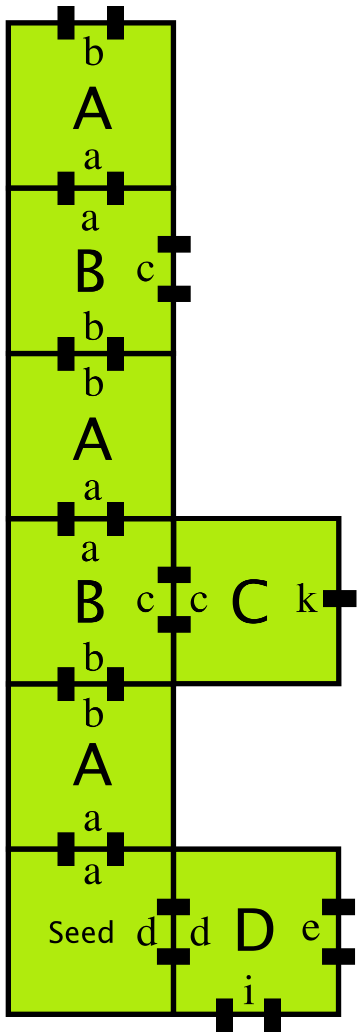
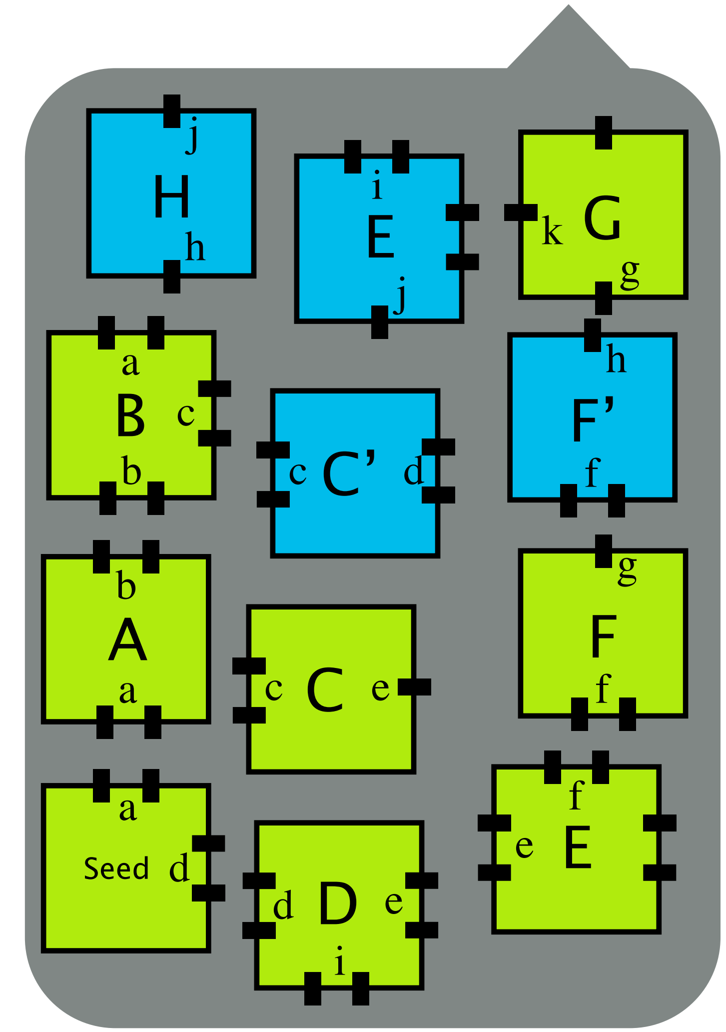
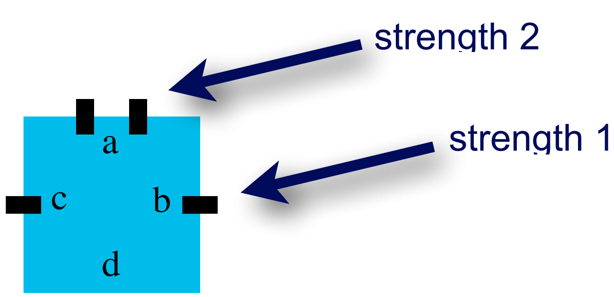


Tile assembly system:
tile set, seed tile, temperature



Temperature = 2



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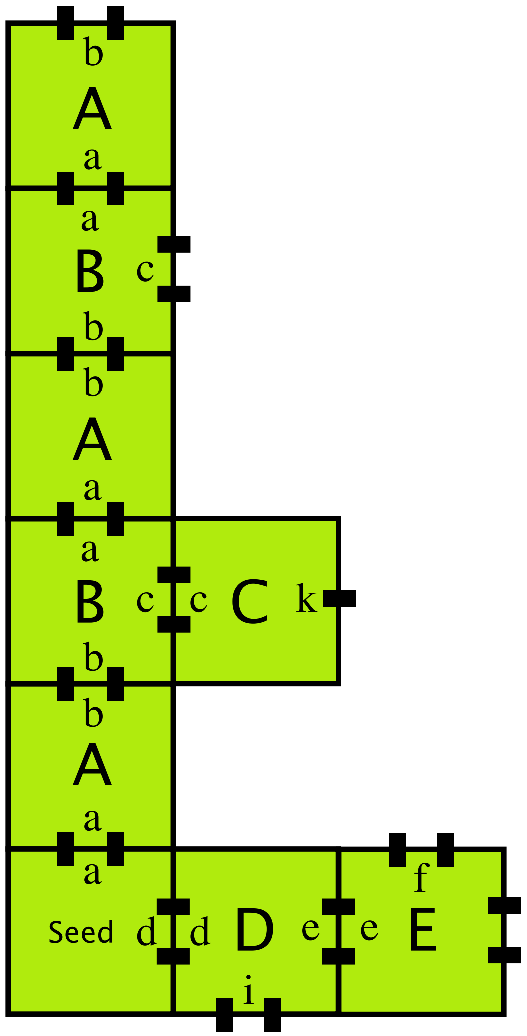

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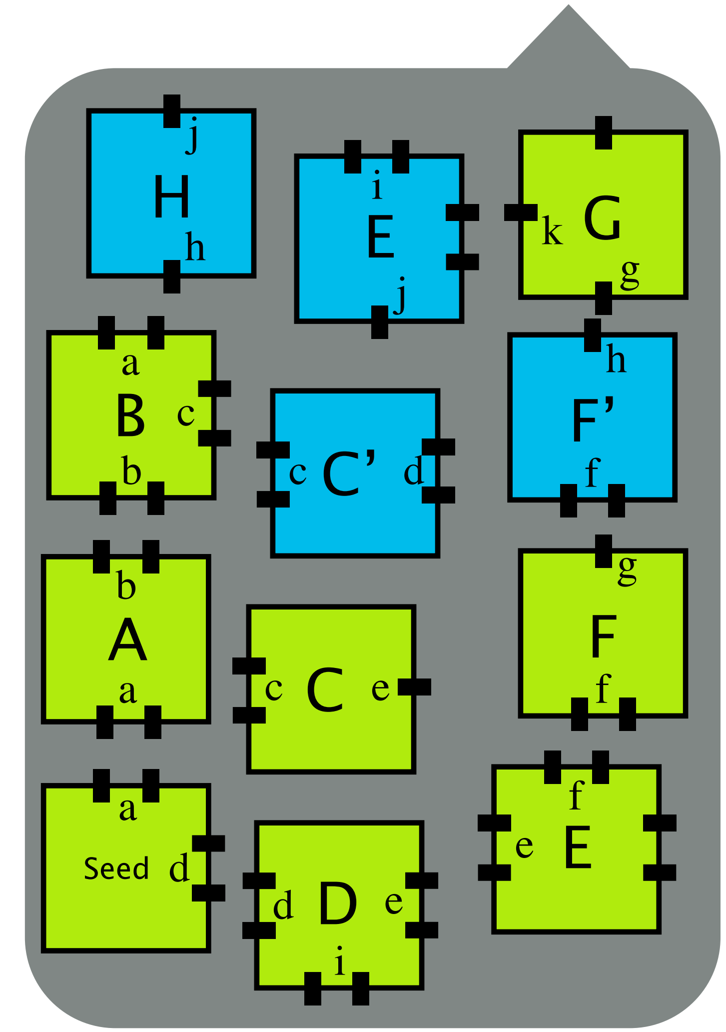

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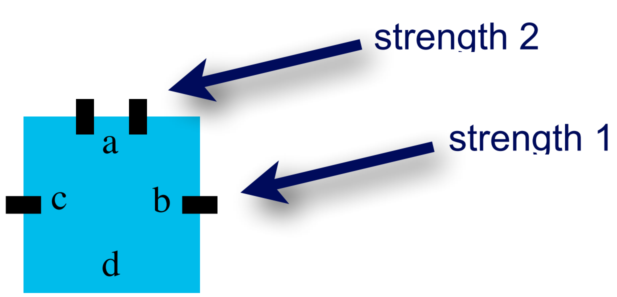

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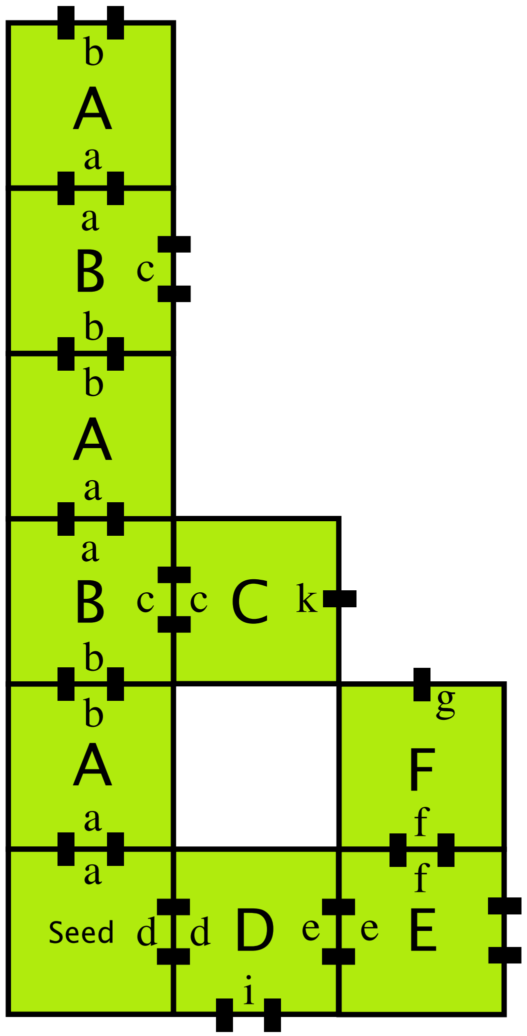

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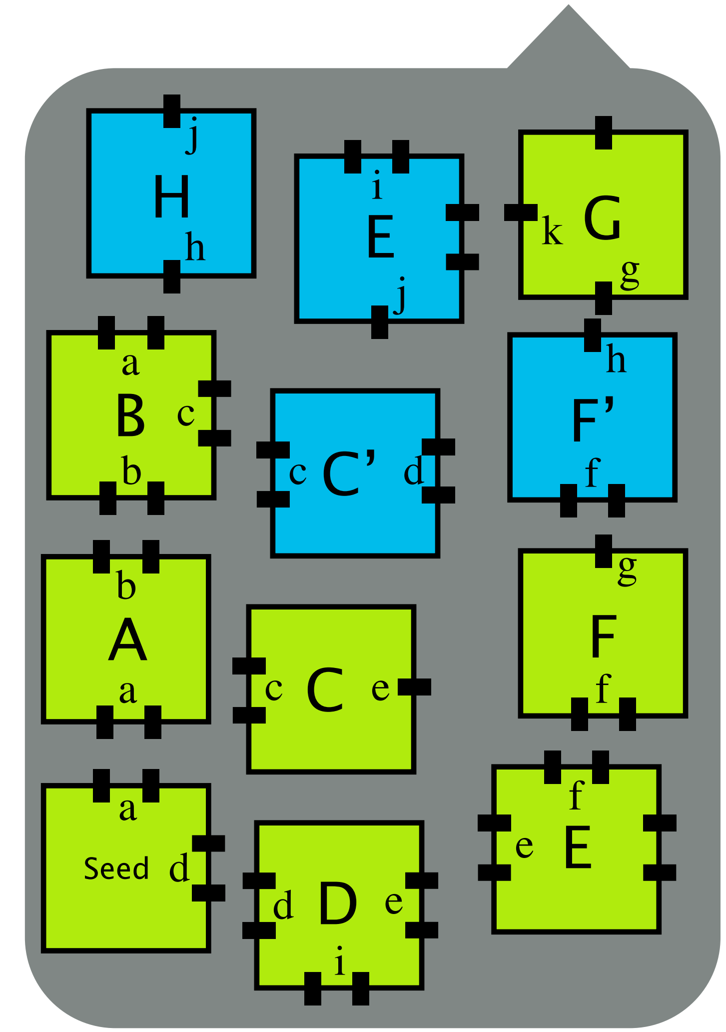

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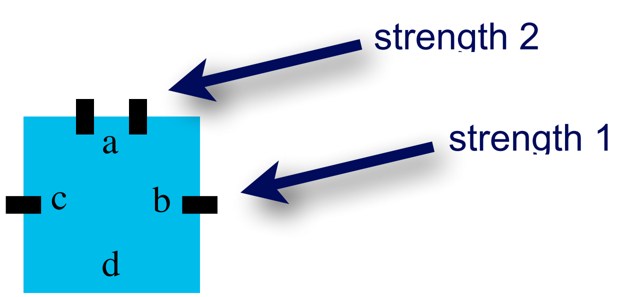

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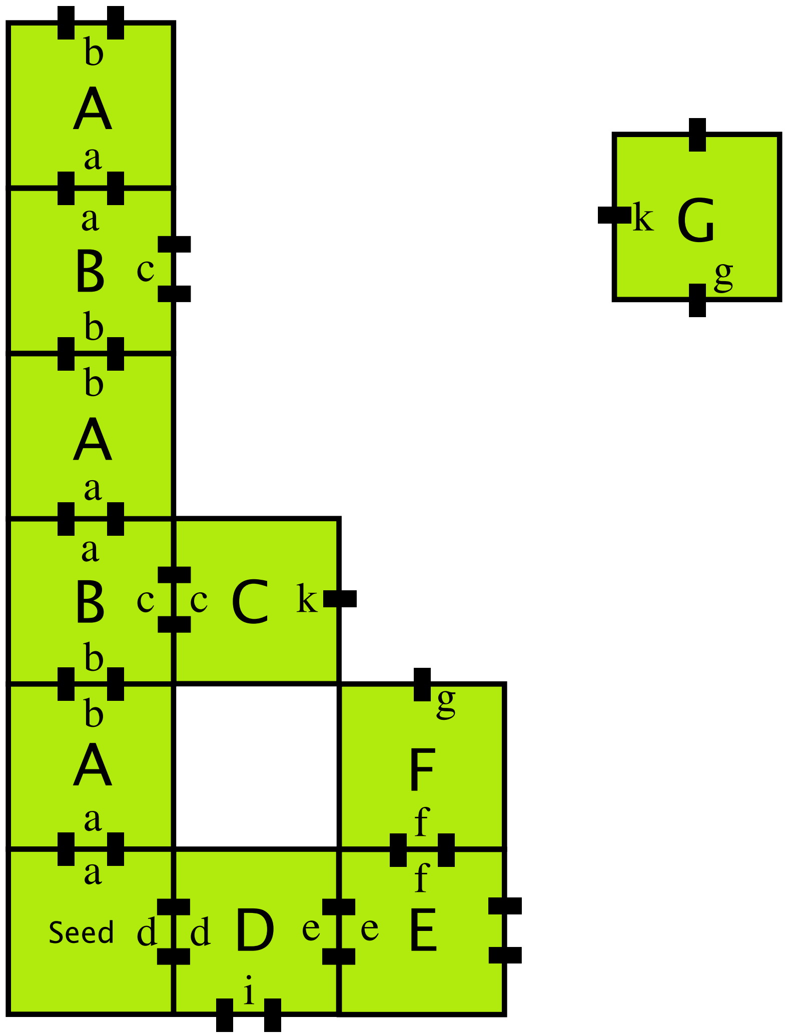

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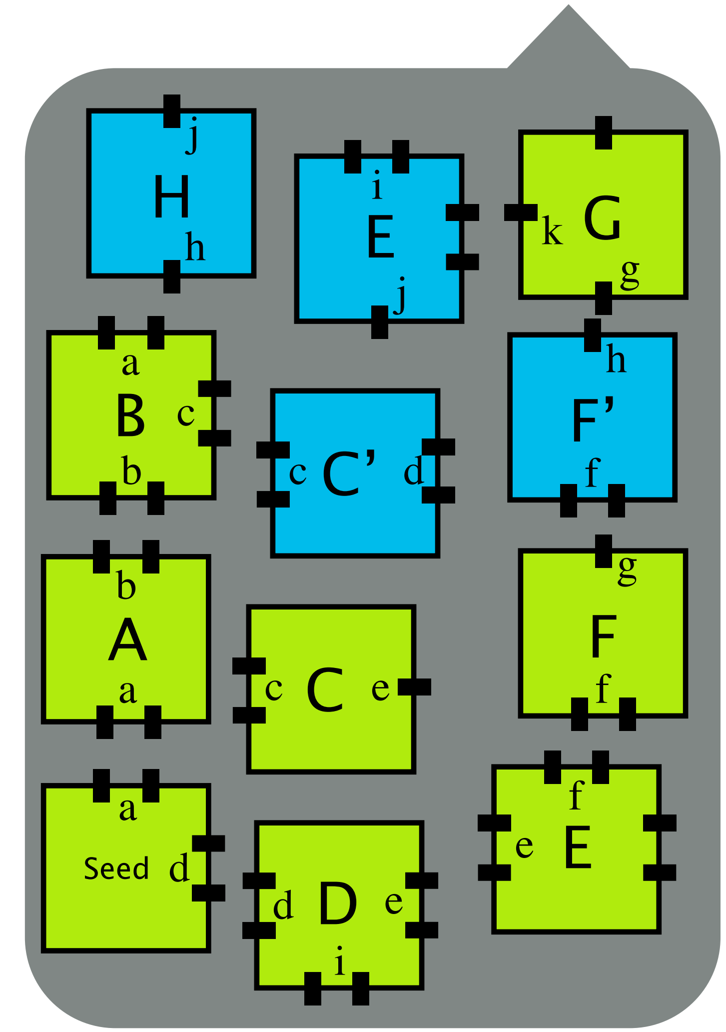

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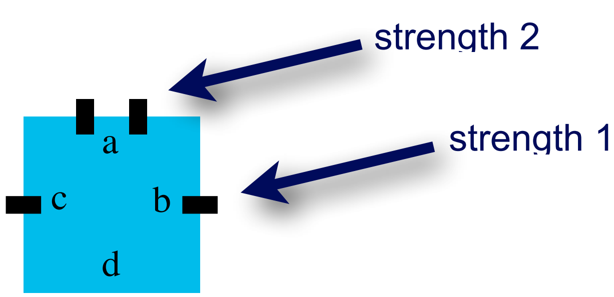

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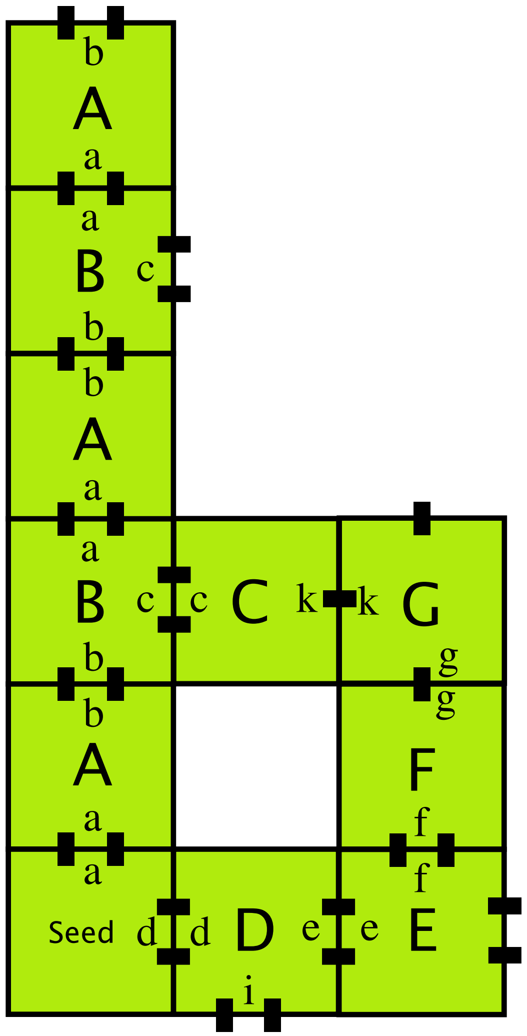

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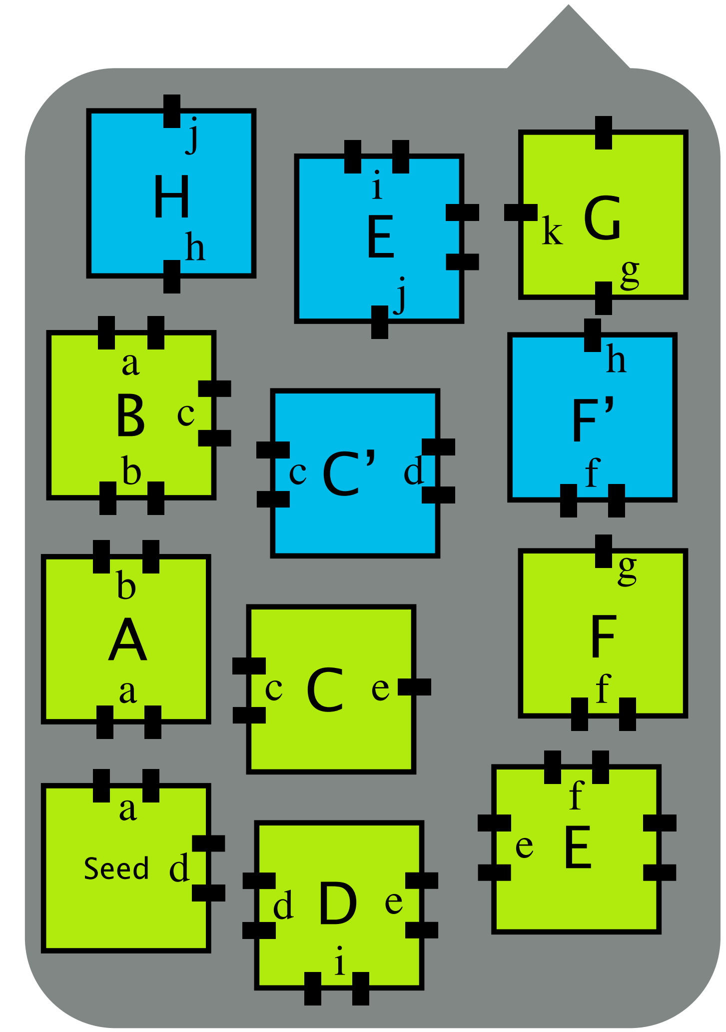

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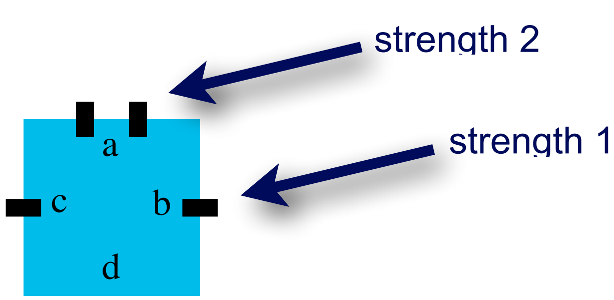

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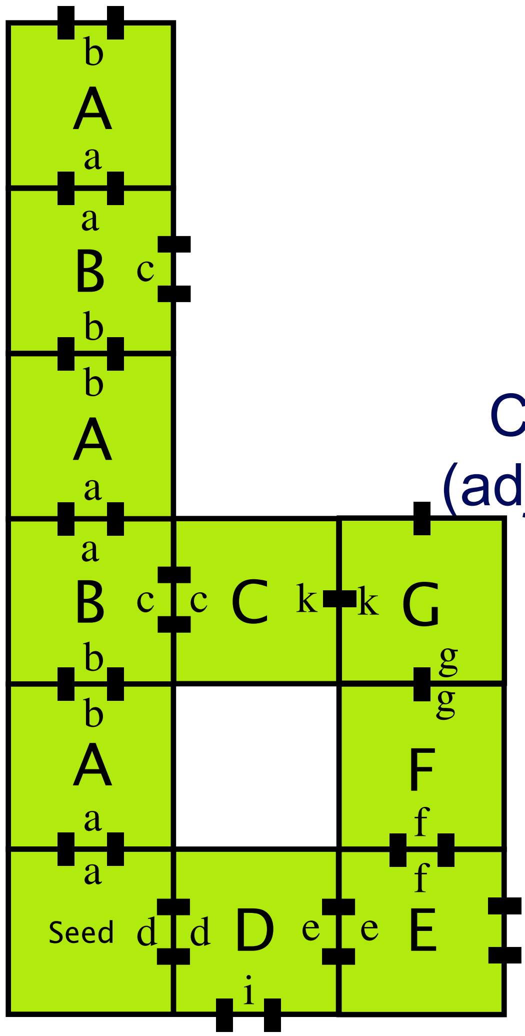

Cooperation
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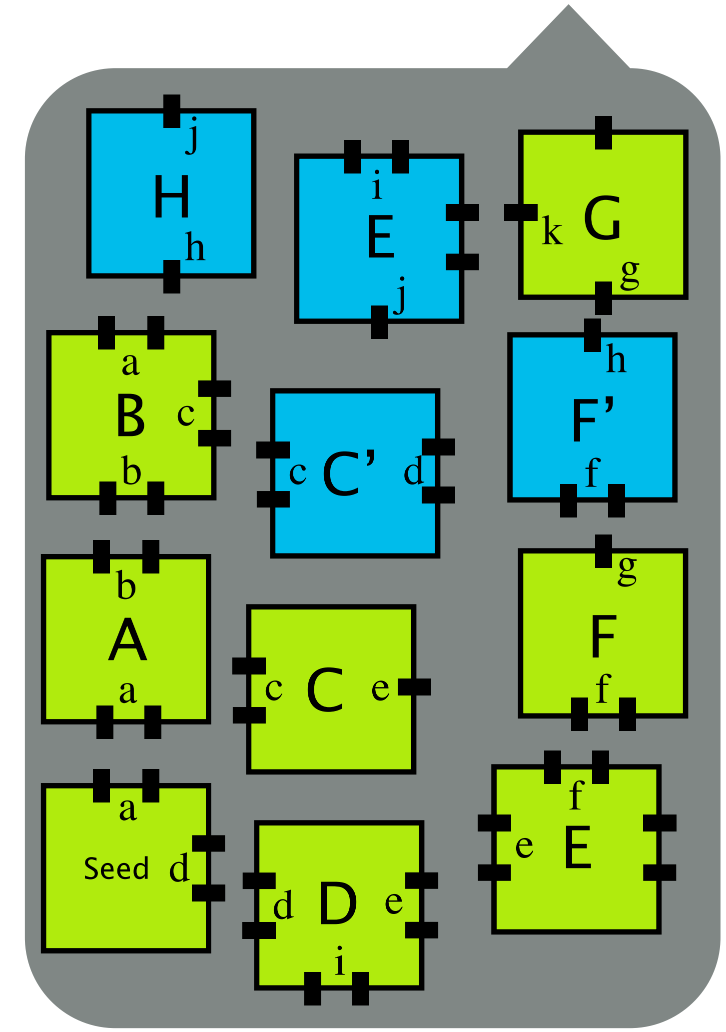

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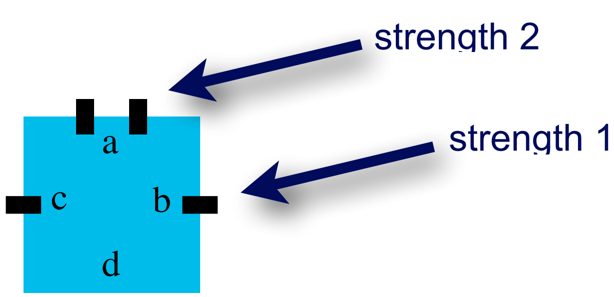

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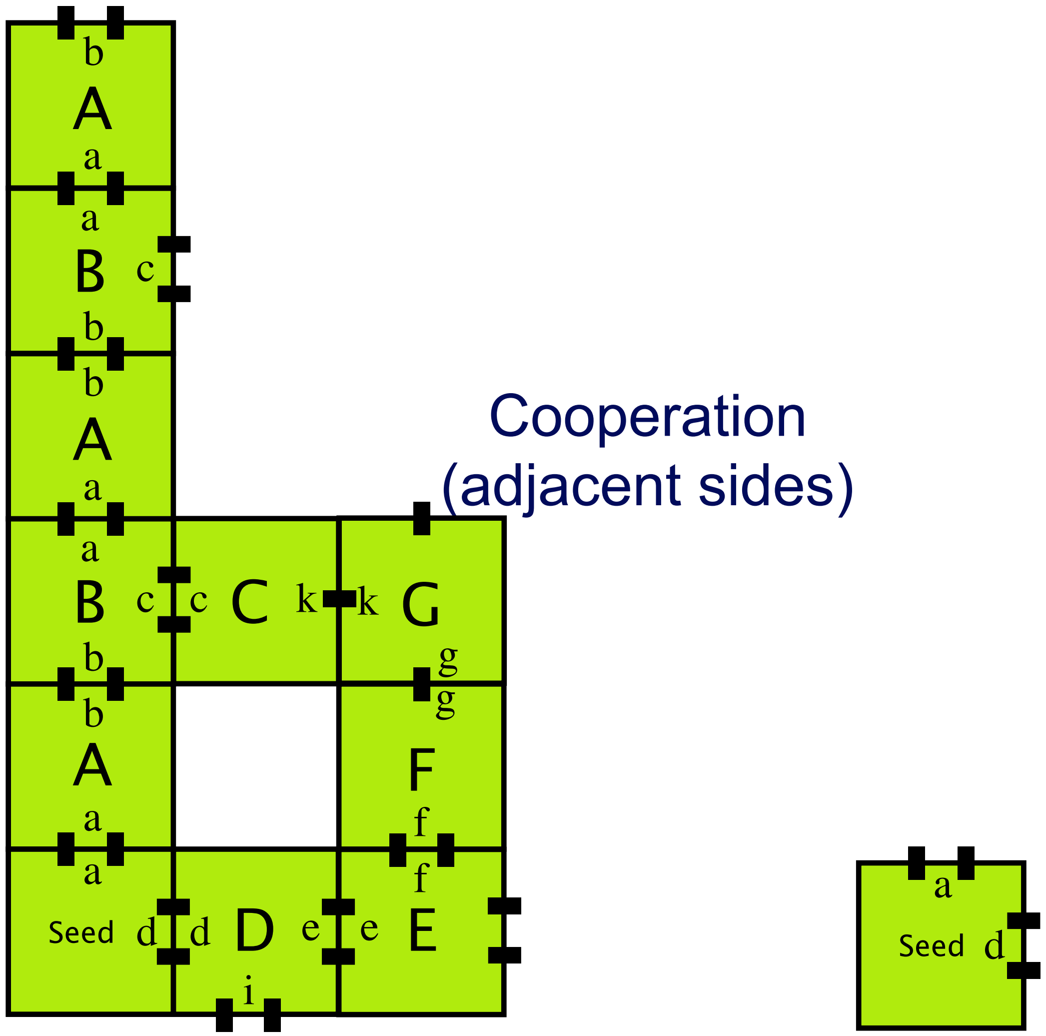

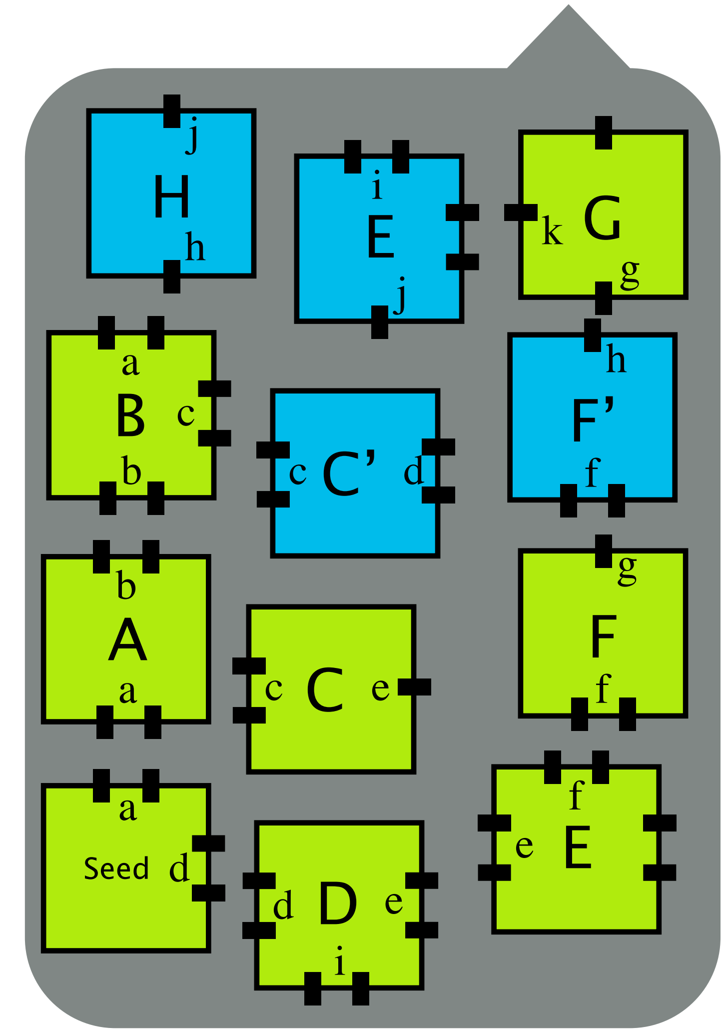

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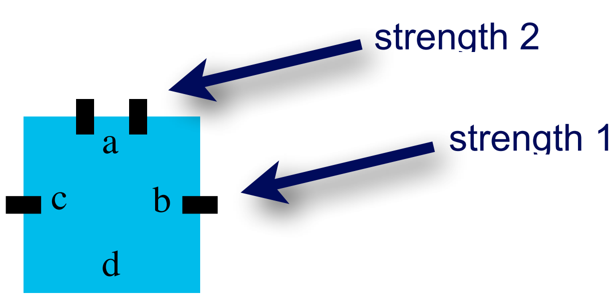

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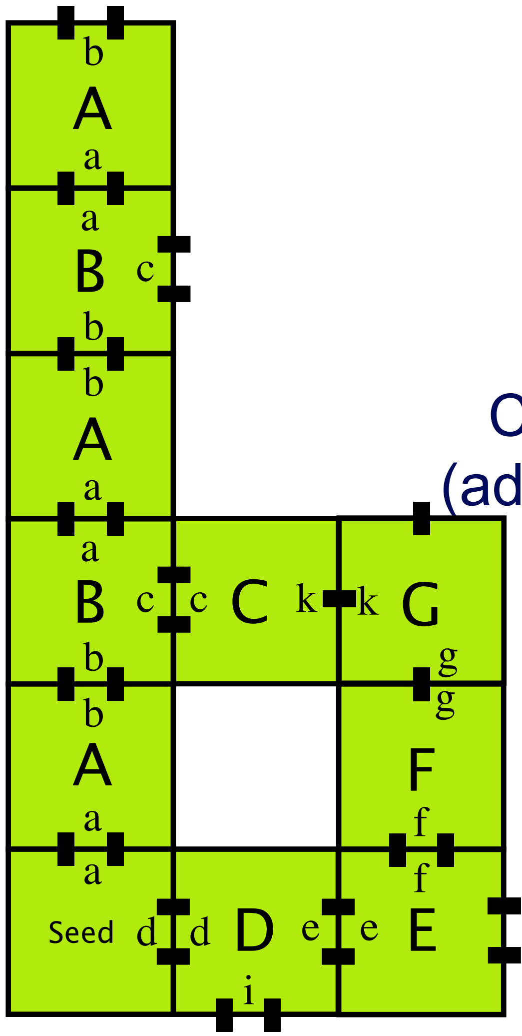

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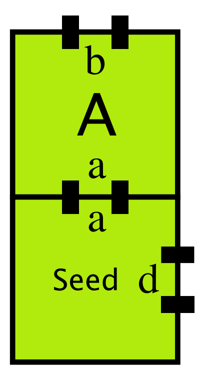

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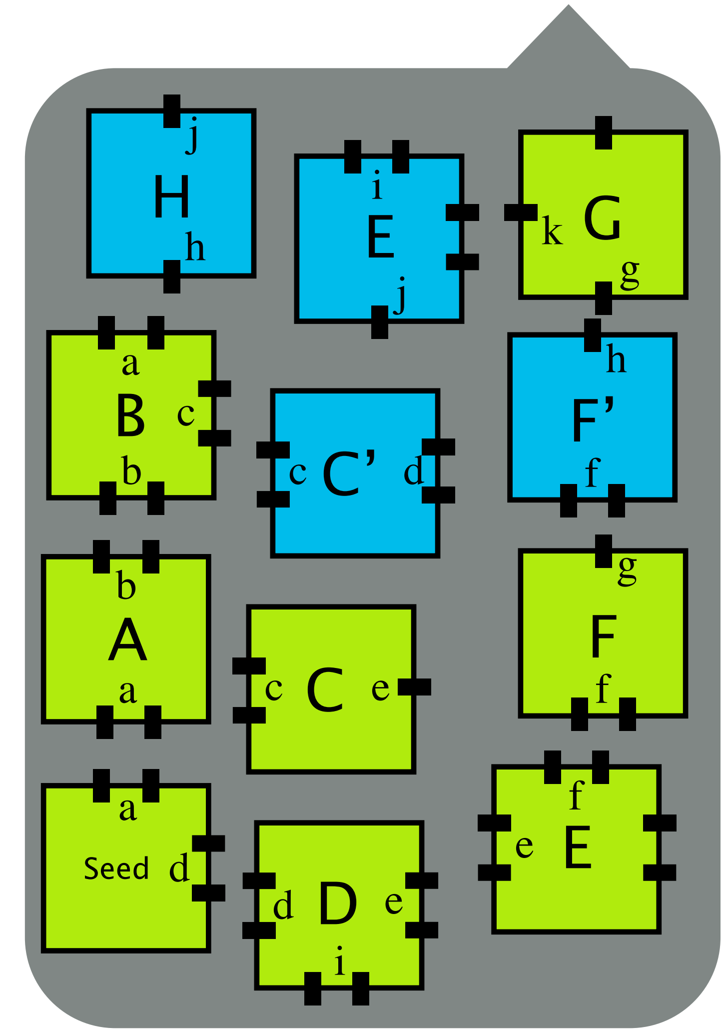

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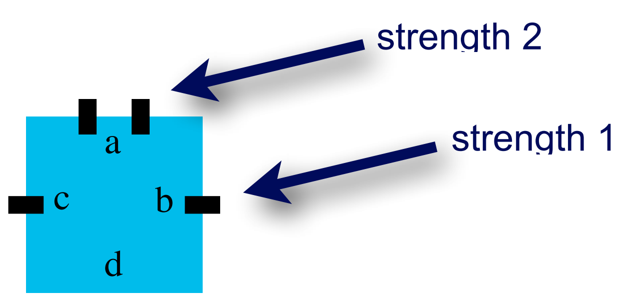

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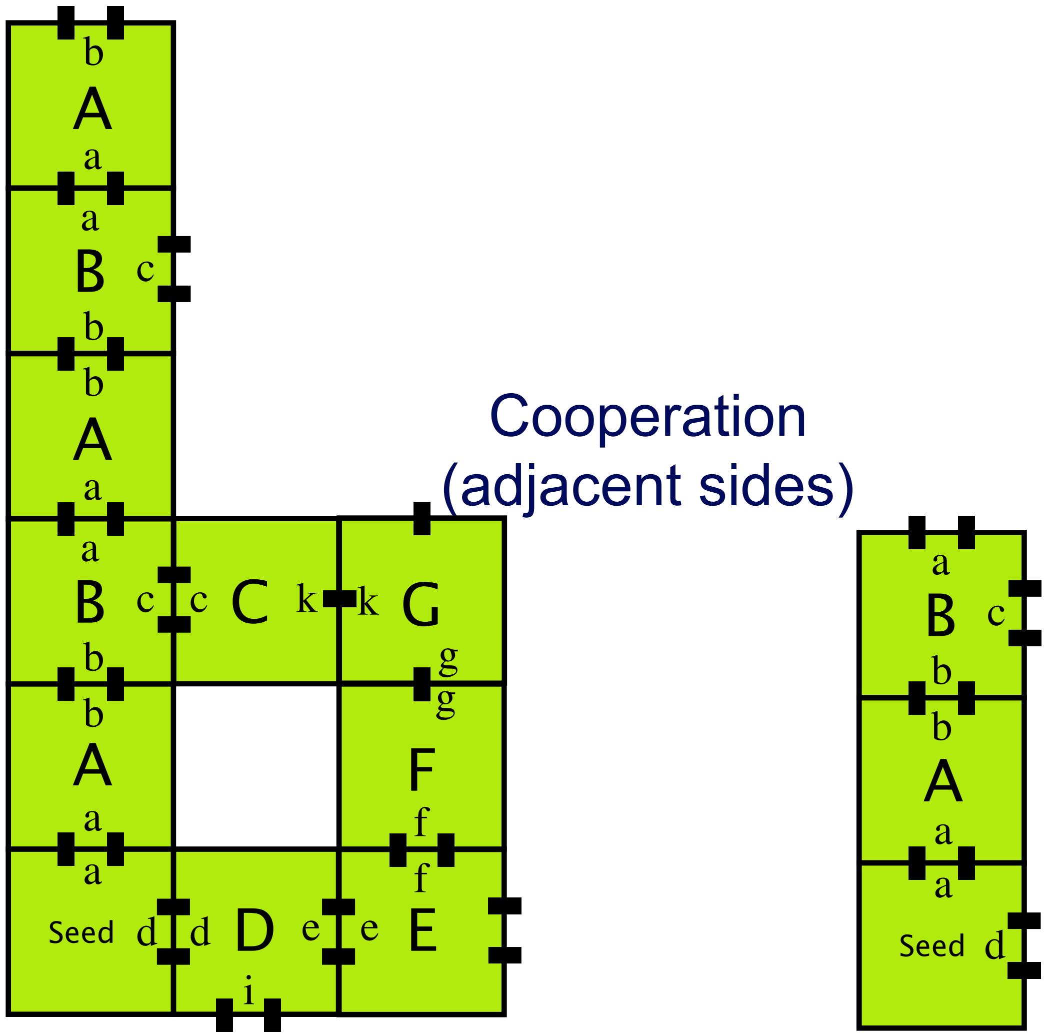

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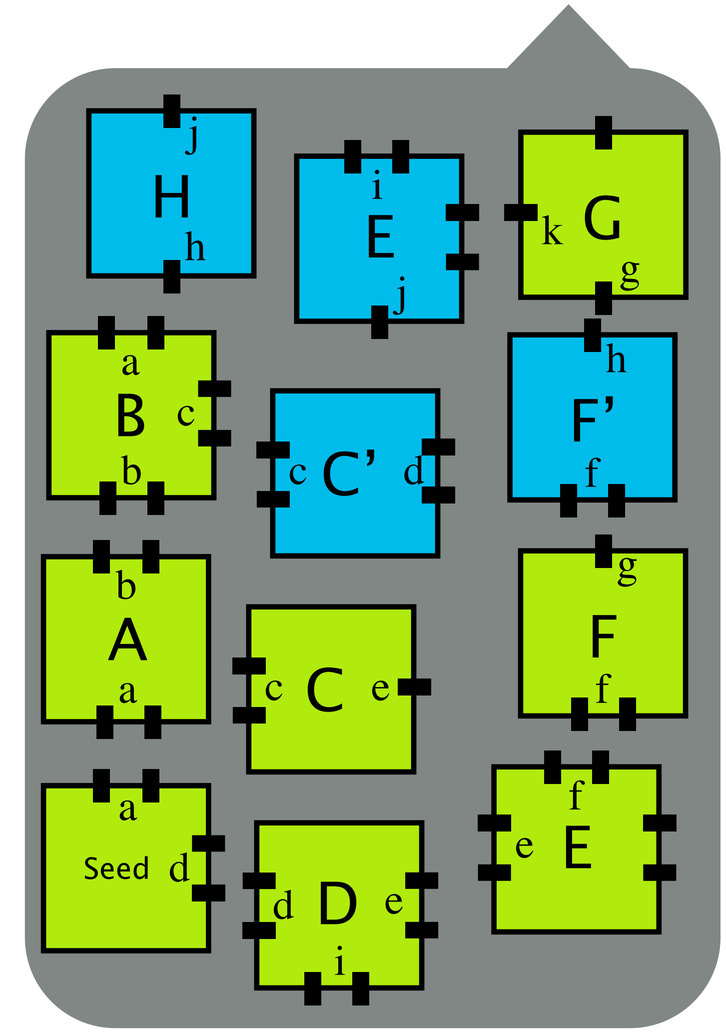

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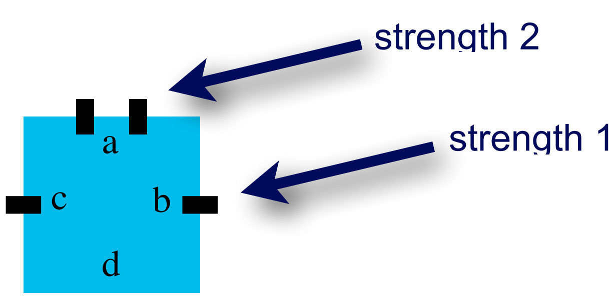

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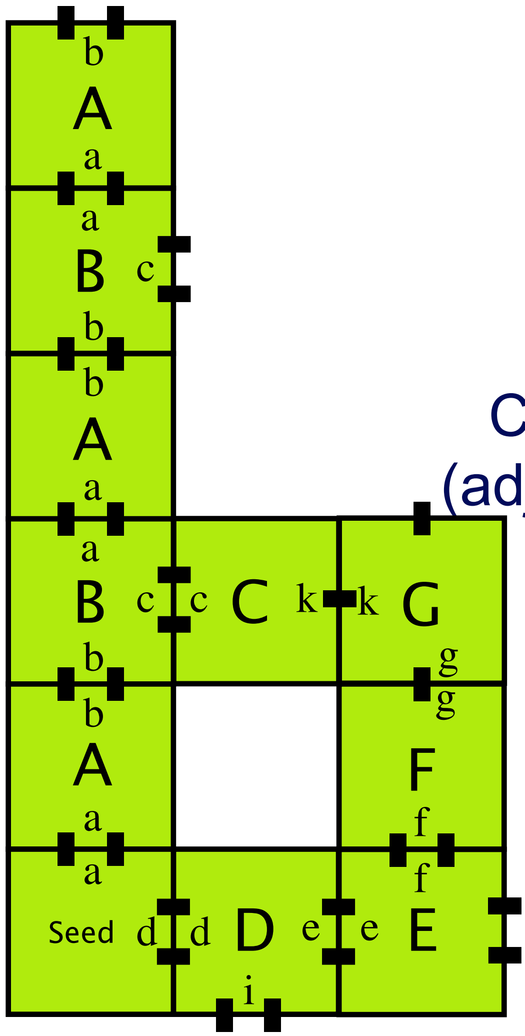

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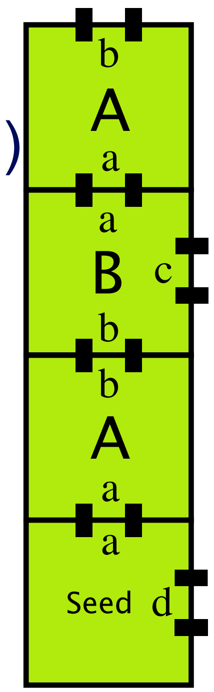

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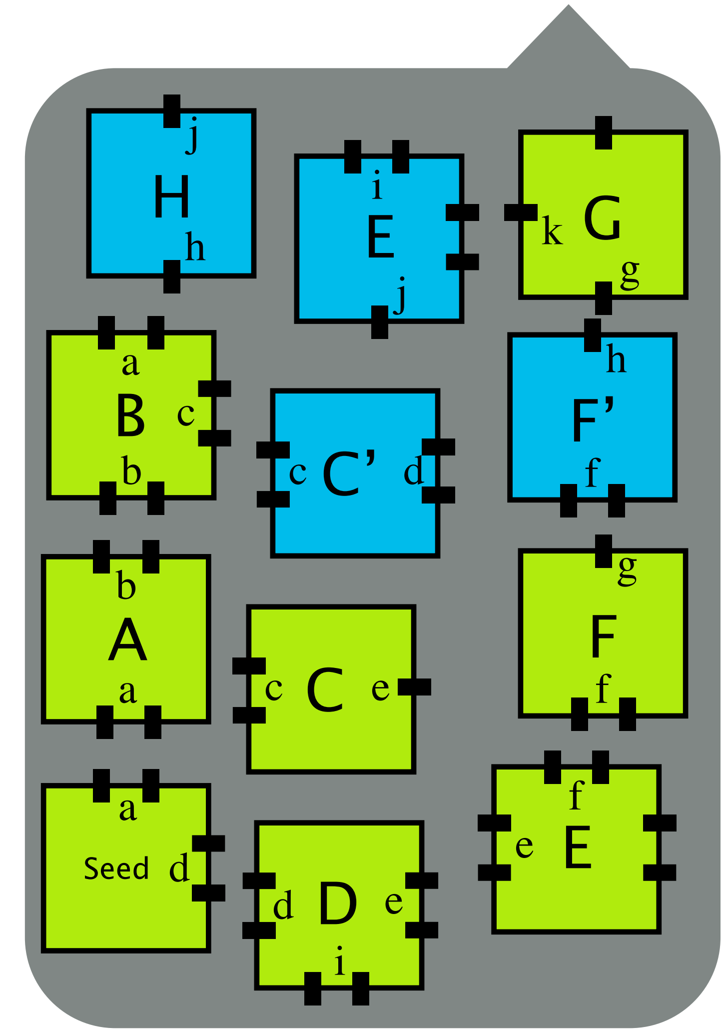

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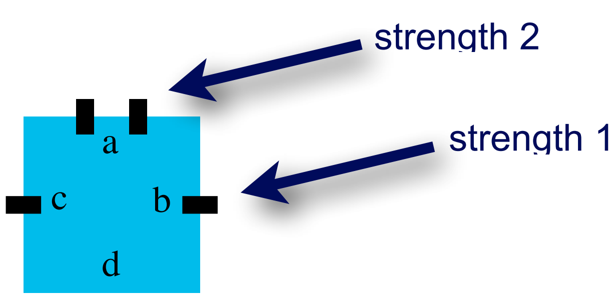

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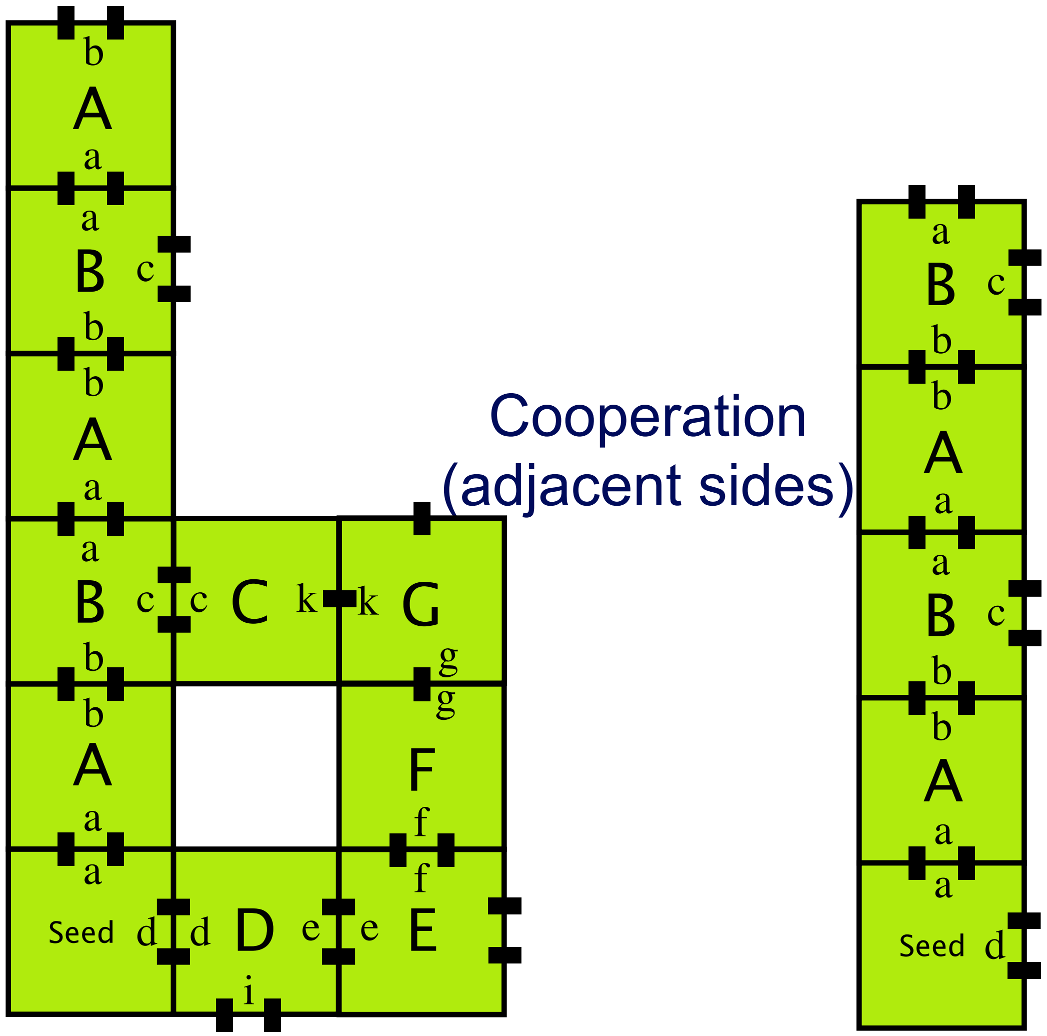

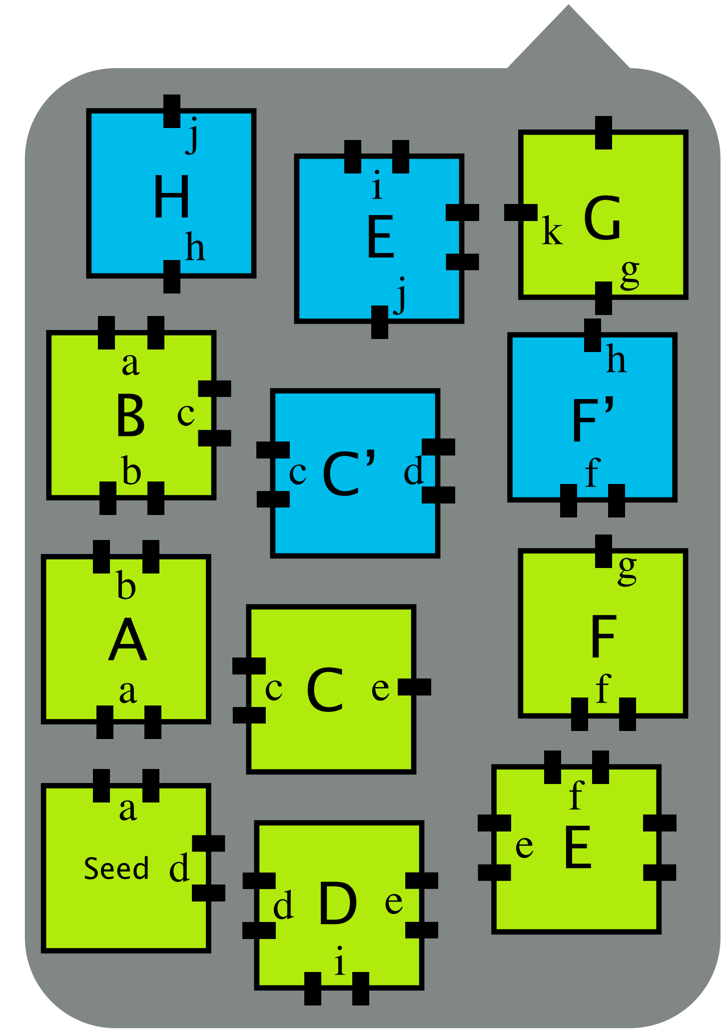

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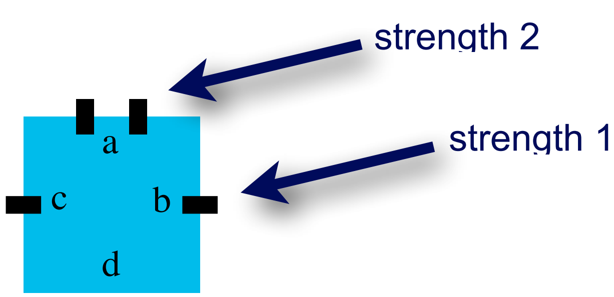

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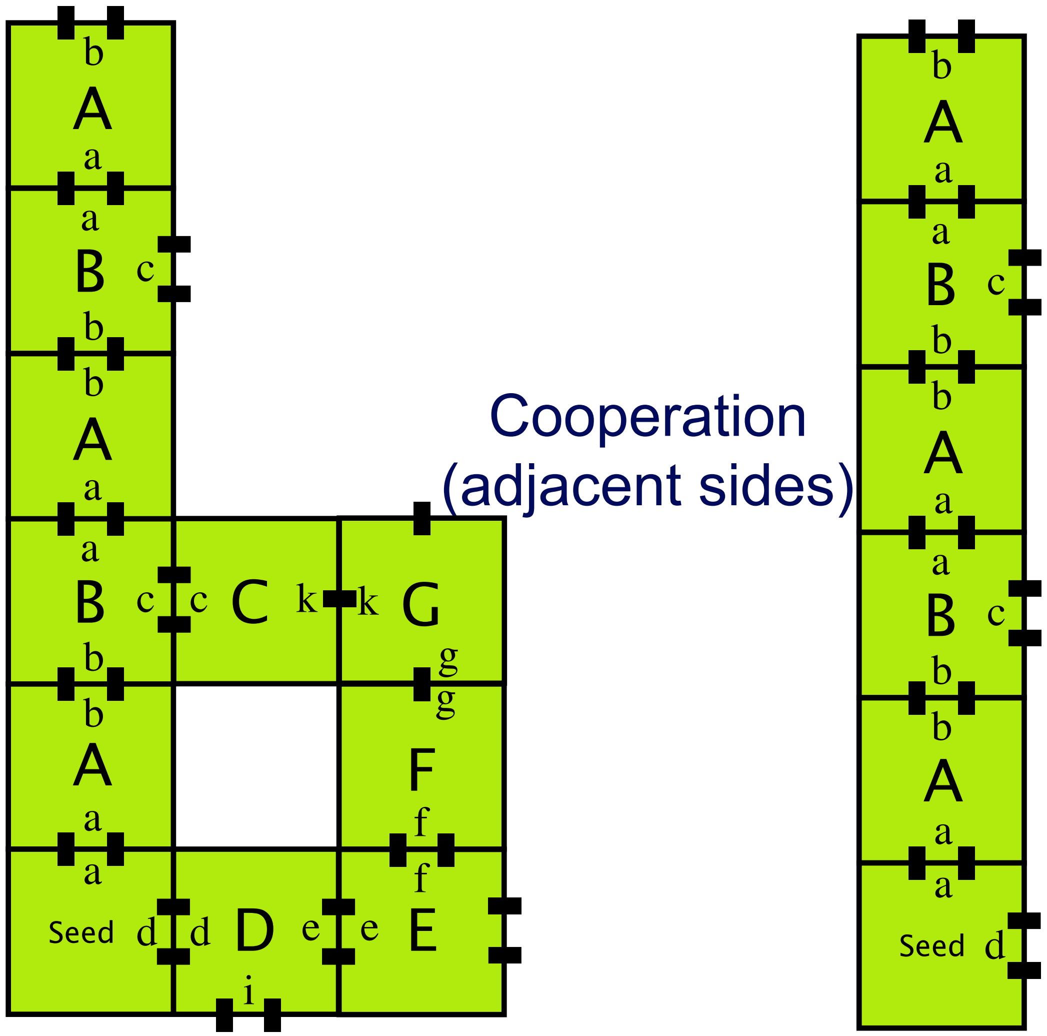

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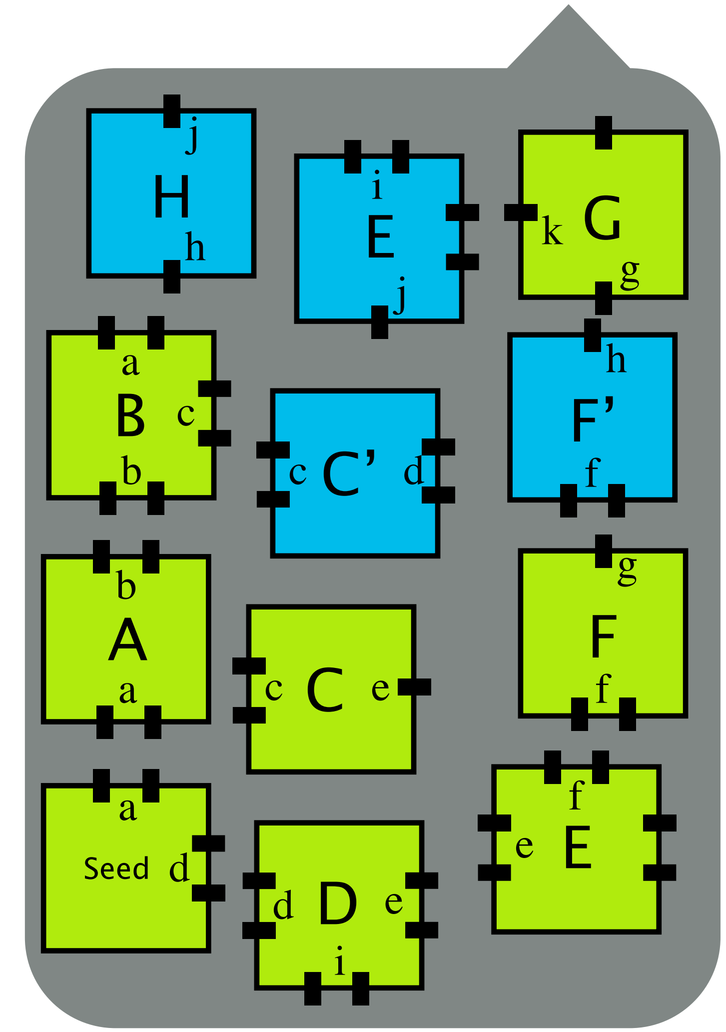

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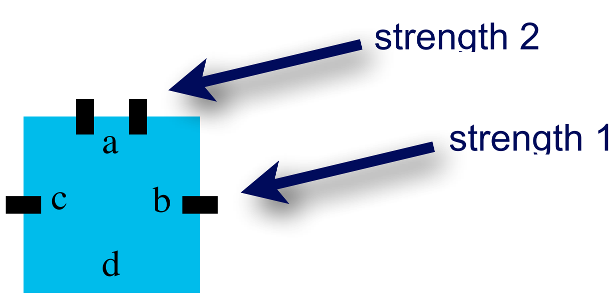

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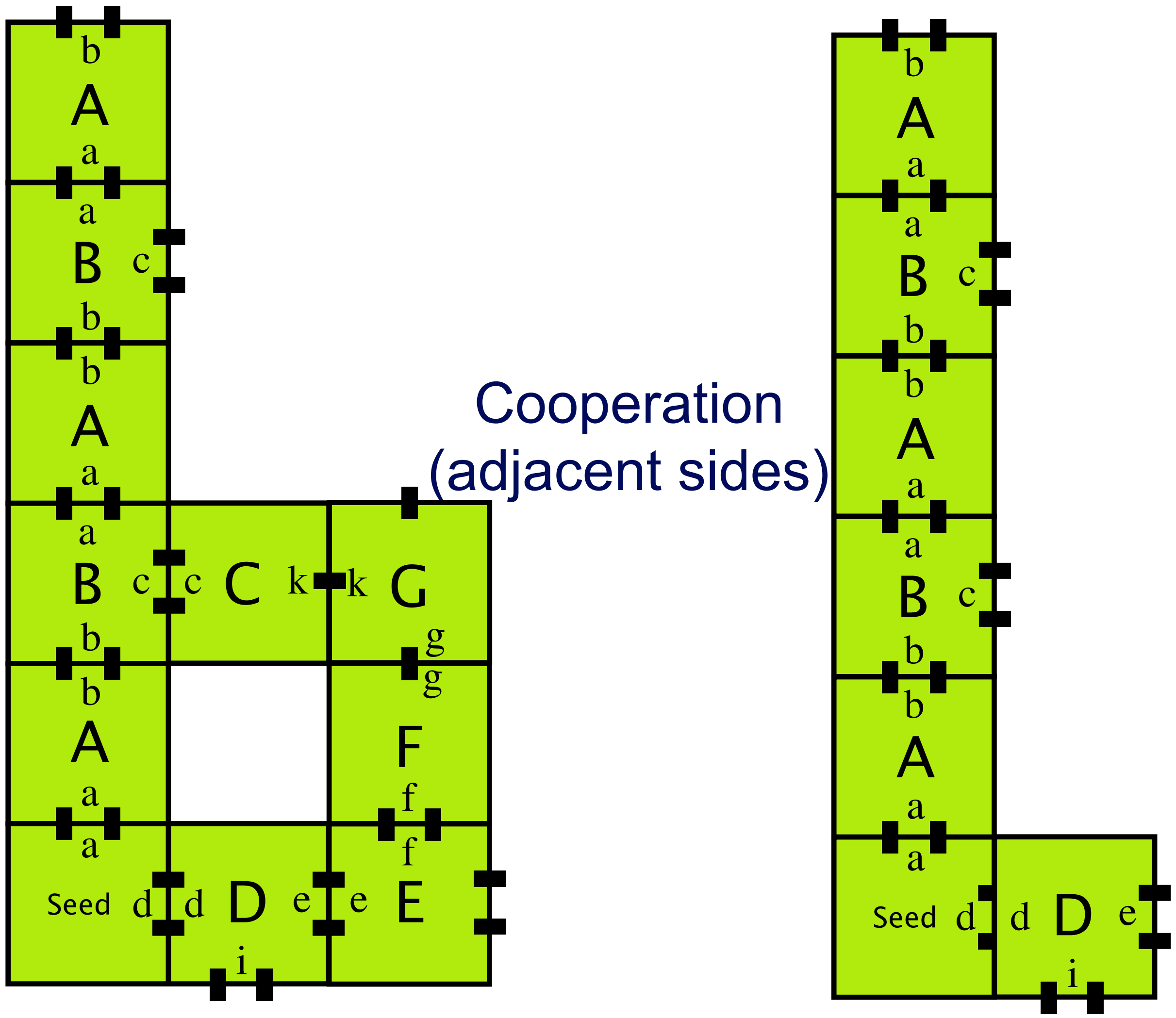

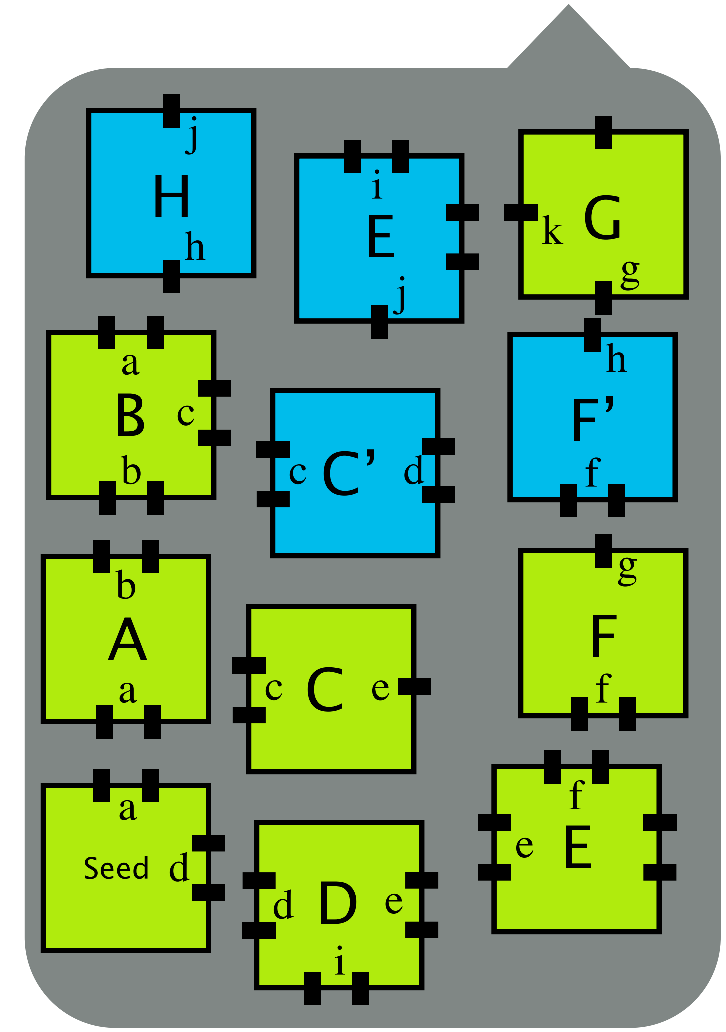

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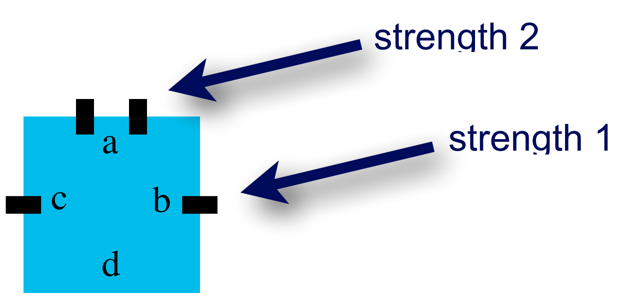

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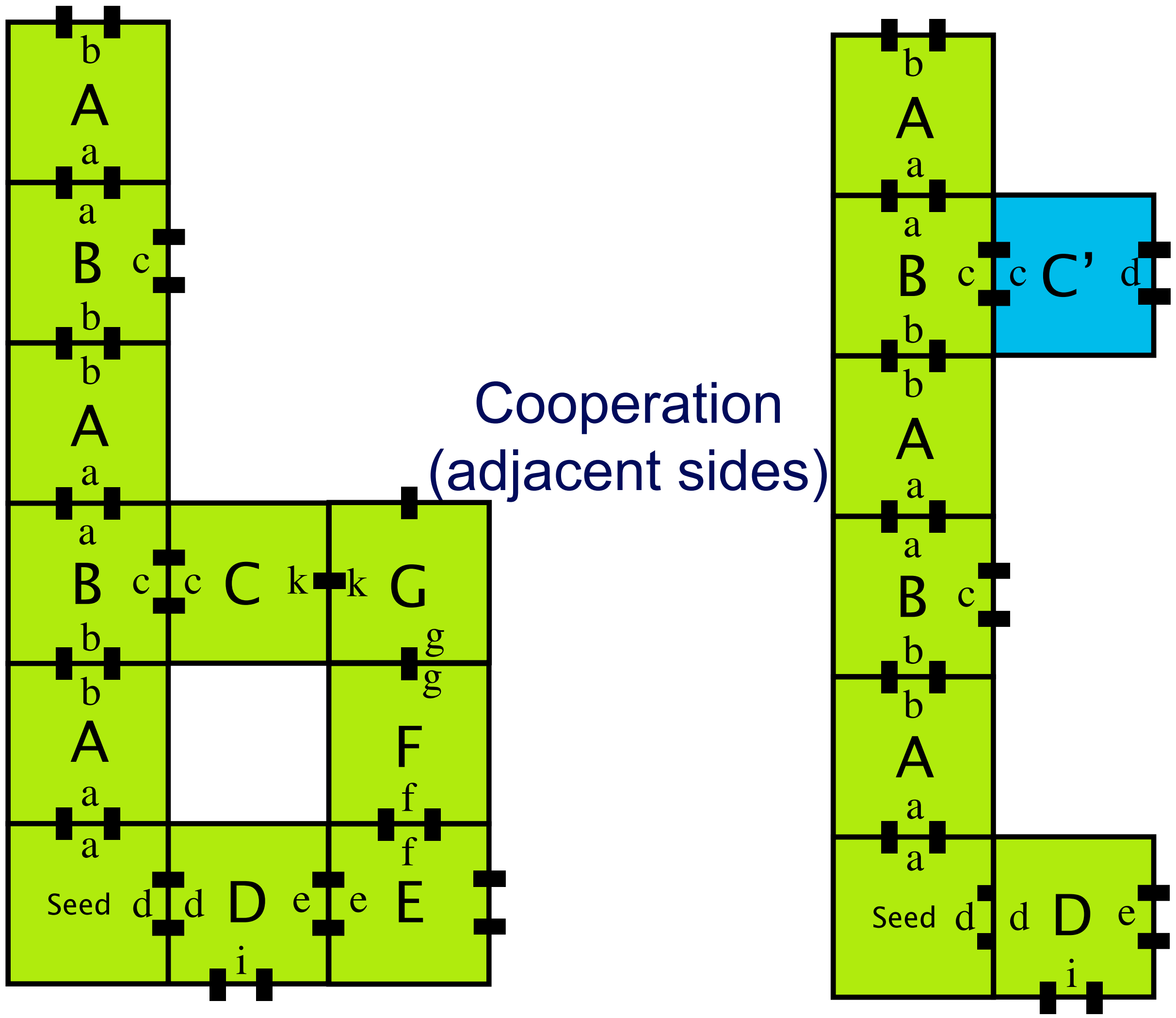

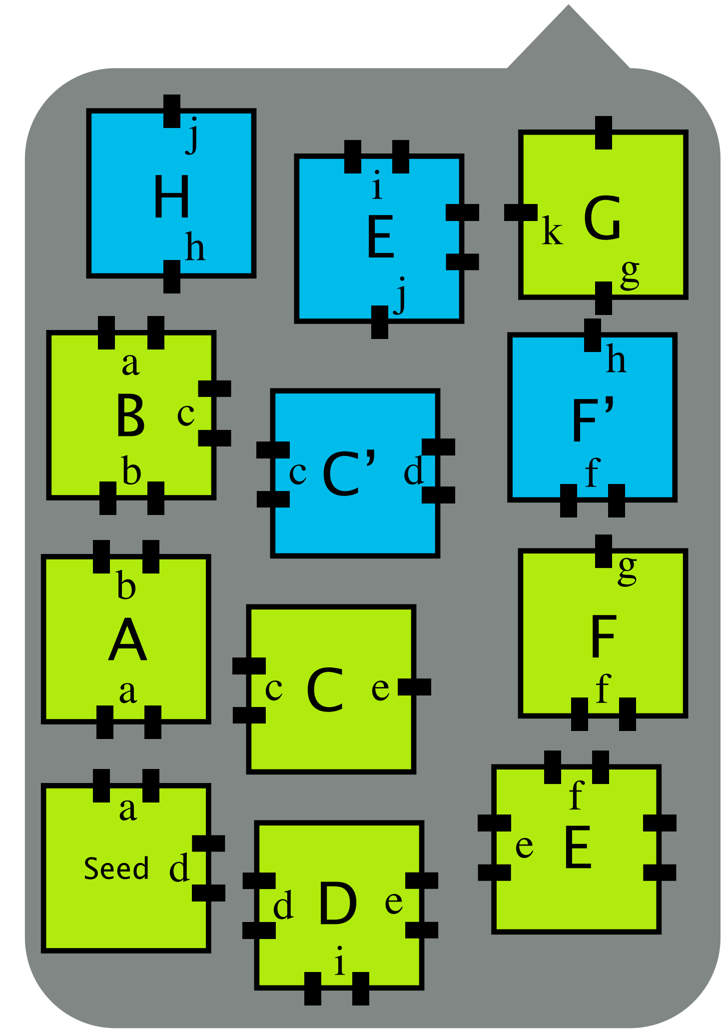

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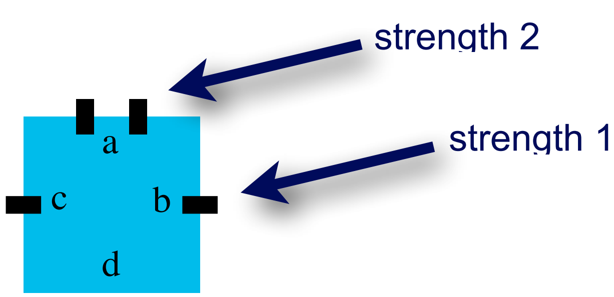

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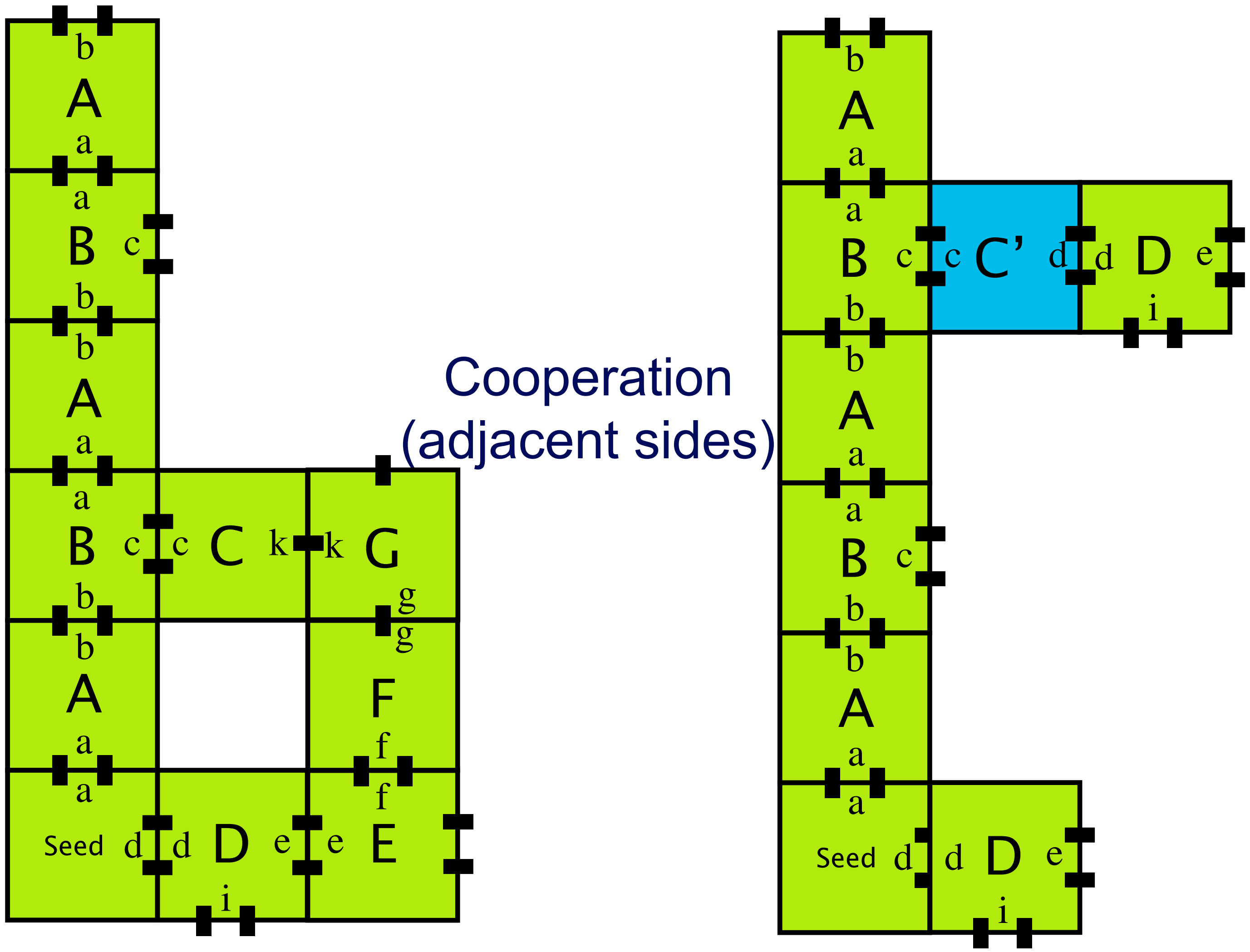

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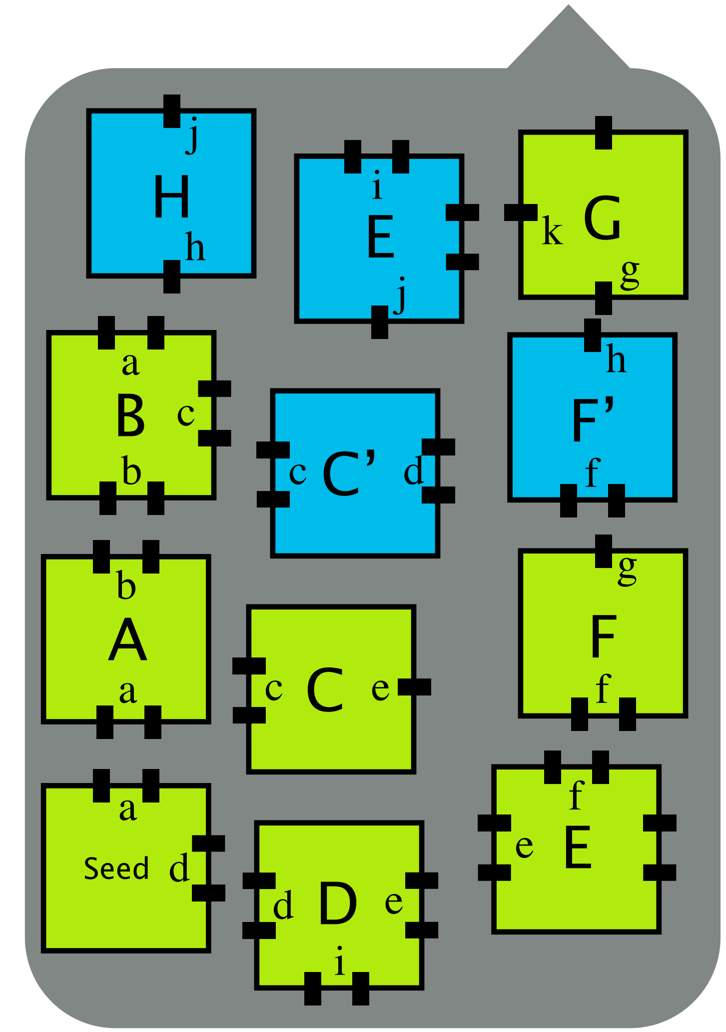

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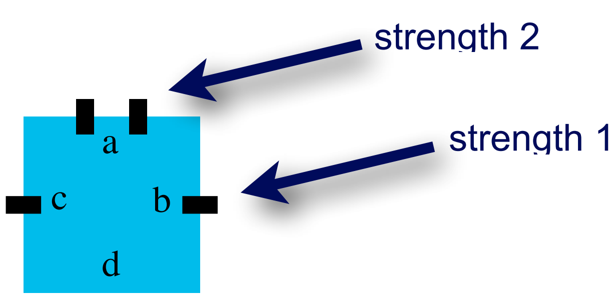

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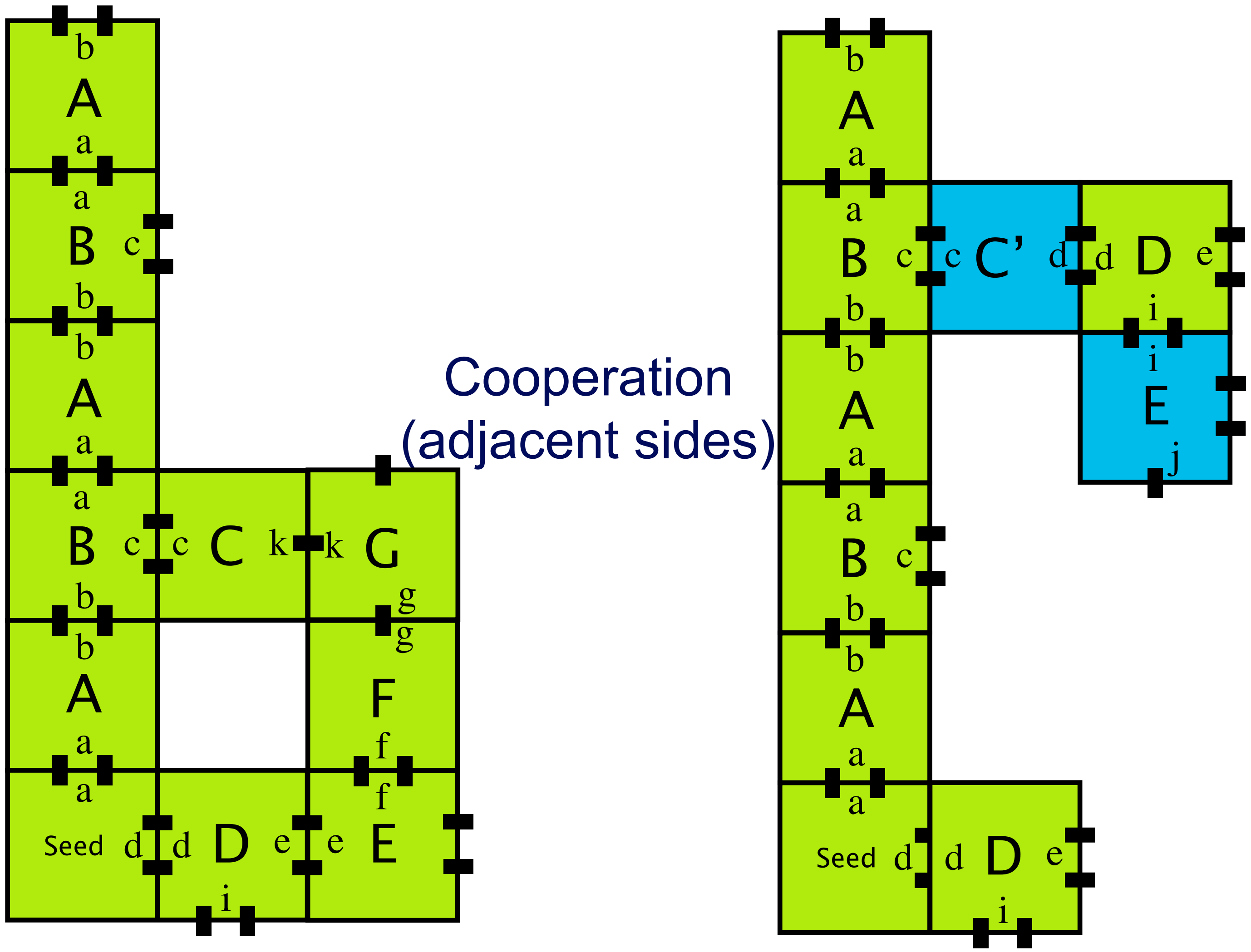

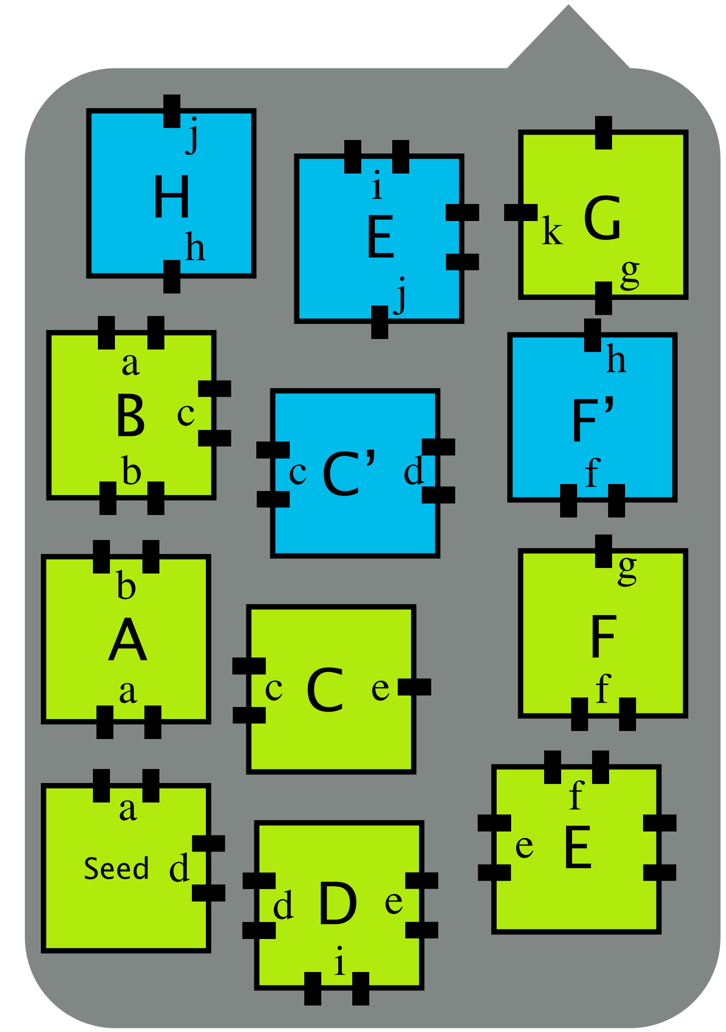

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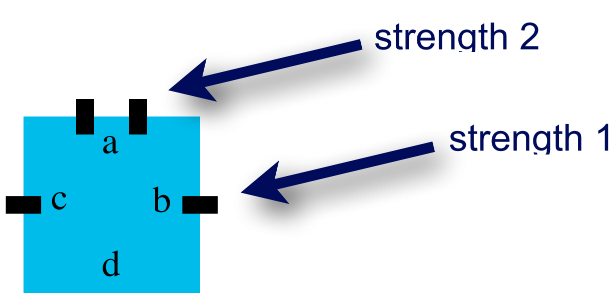

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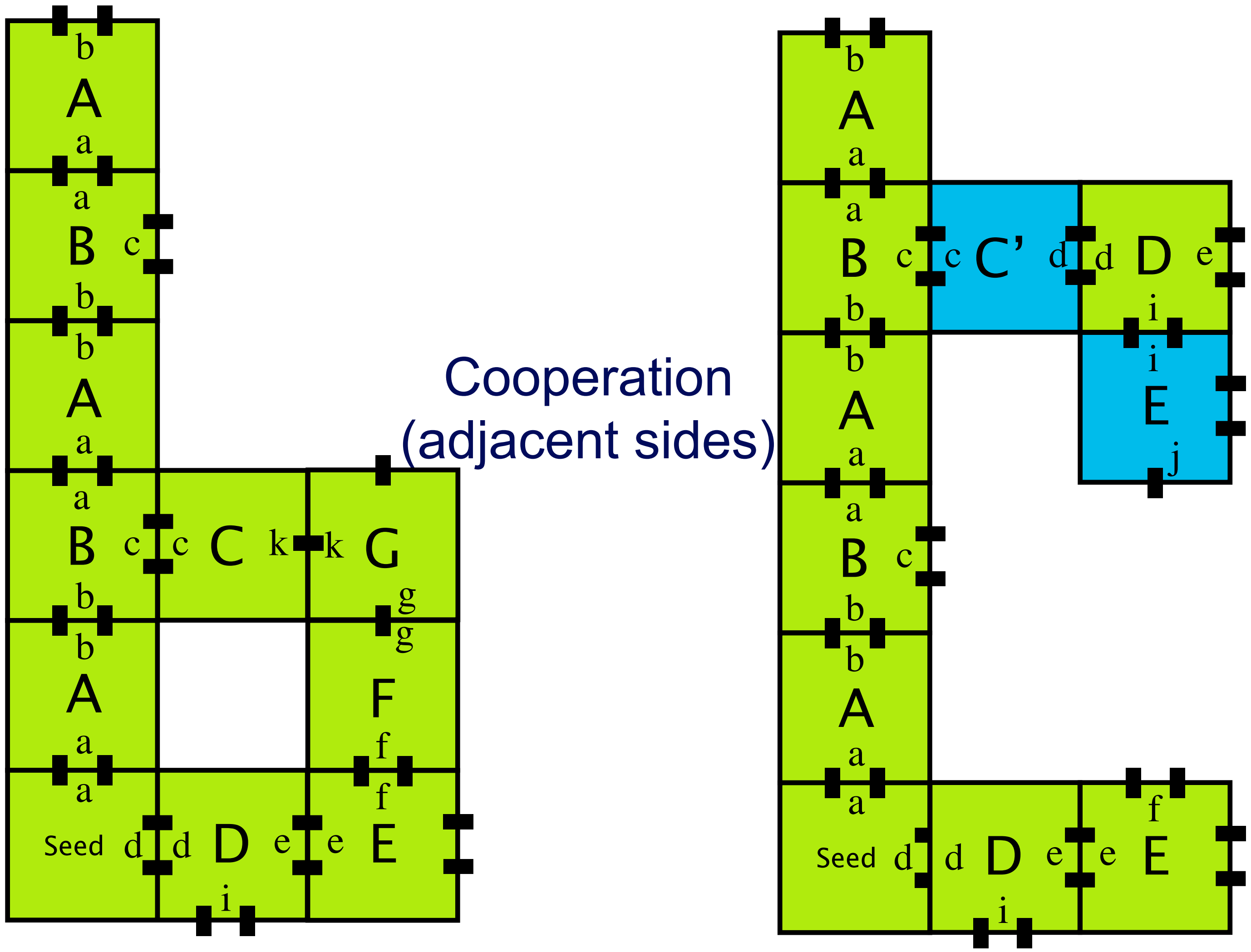

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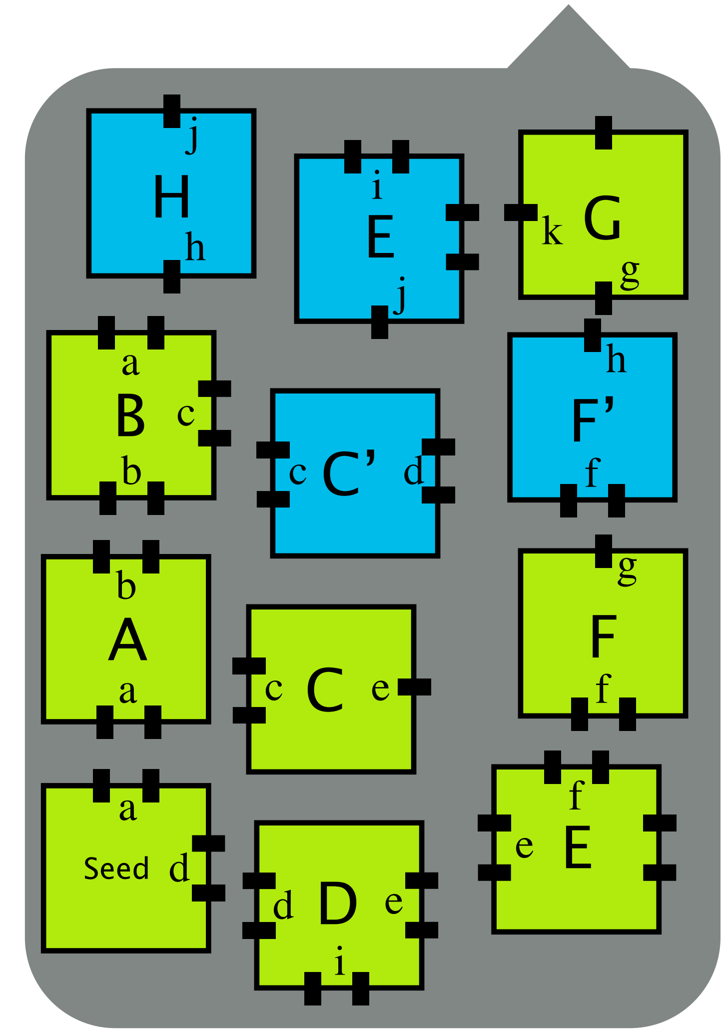

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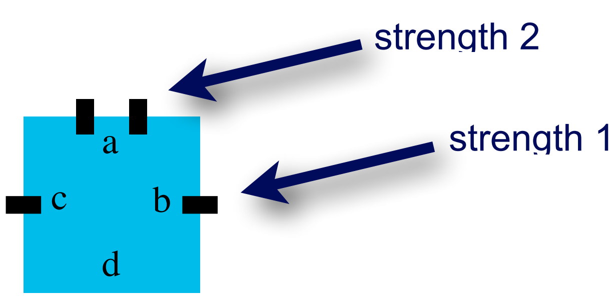

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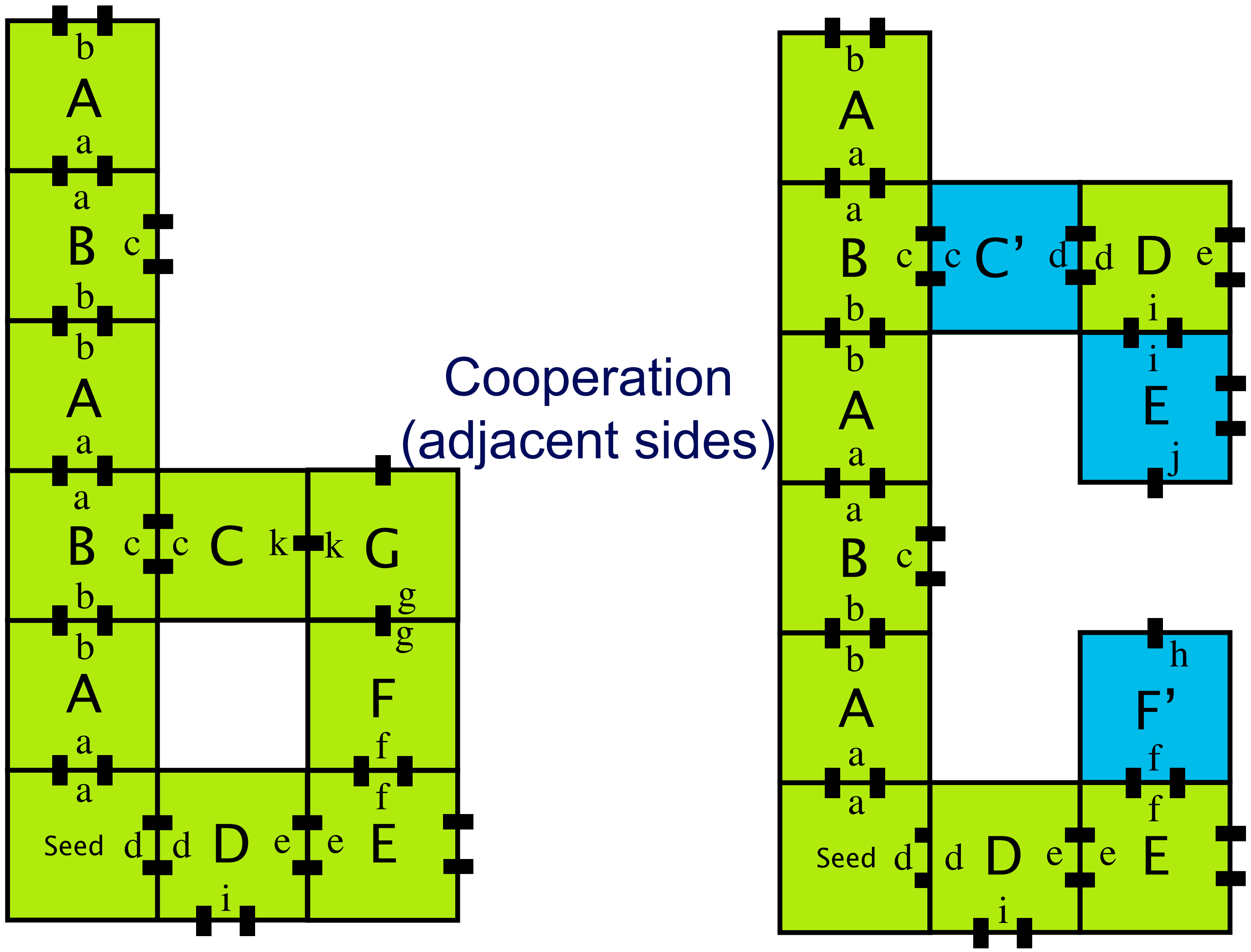

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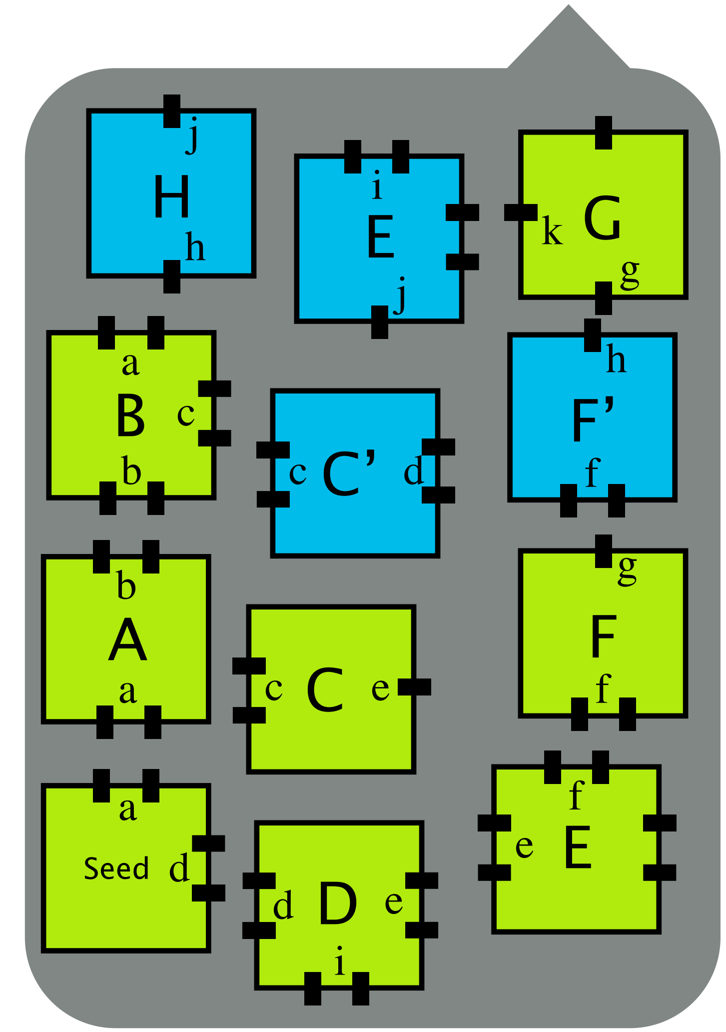

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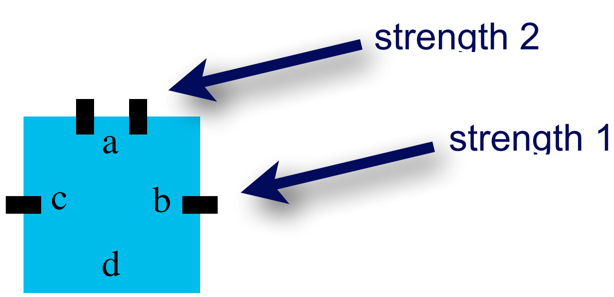

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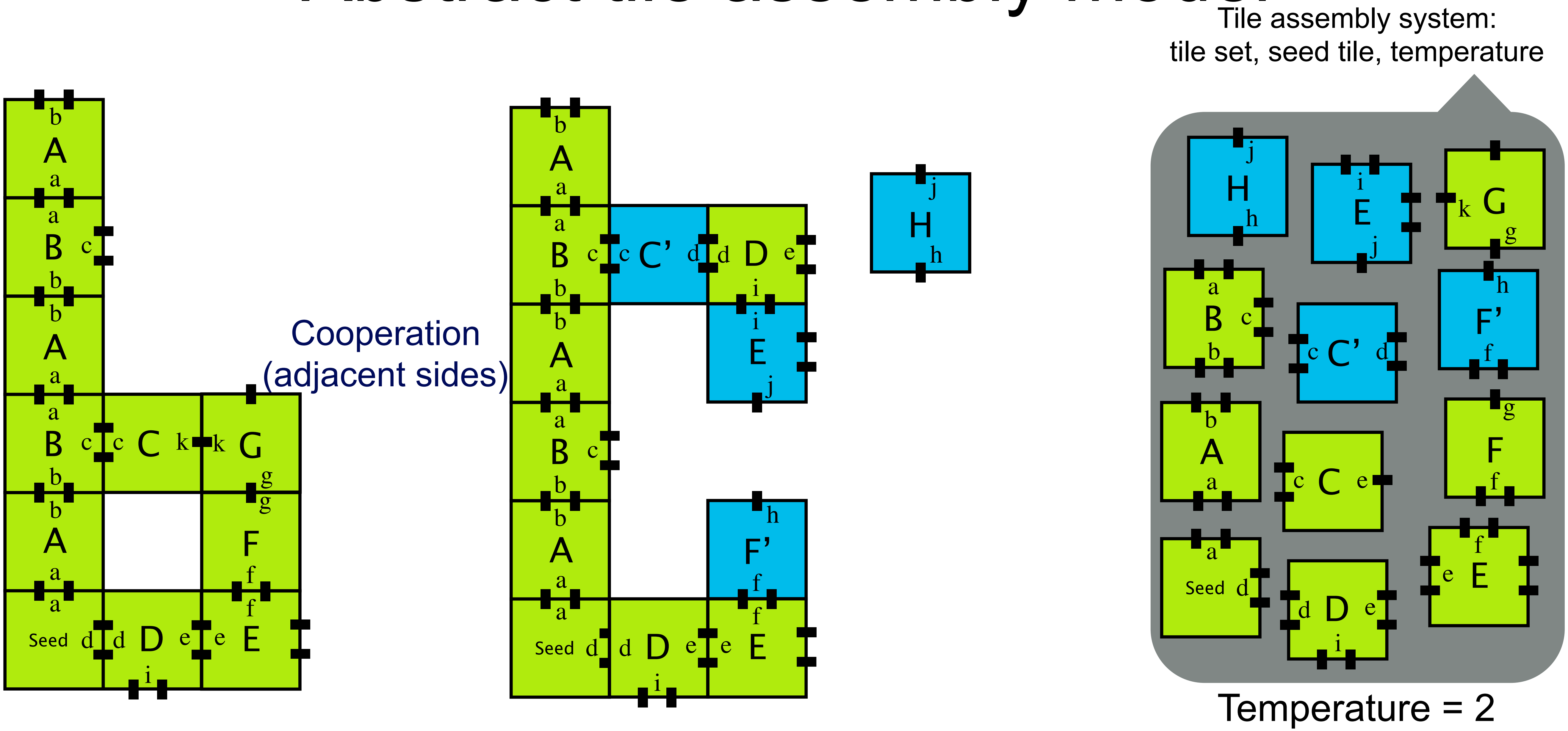
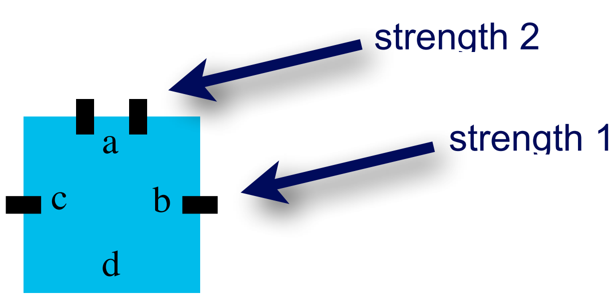
Abstract tile assembly model





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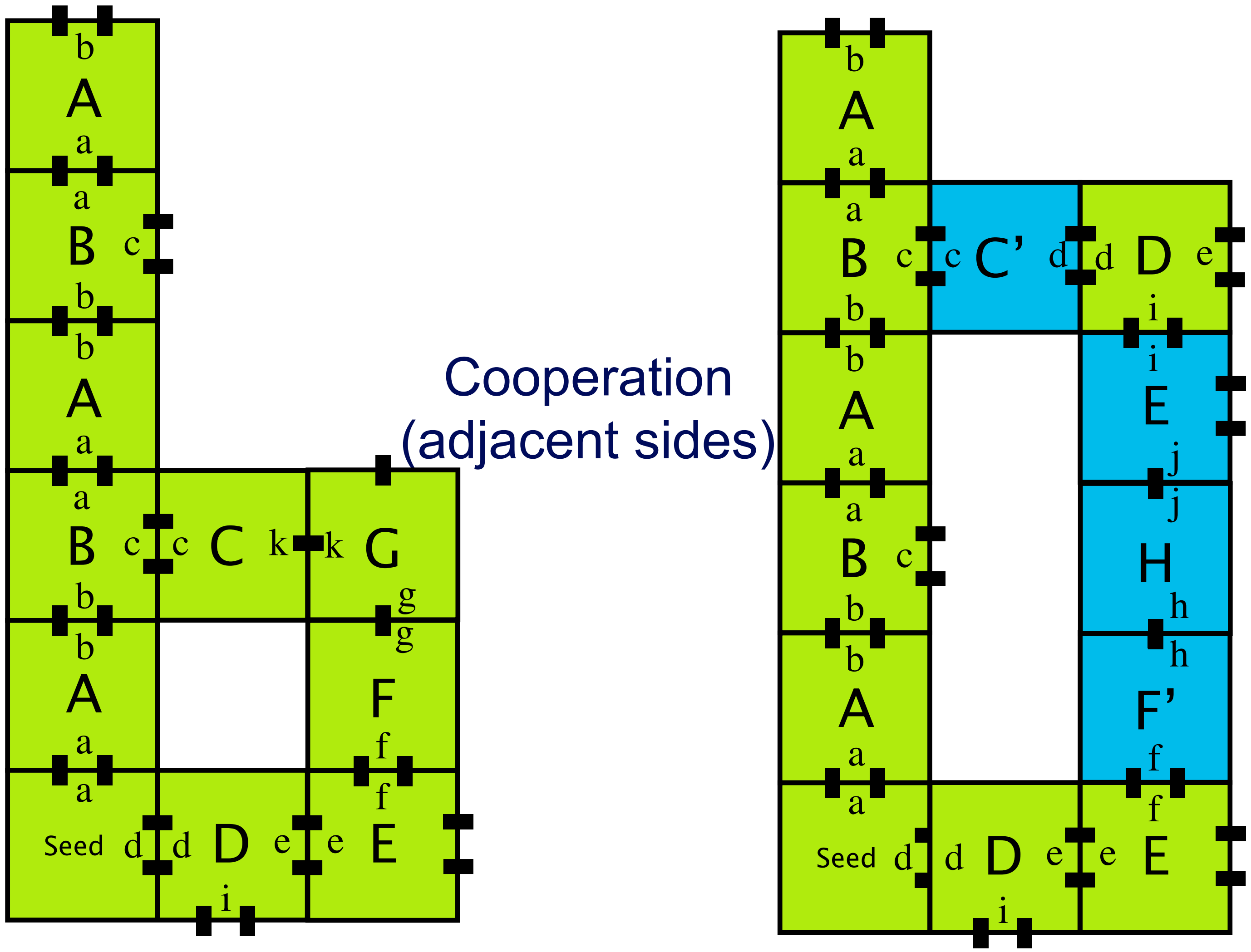
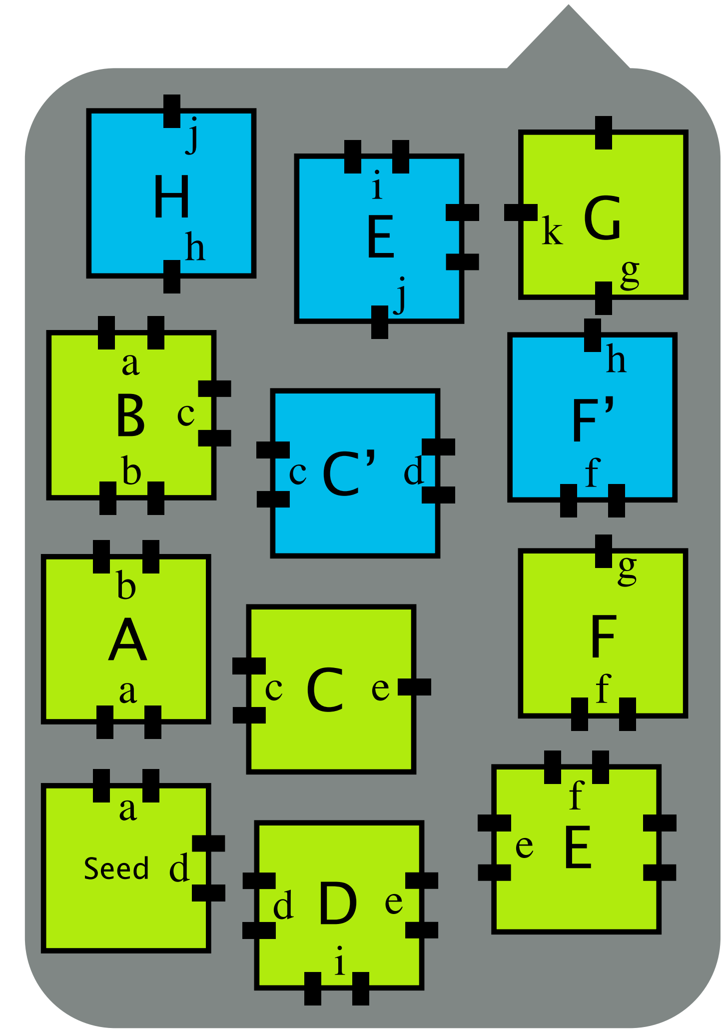
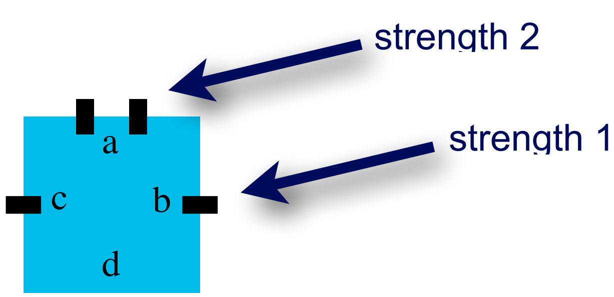


Tile assembly system:
tile set, seed tile, temperature



Temperature = 2



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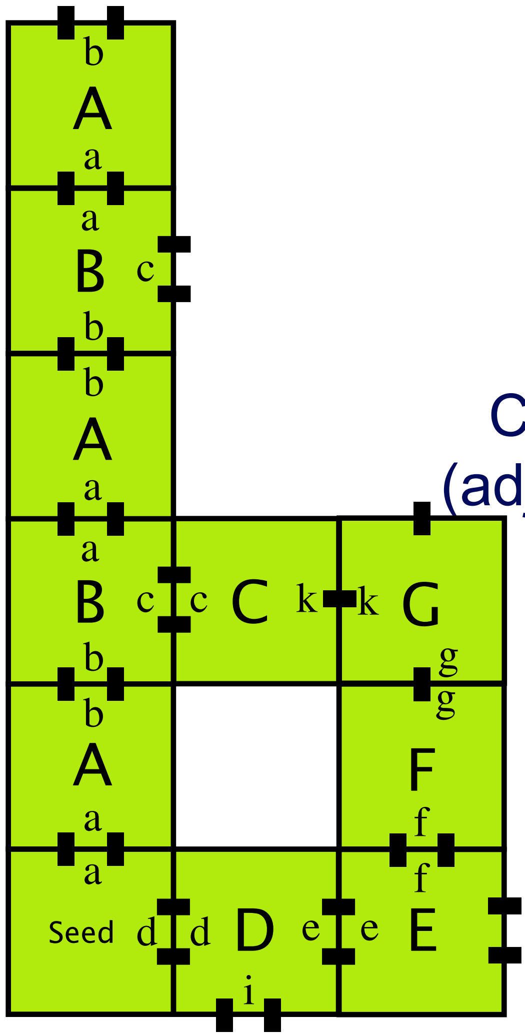
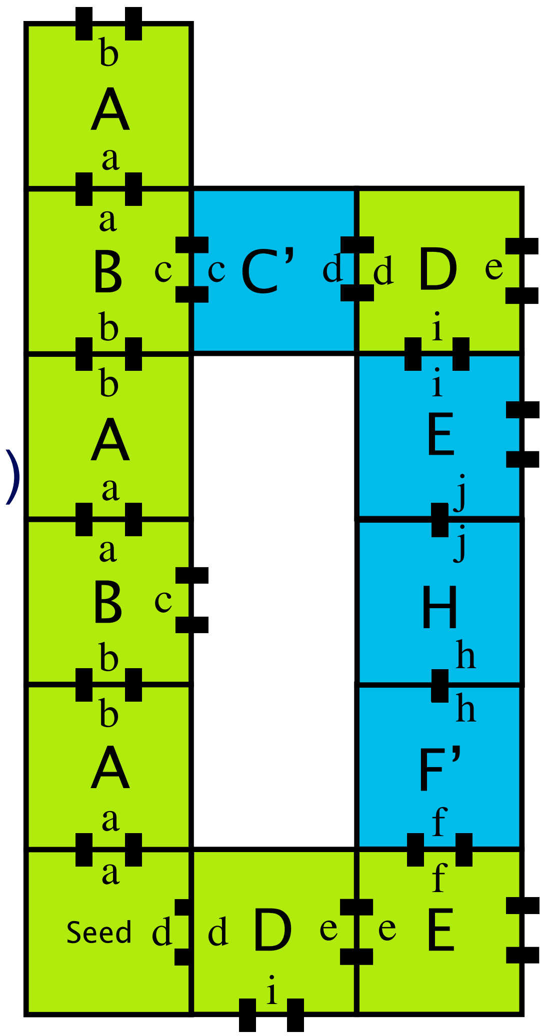


Cooperation (adjacent sides)



Cooperation (opposite sides)

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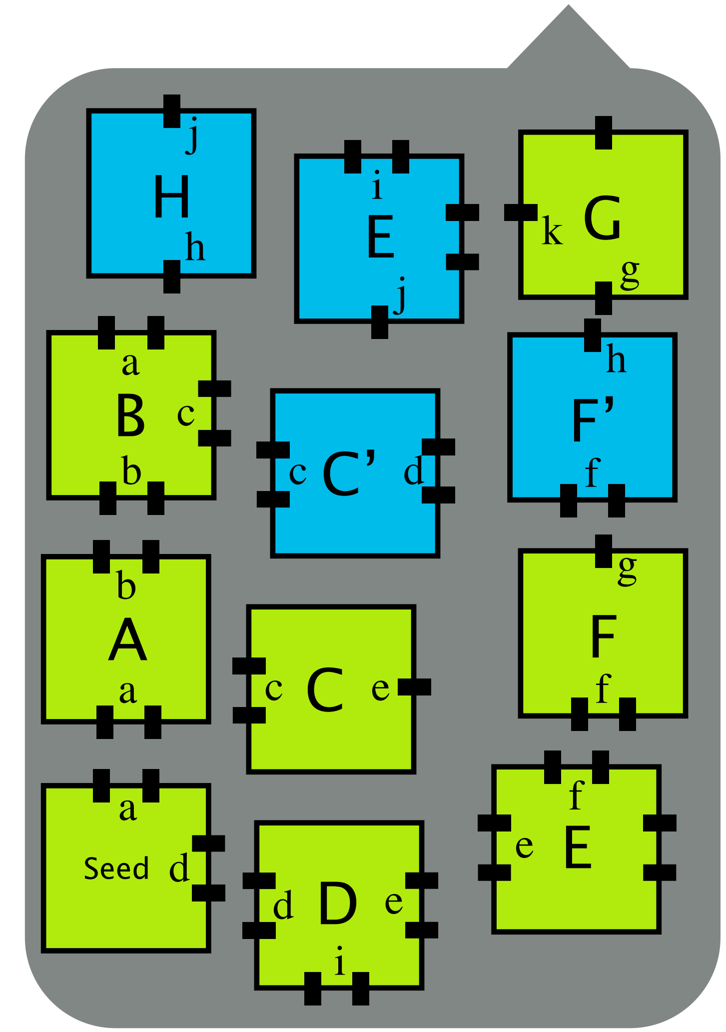

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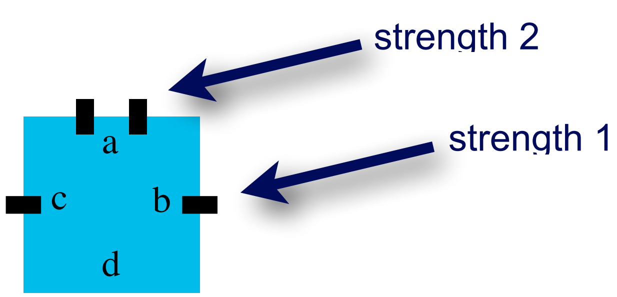

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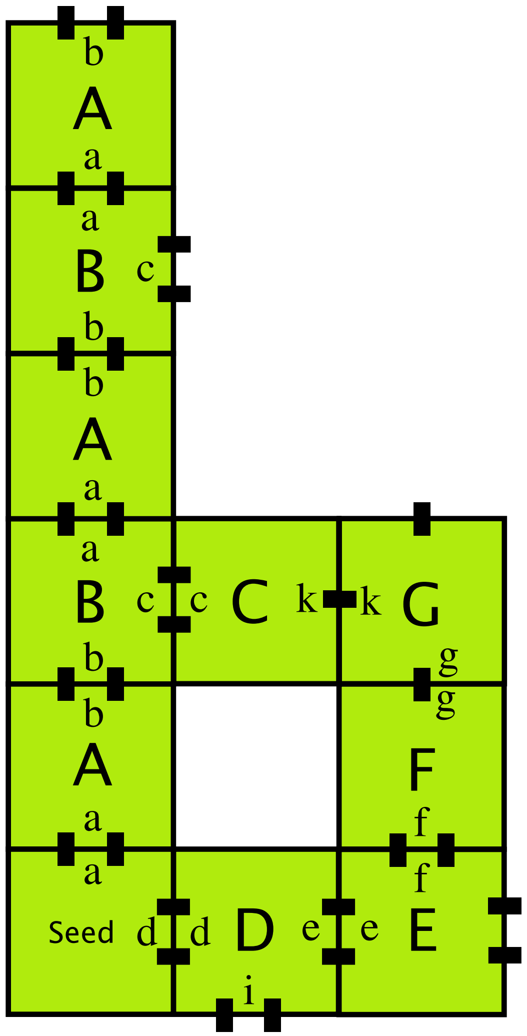

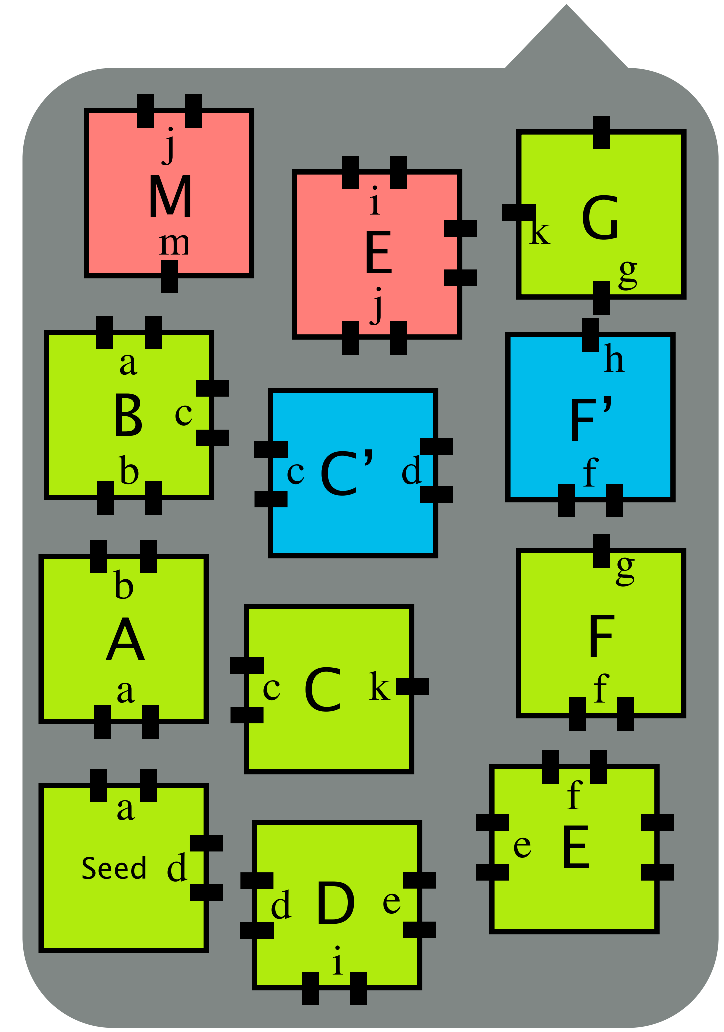

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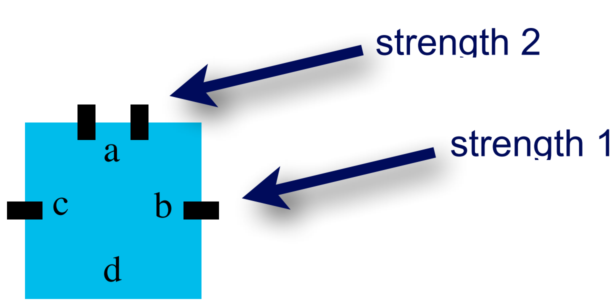

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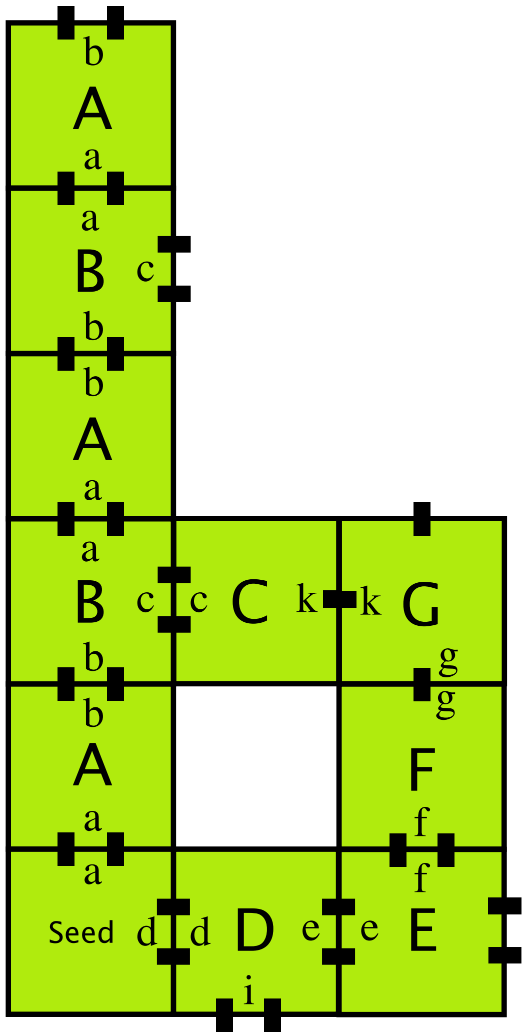

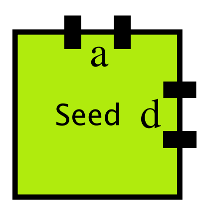

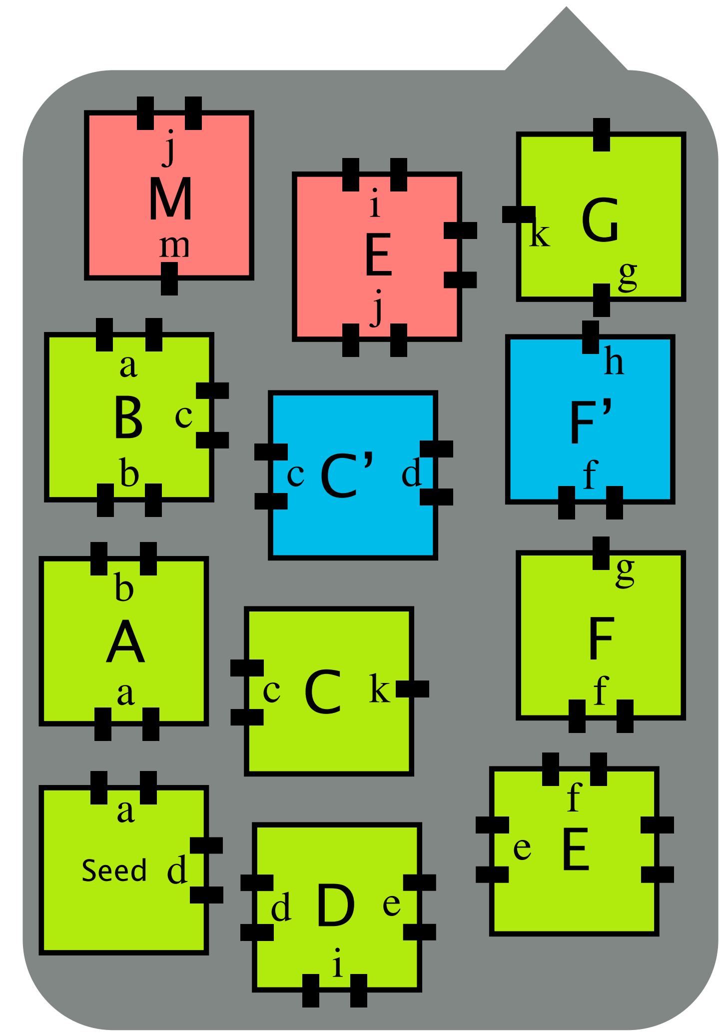

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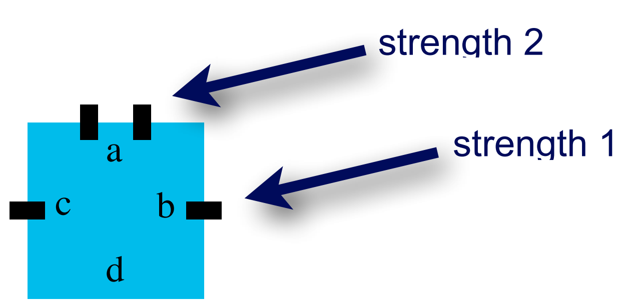

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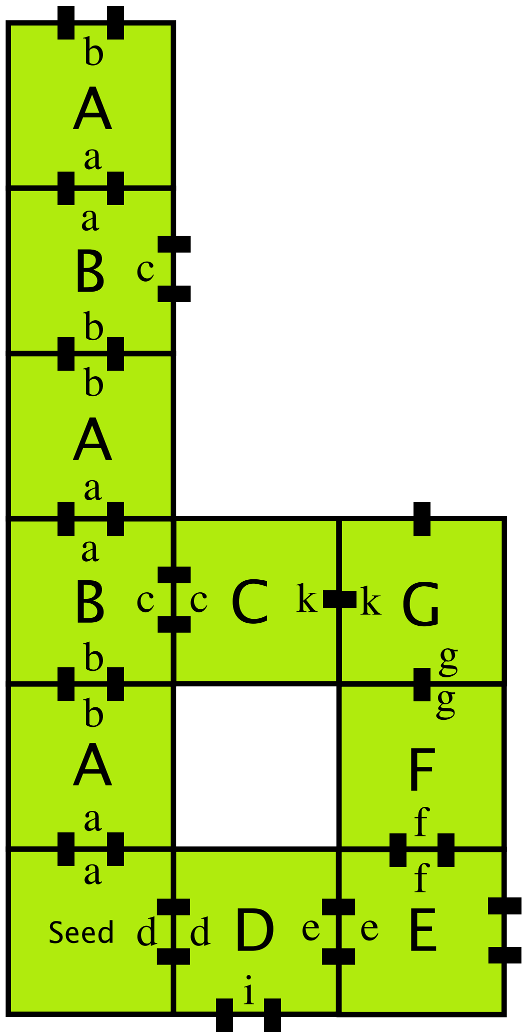

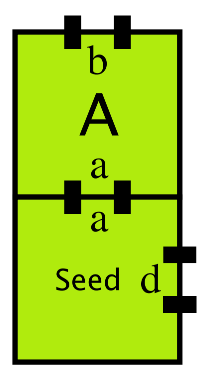

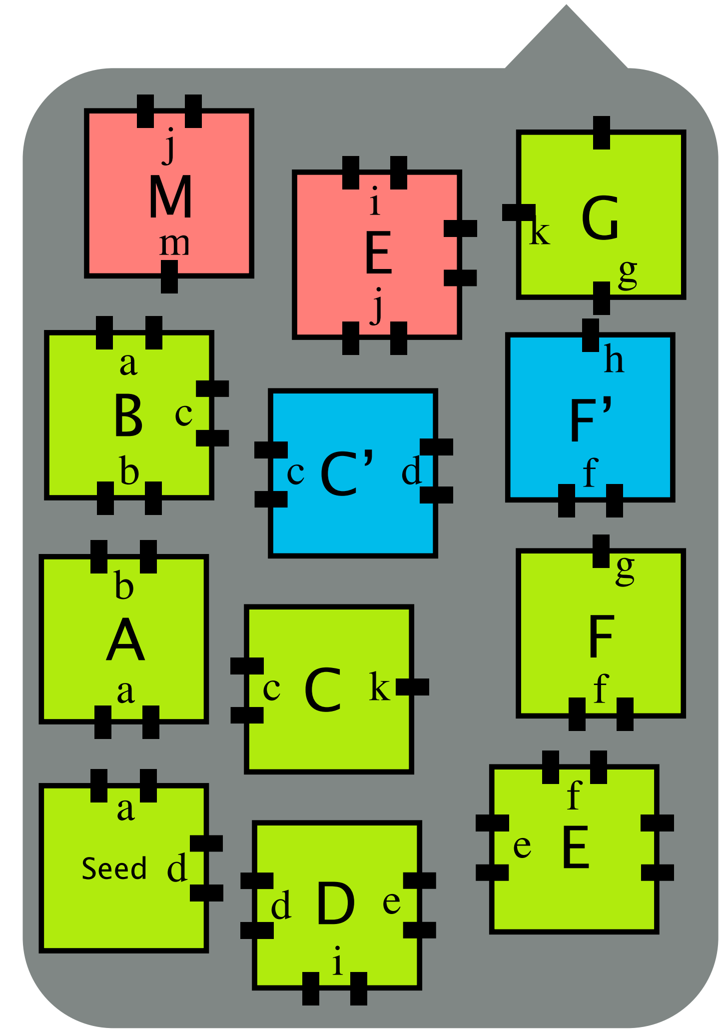

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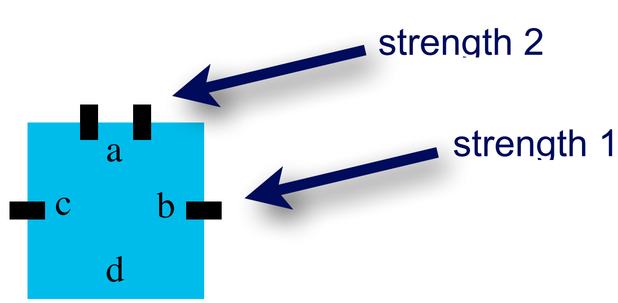

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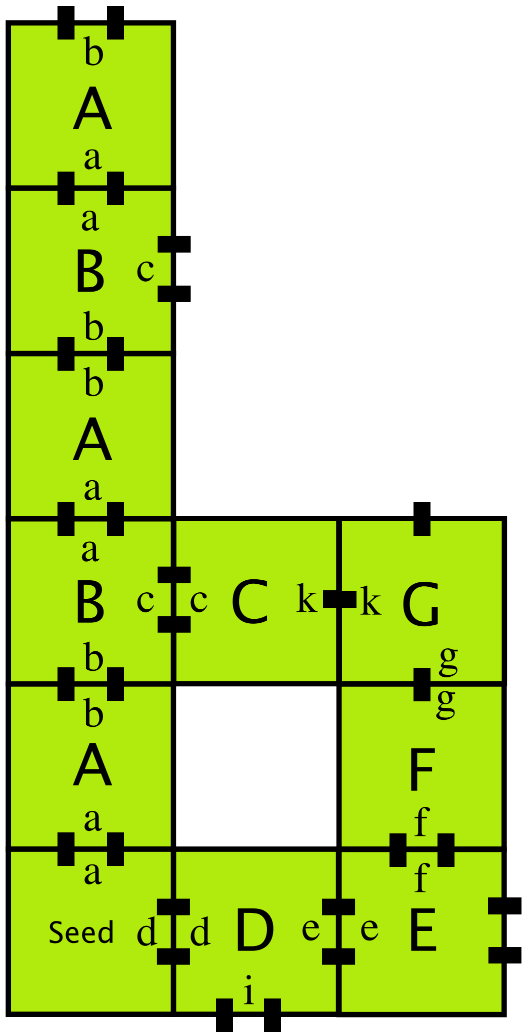

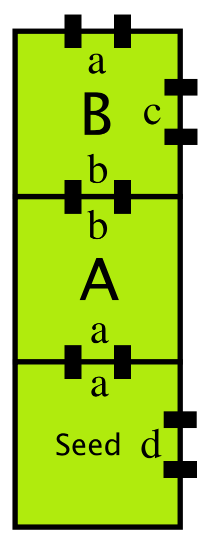

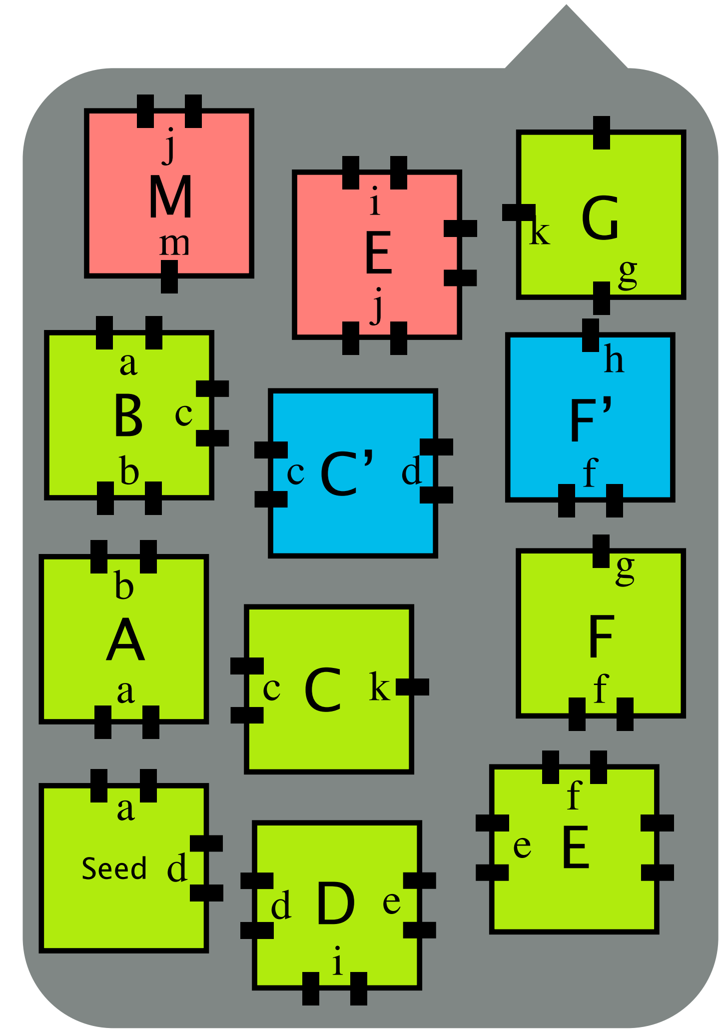

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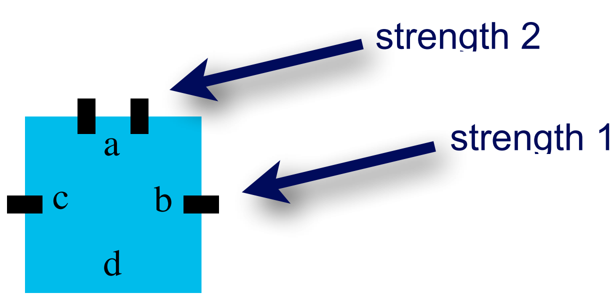

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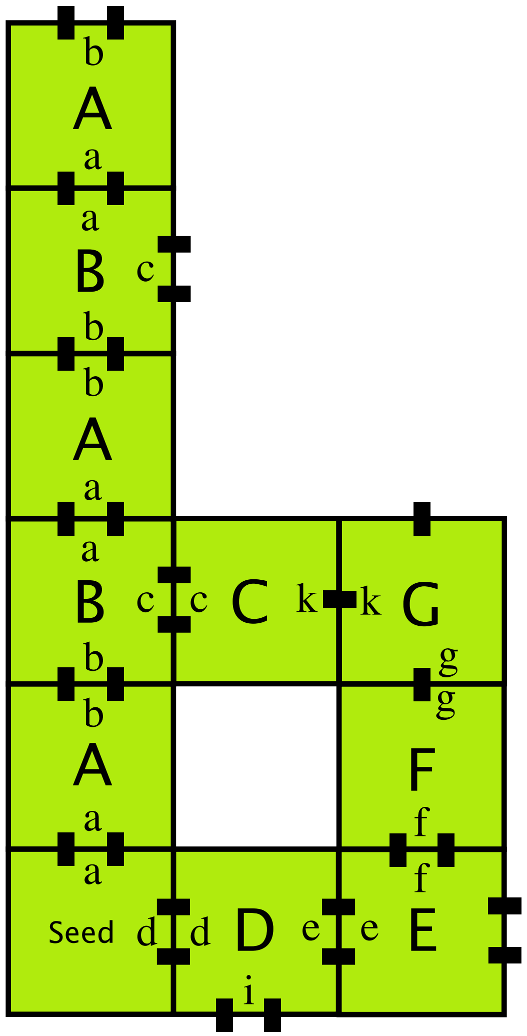

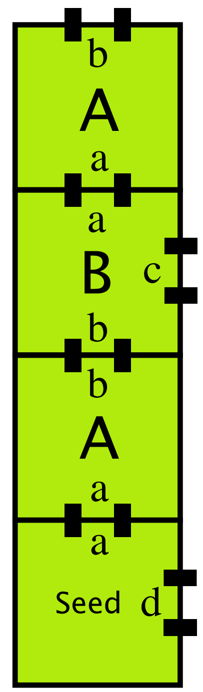

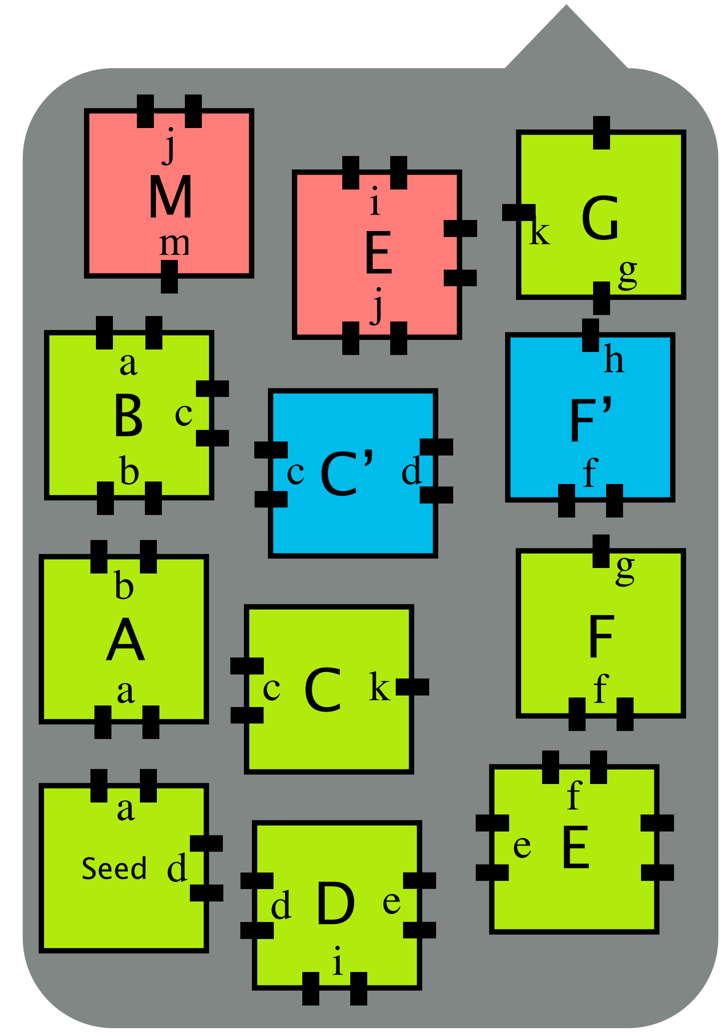

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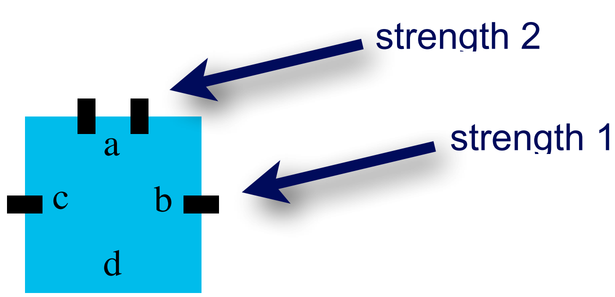

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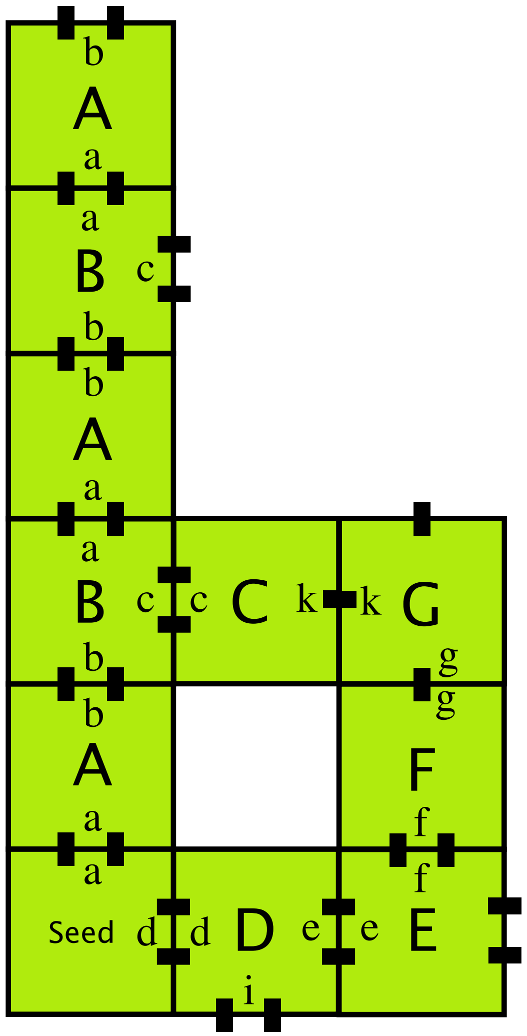

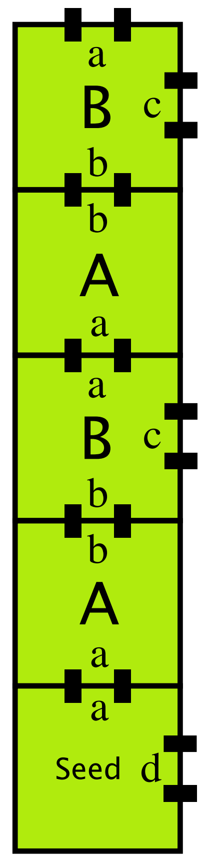

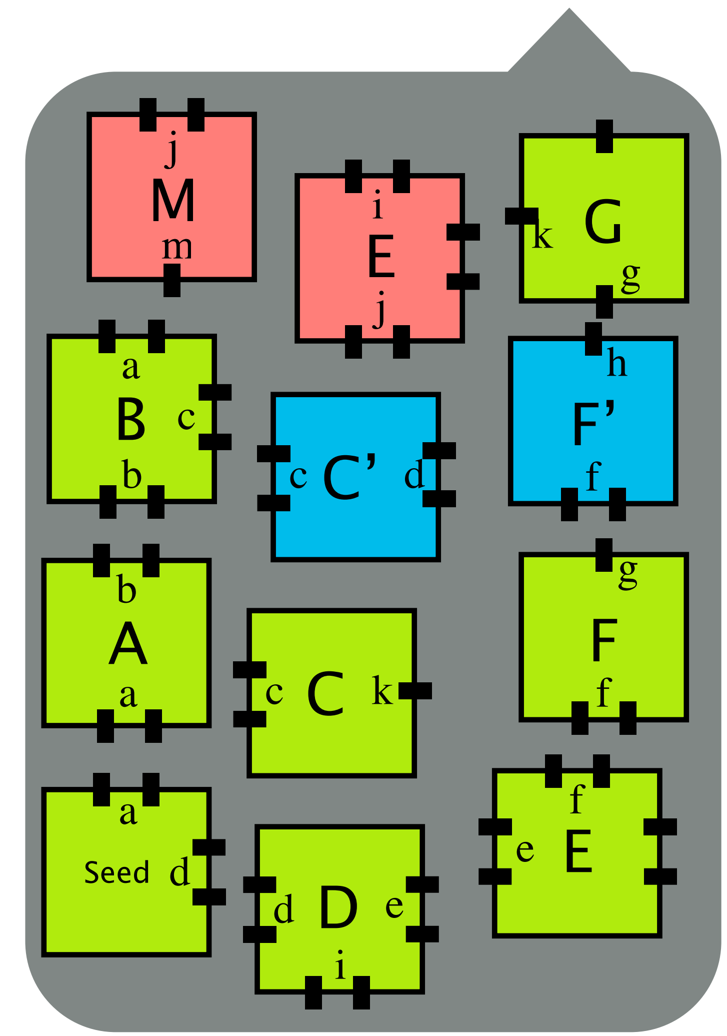

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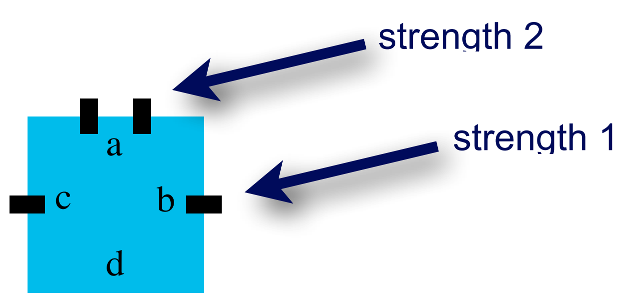

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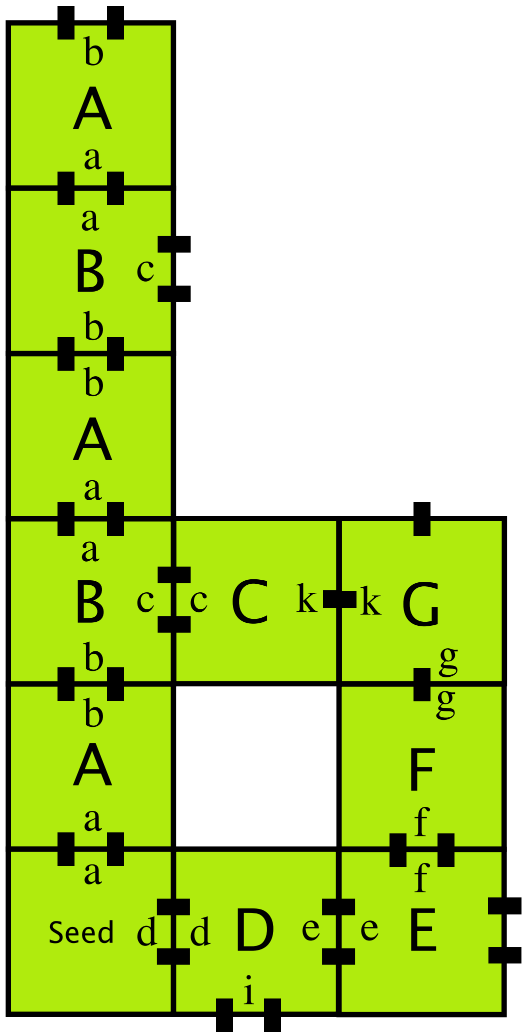

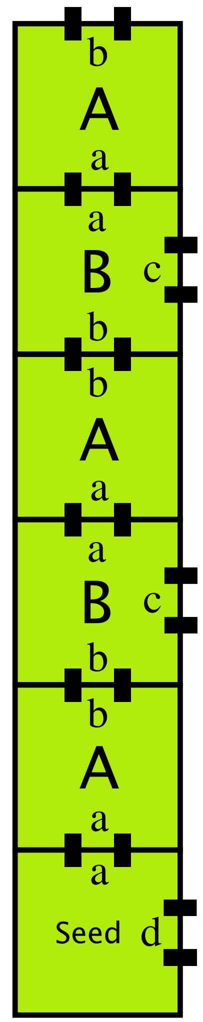

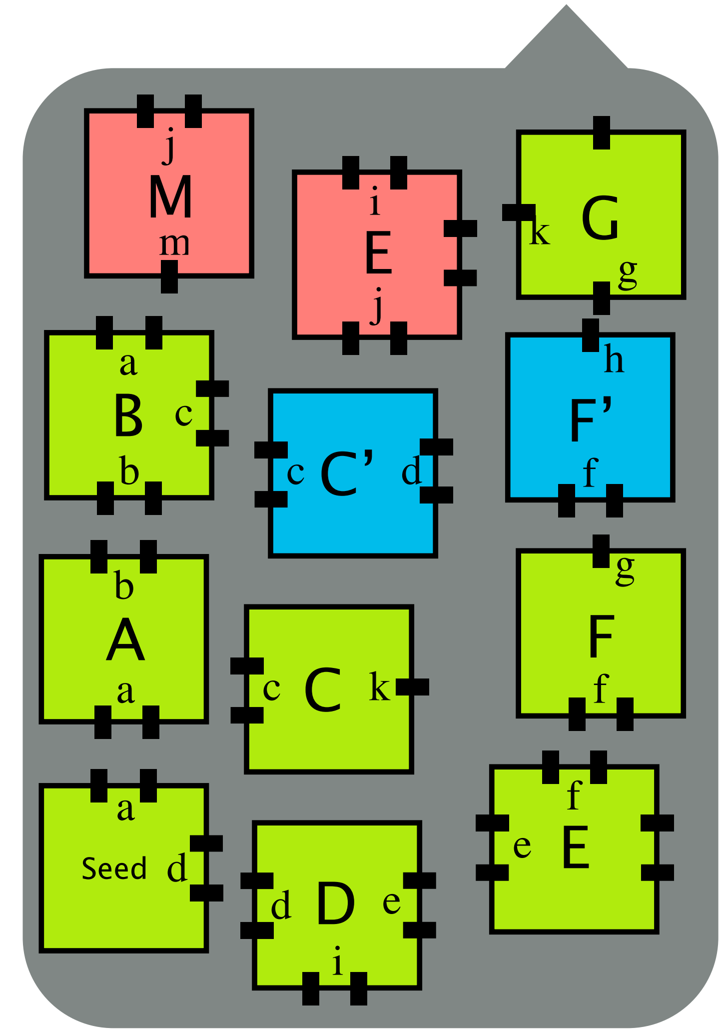

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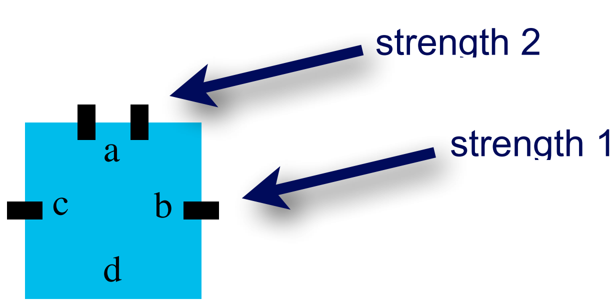

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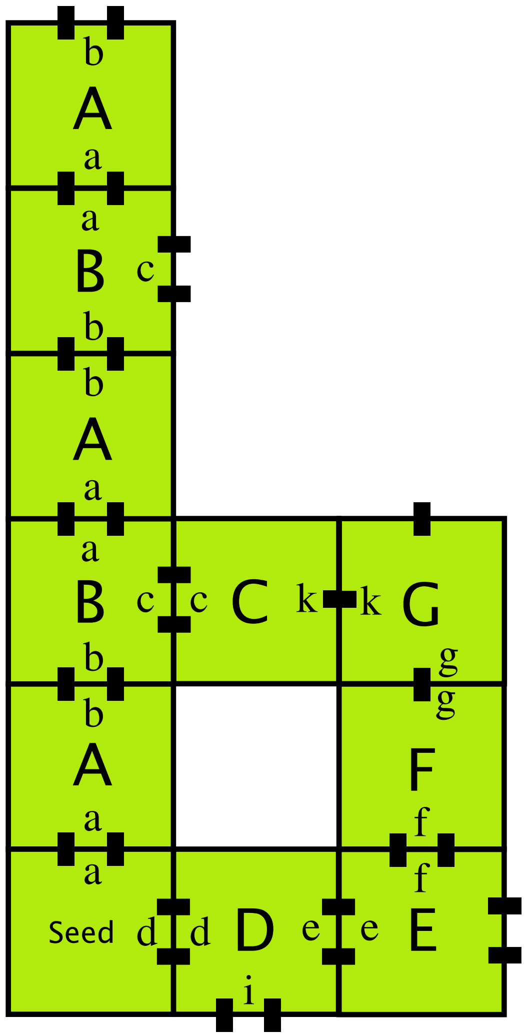

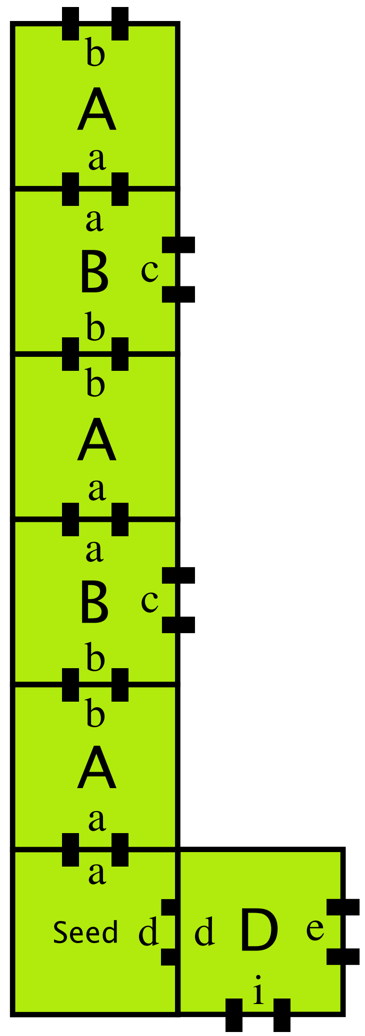

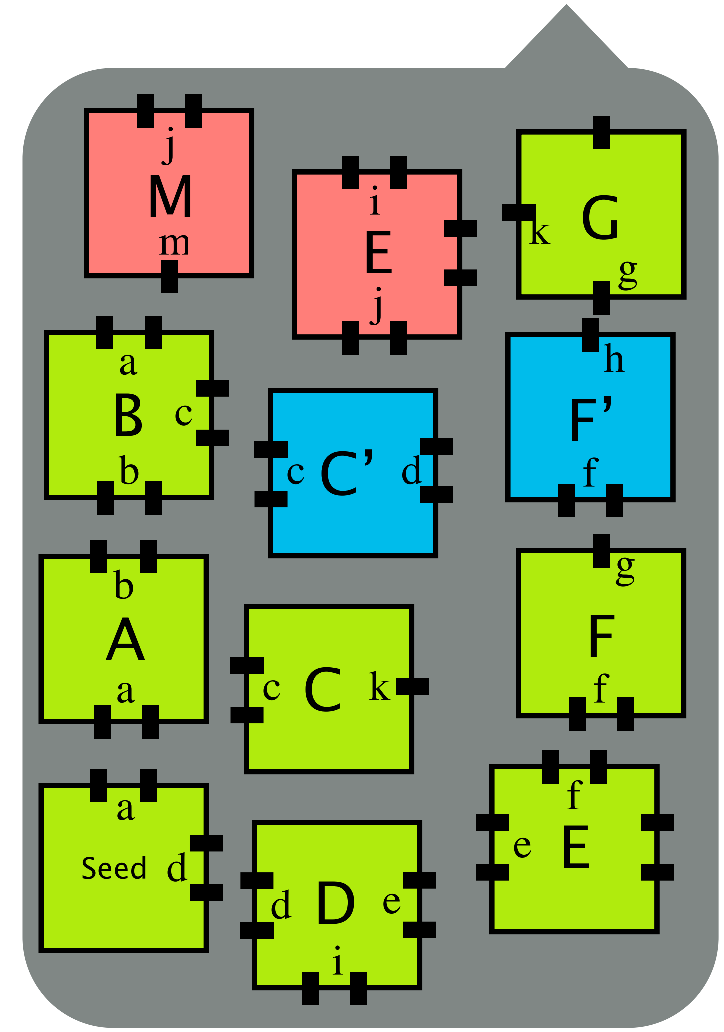

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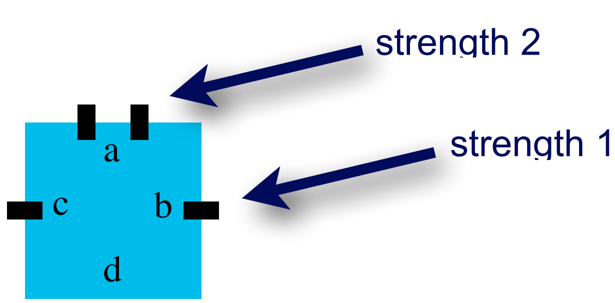

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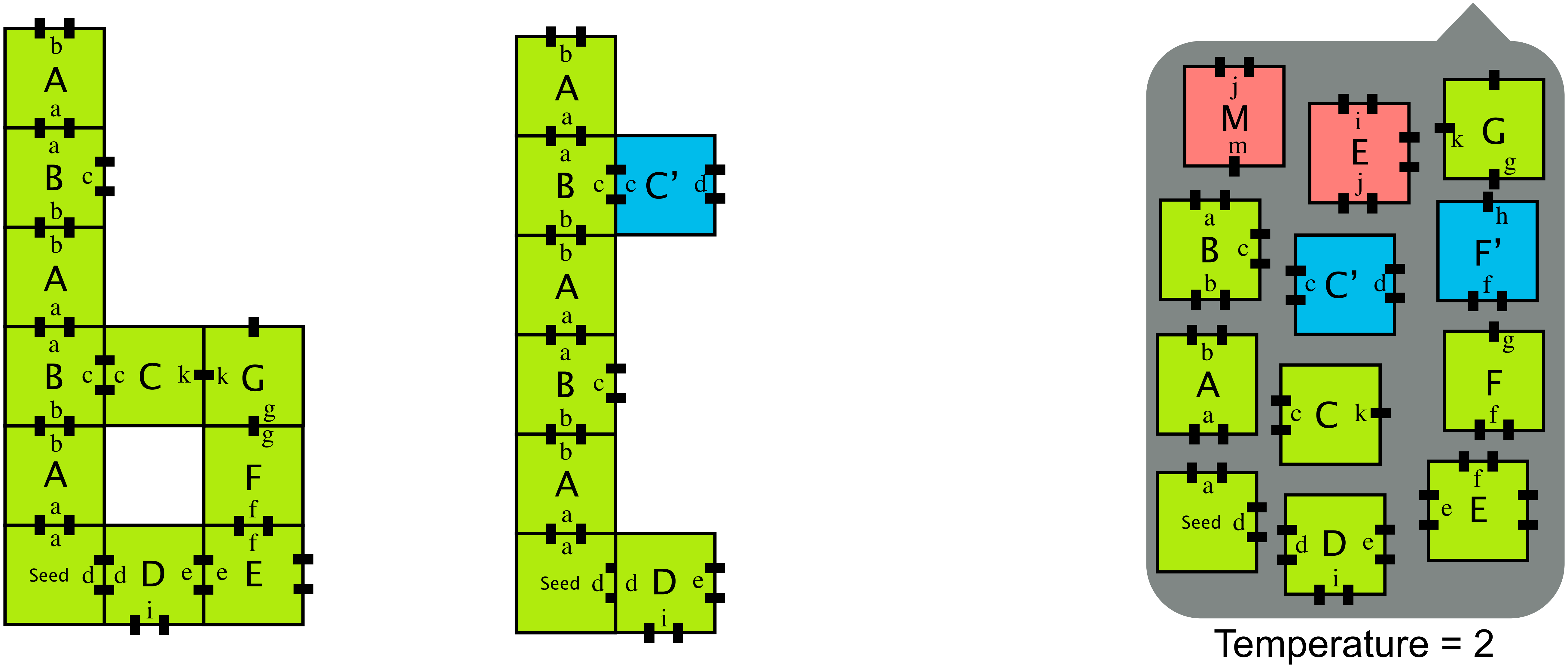

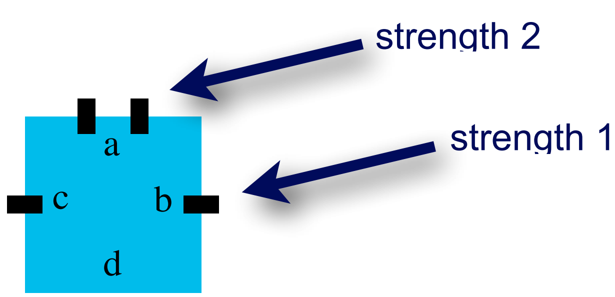

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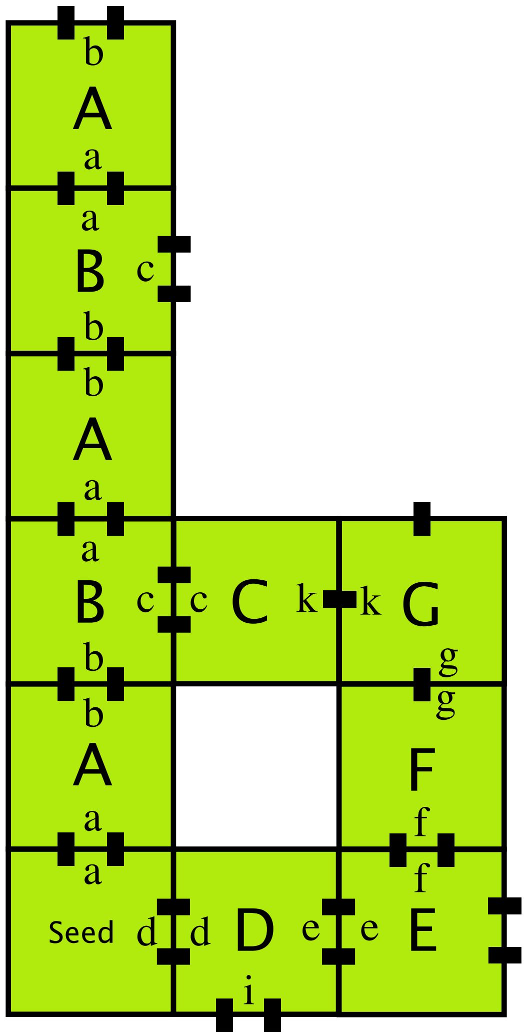

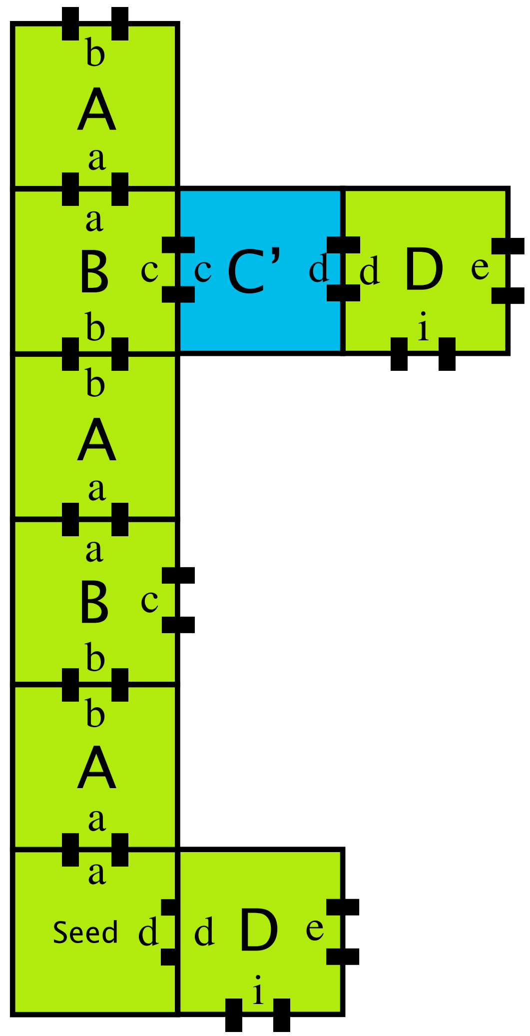

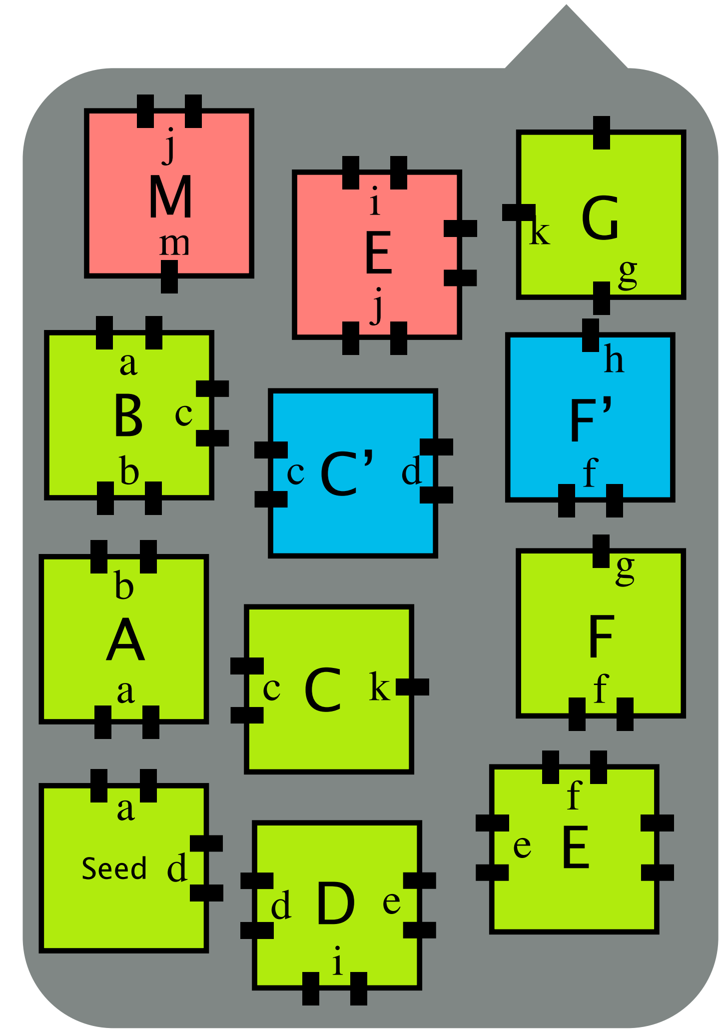

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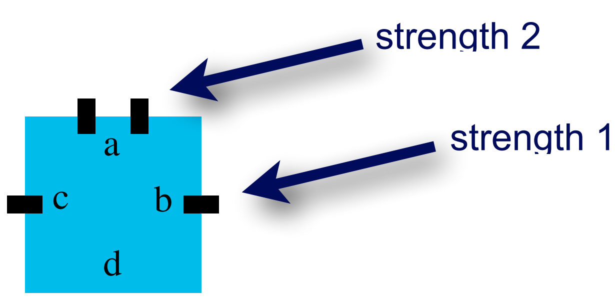

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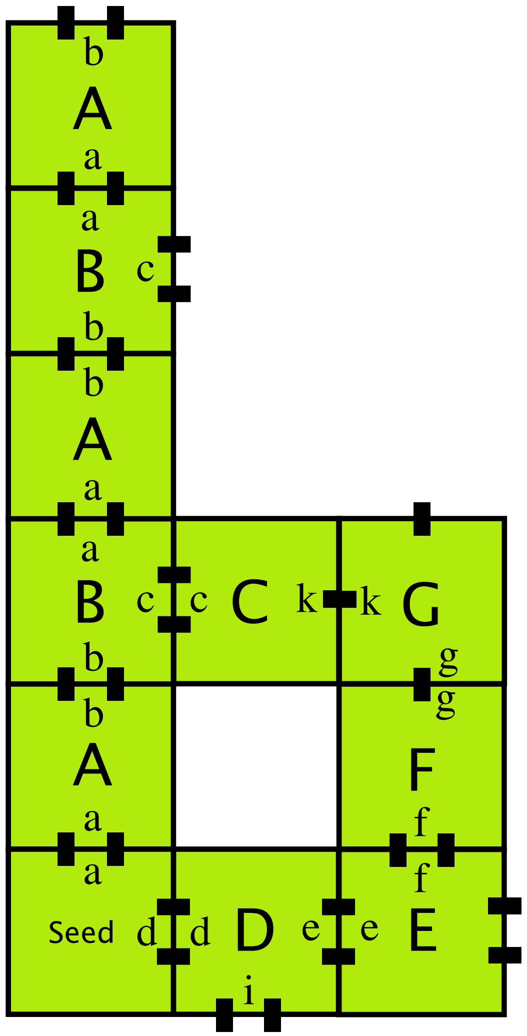

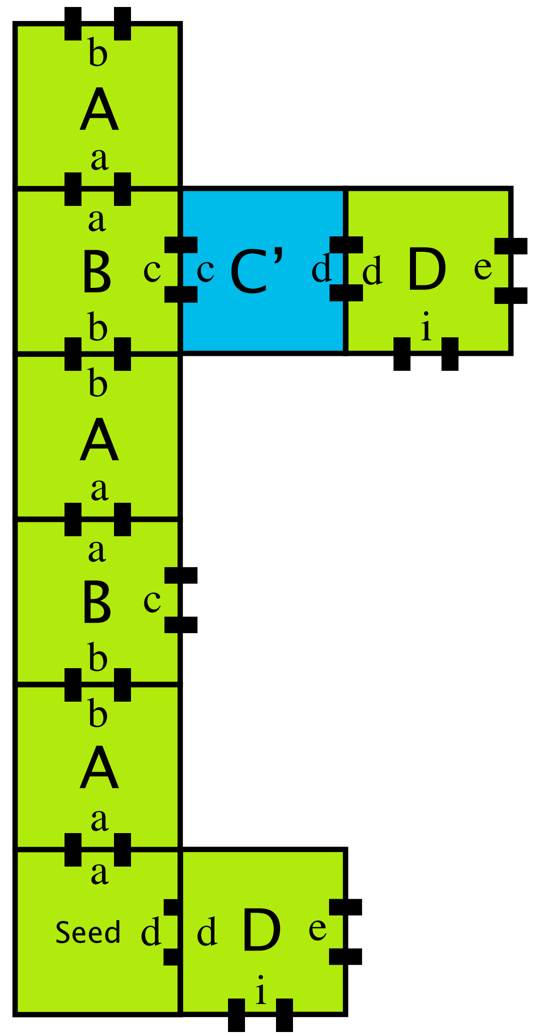

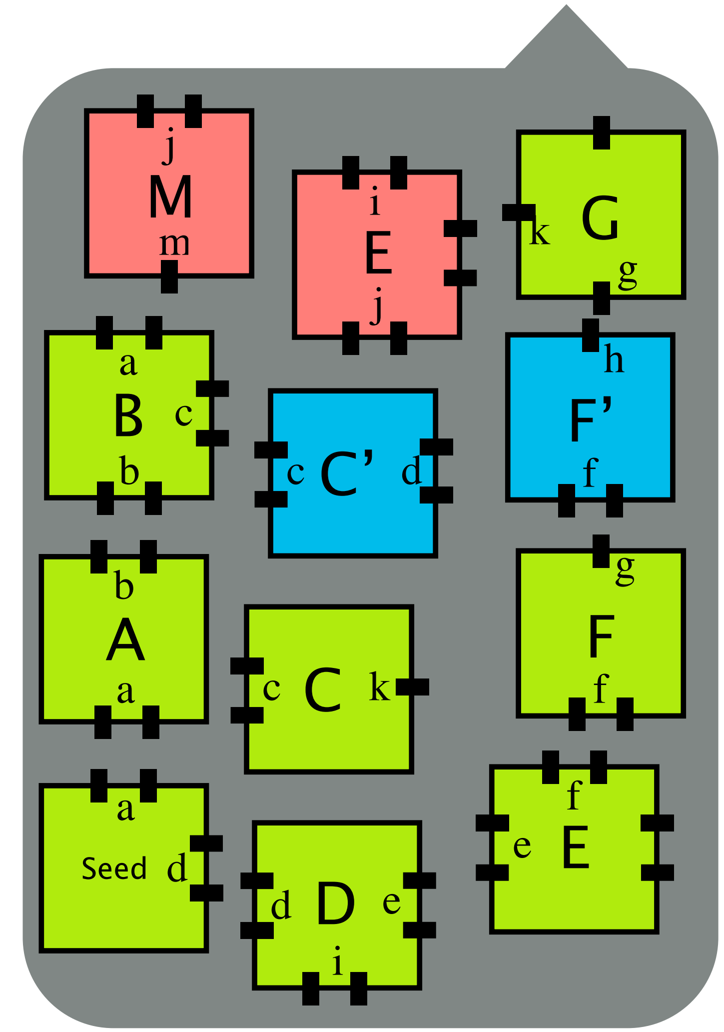

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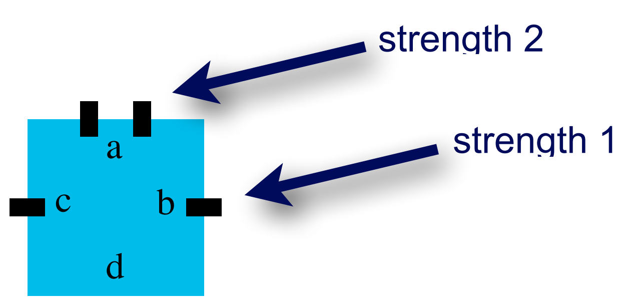
Order of tile placement is nondeterminisitc (asynchronous growth).
More than one tile type can go at a given position



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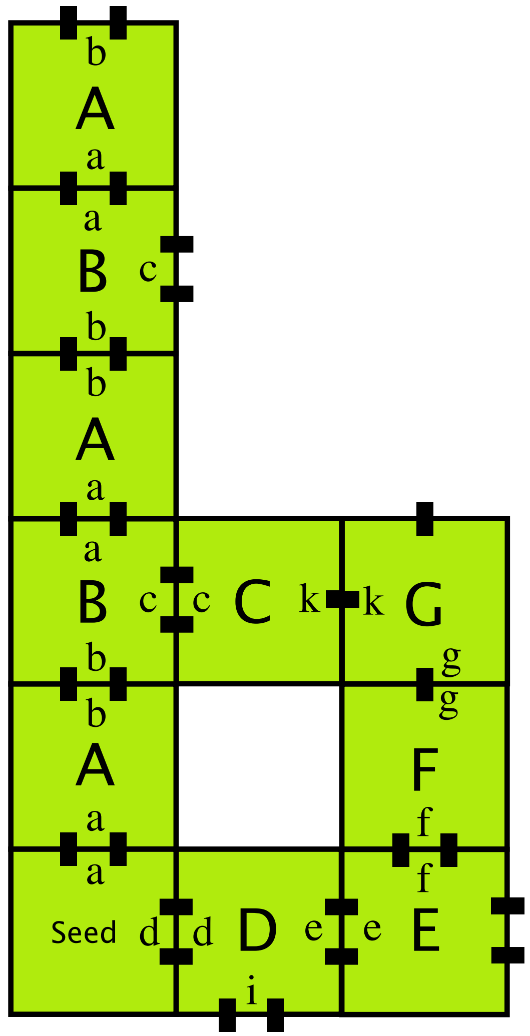

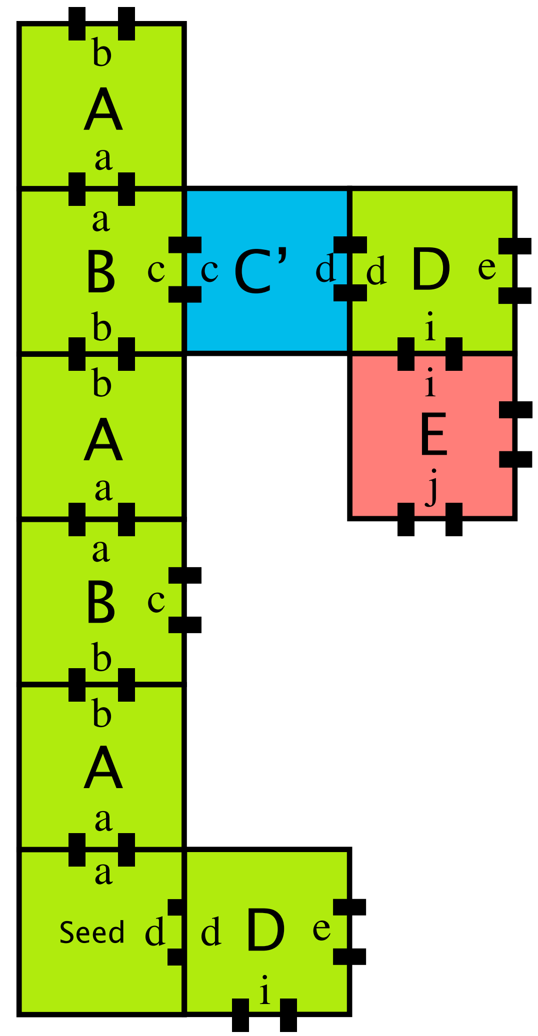

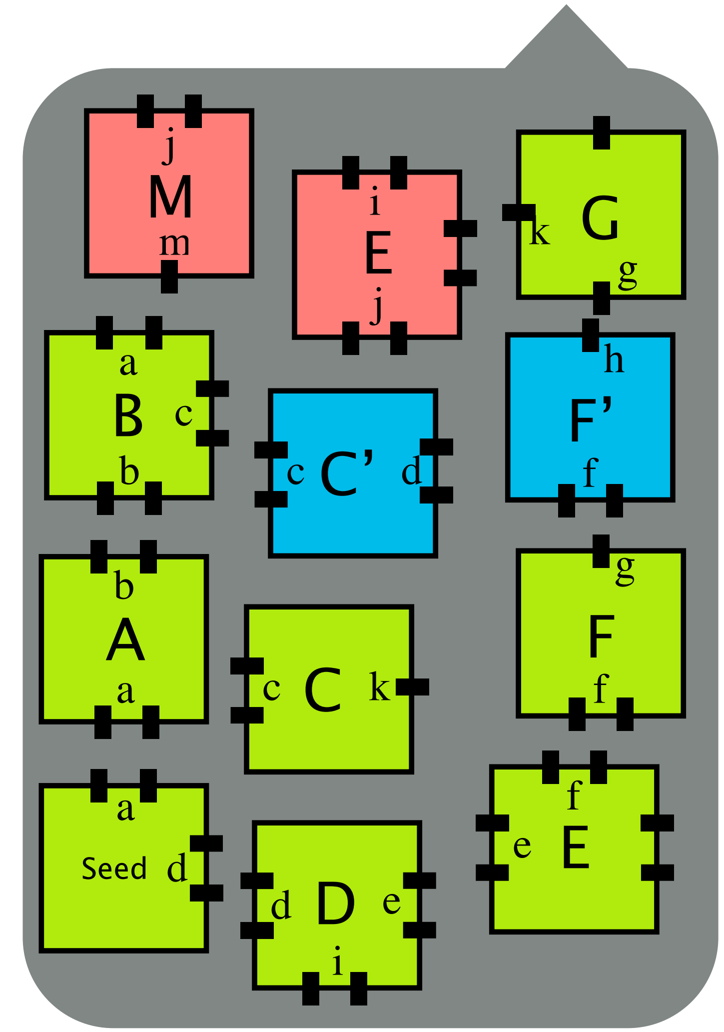

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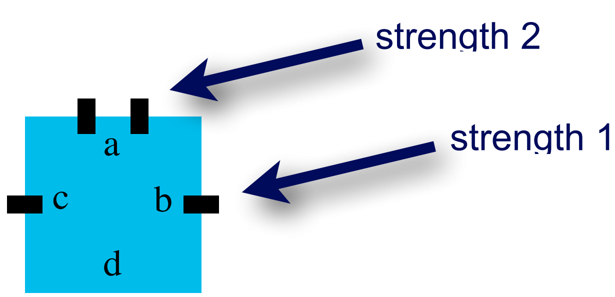

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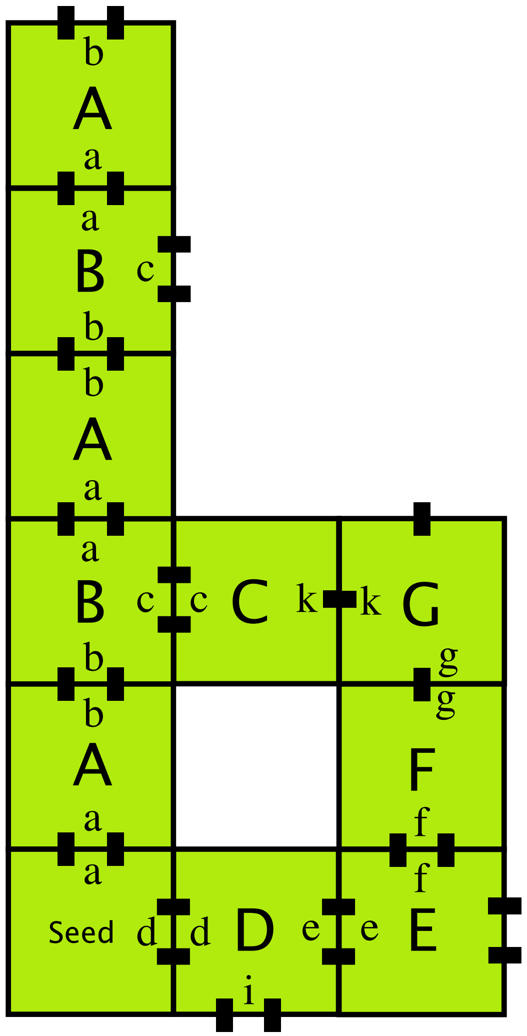

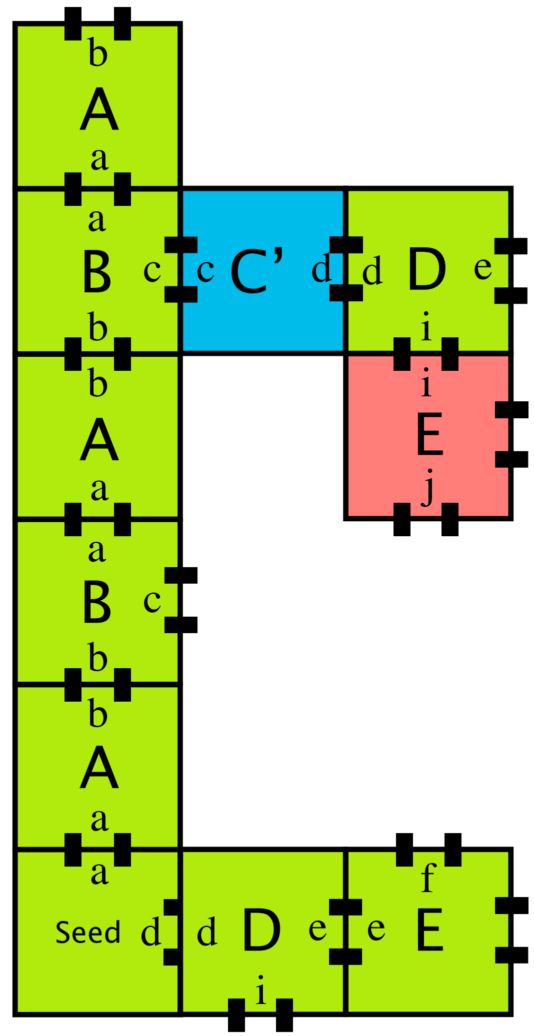

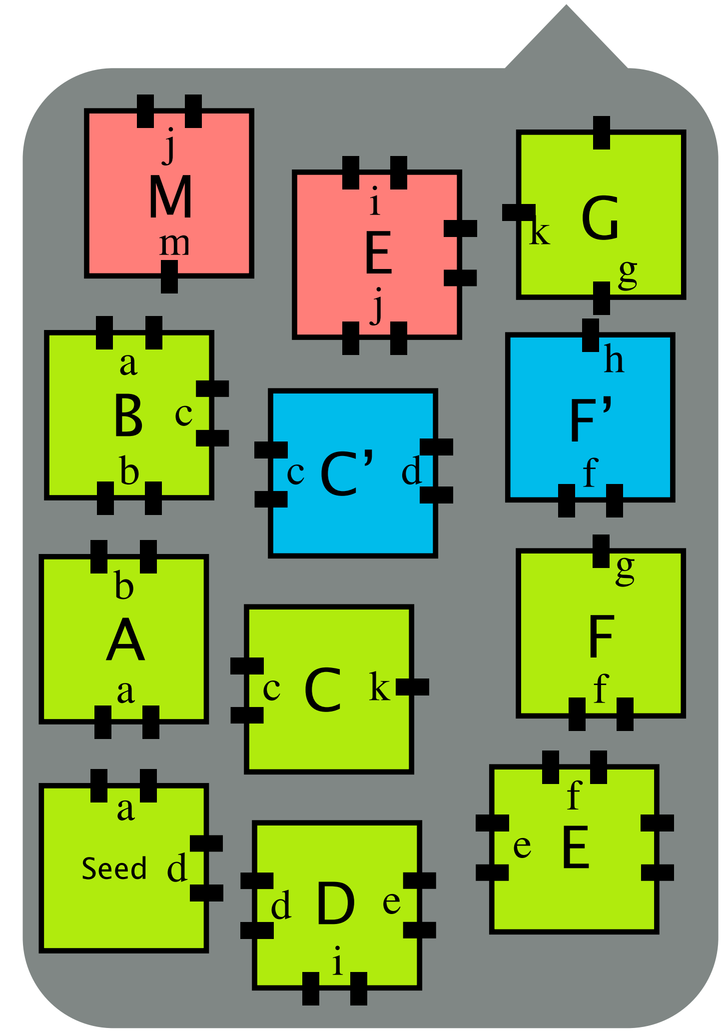

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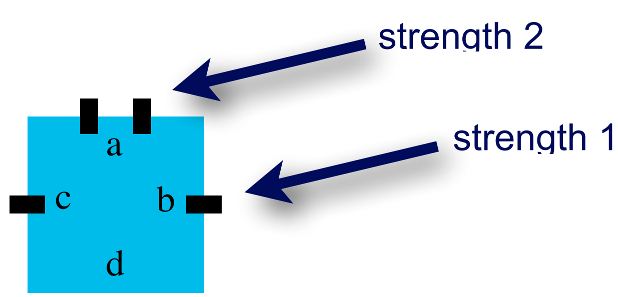

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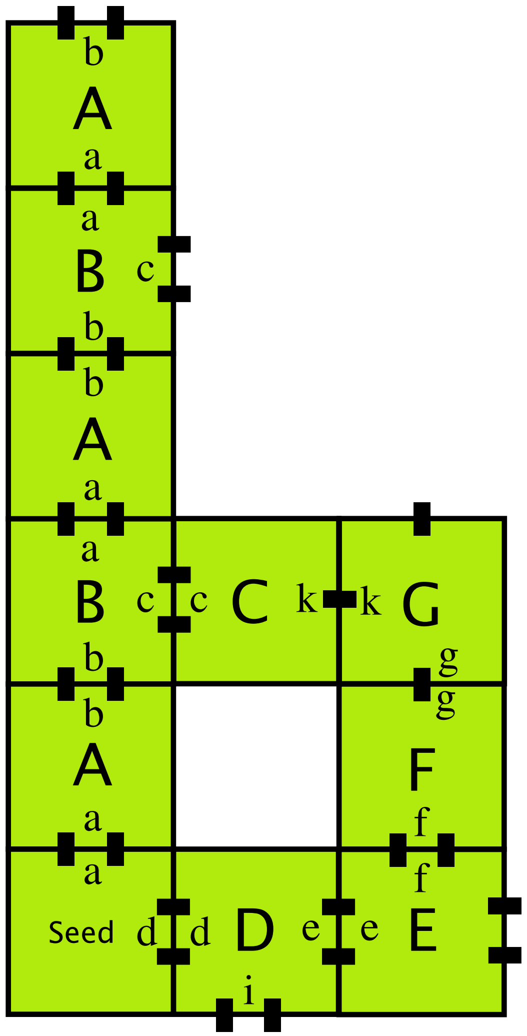
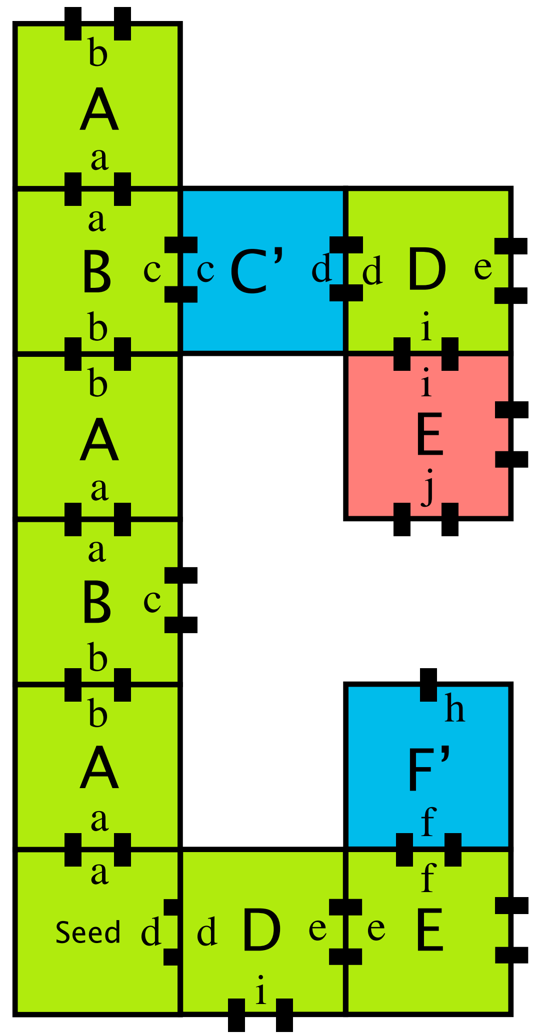
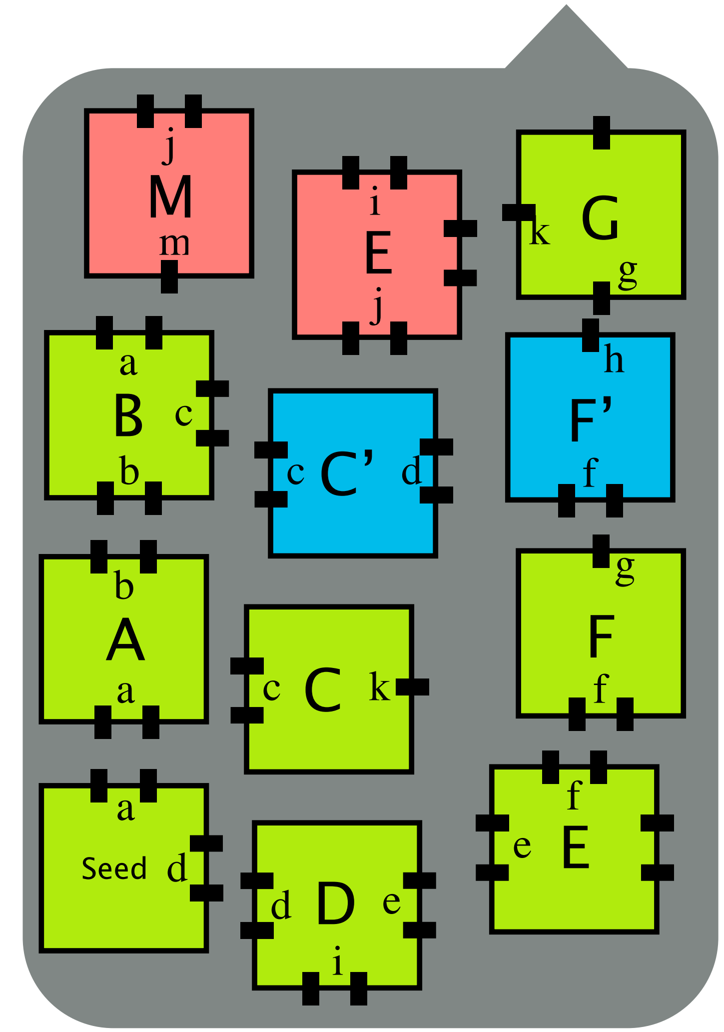
Abstract tile assembly model

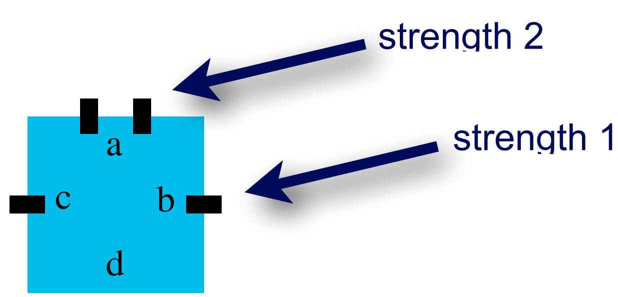






Temperature = 2

Order of tile placement is nondeterminisitc (asynchronous growth).
More than one tile type can go at a given position



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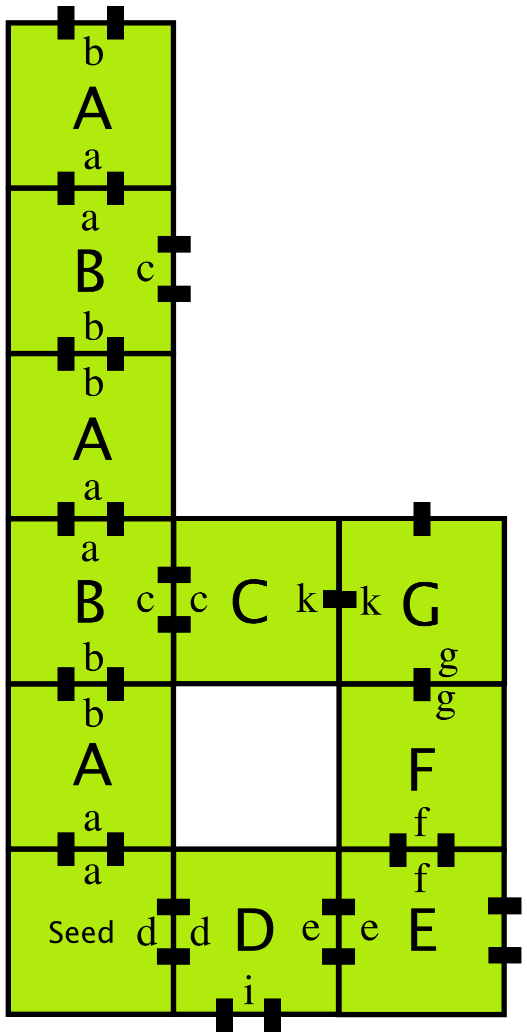

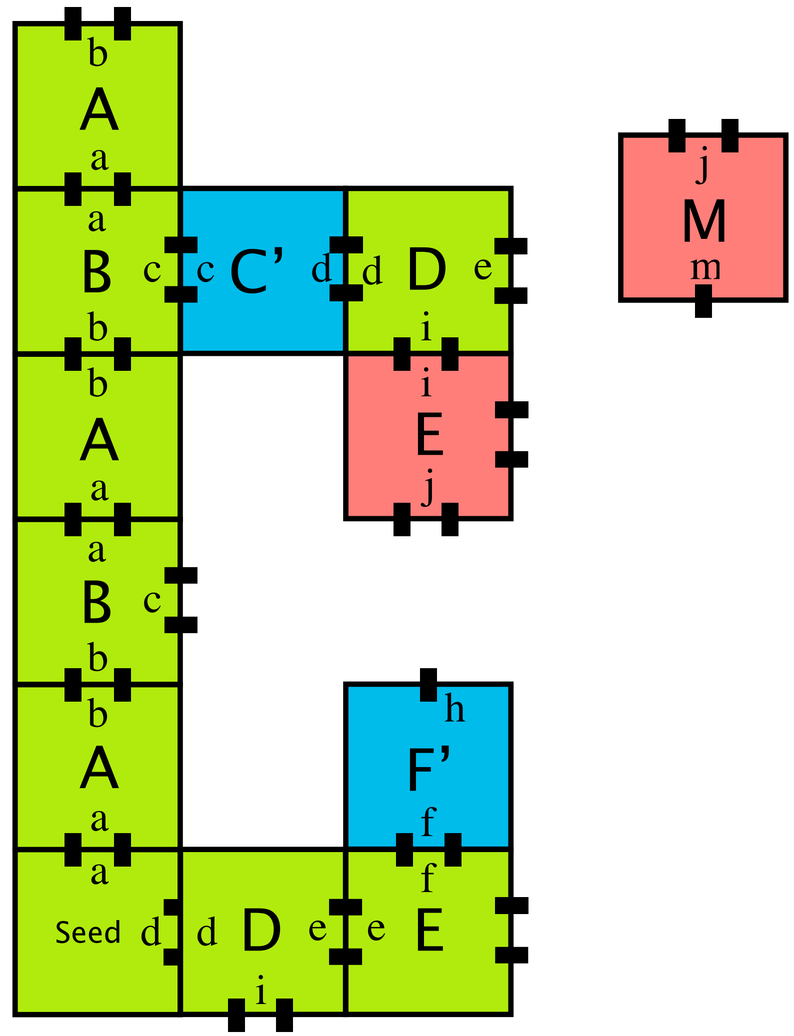

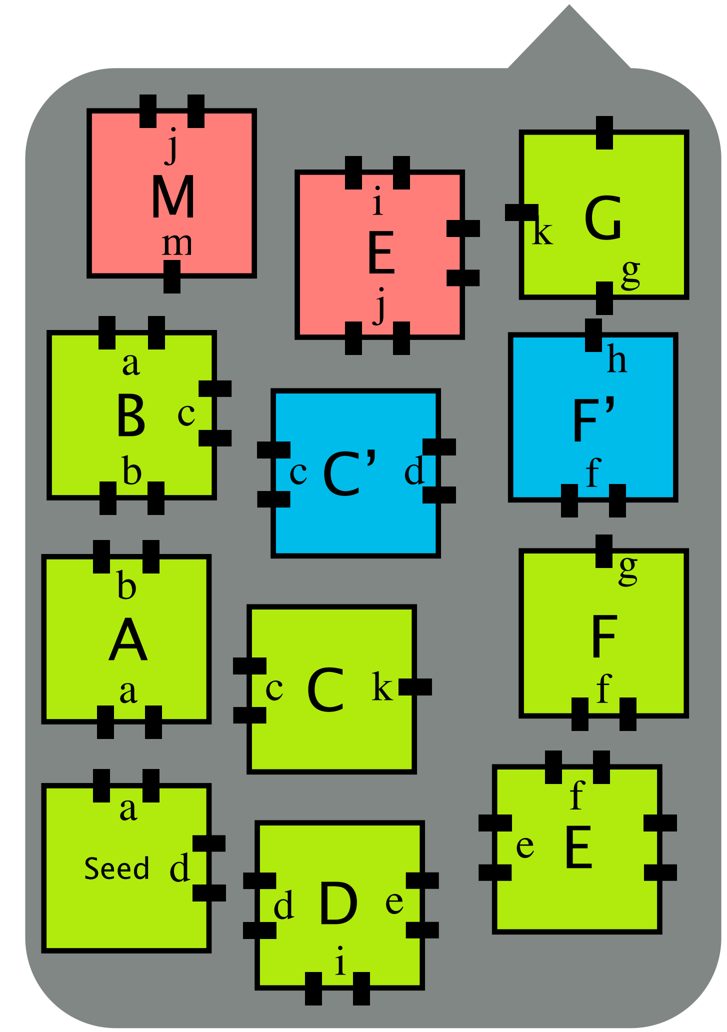

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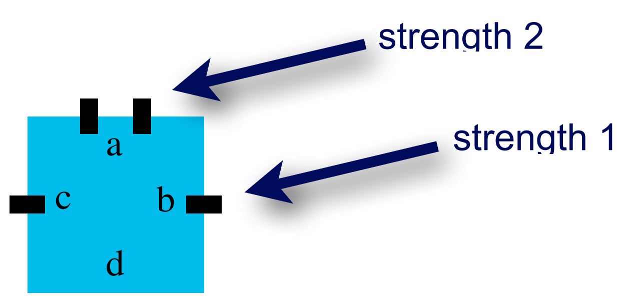

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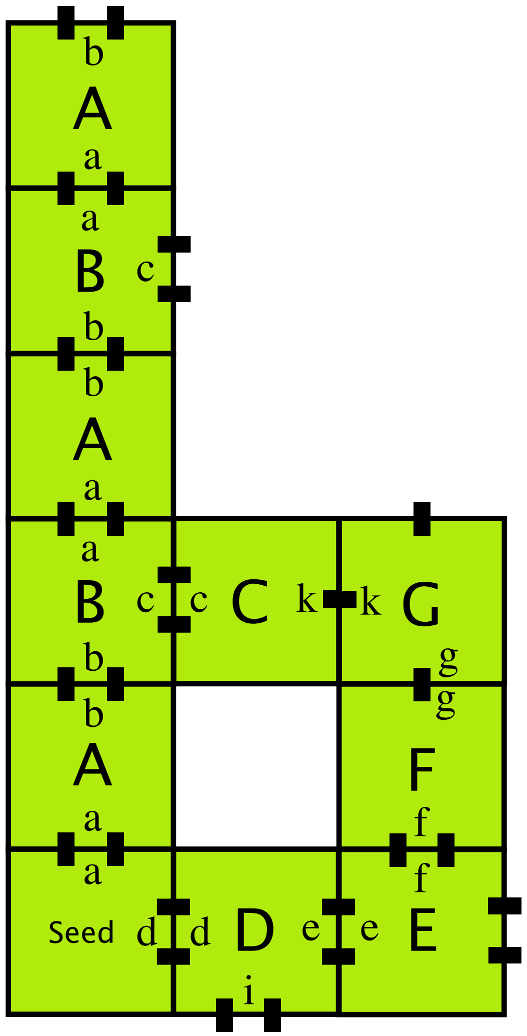

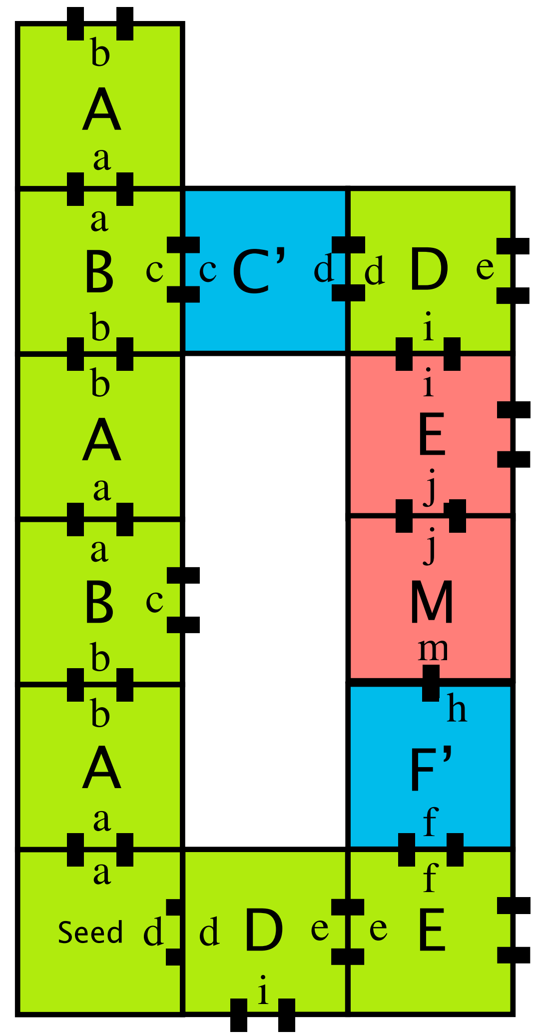

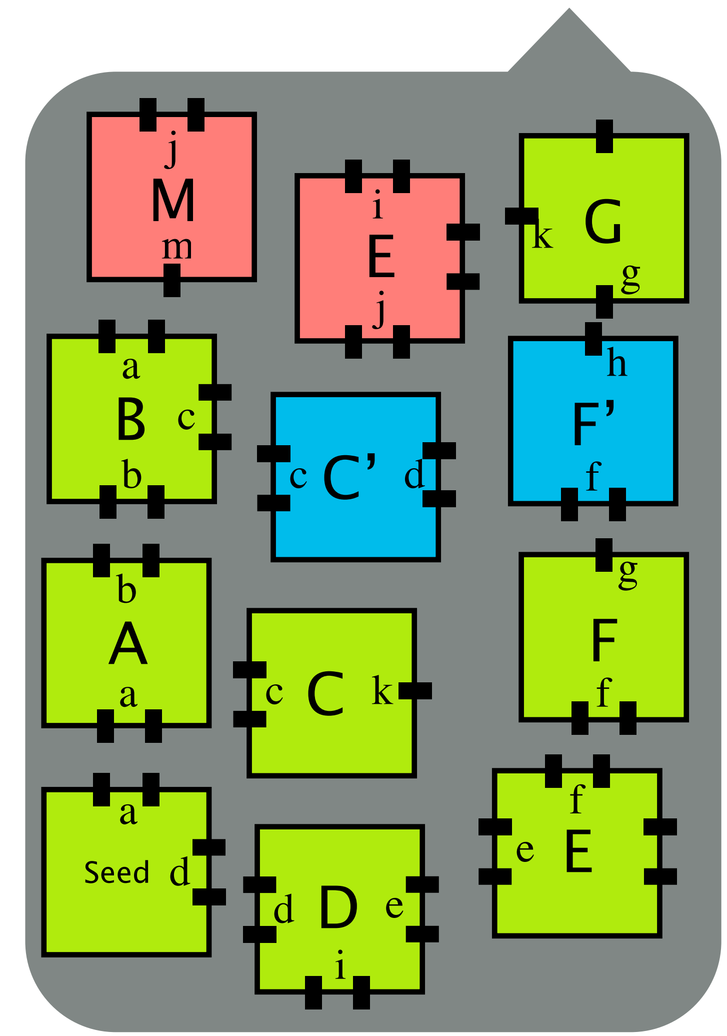

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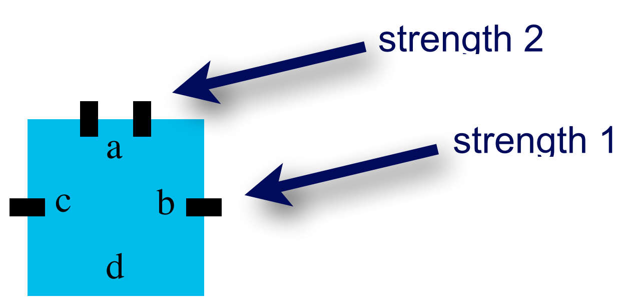

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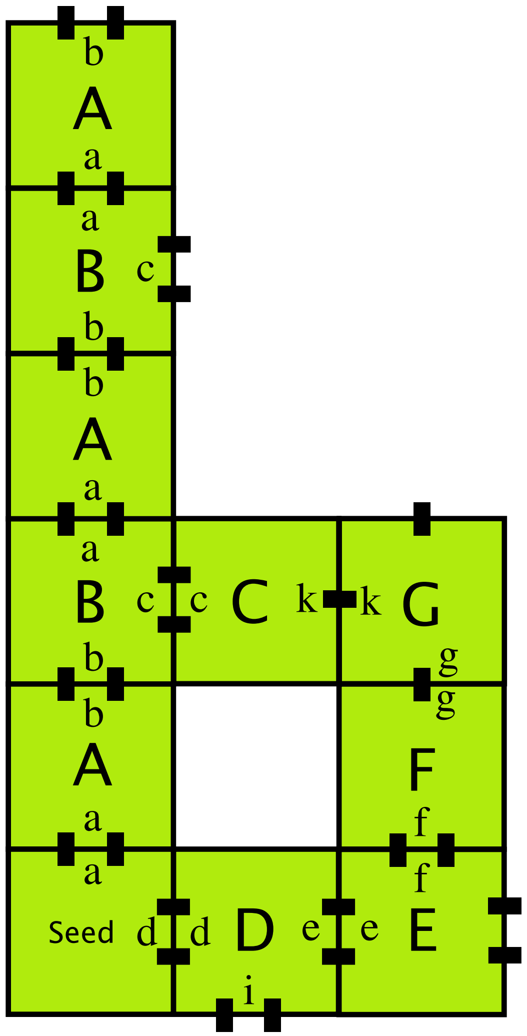
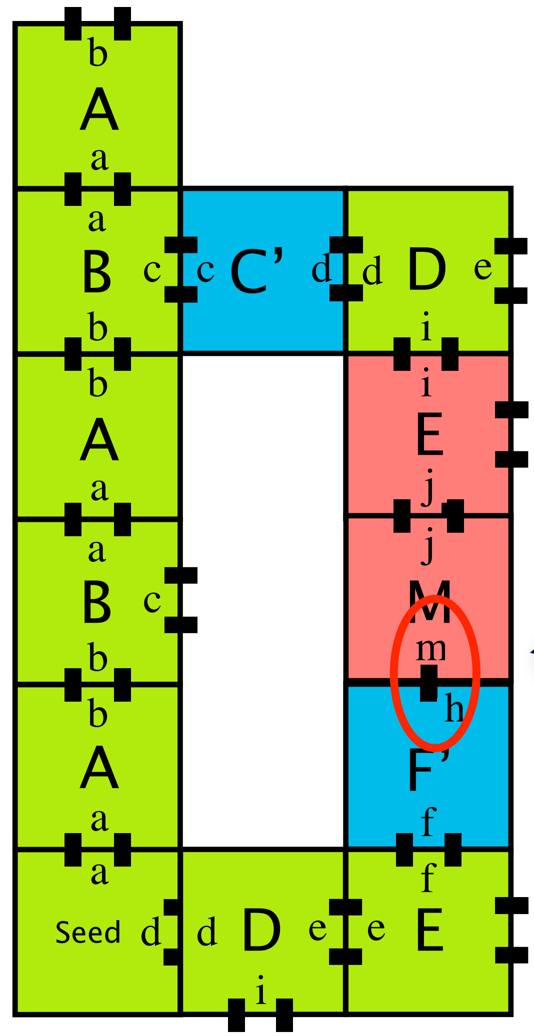
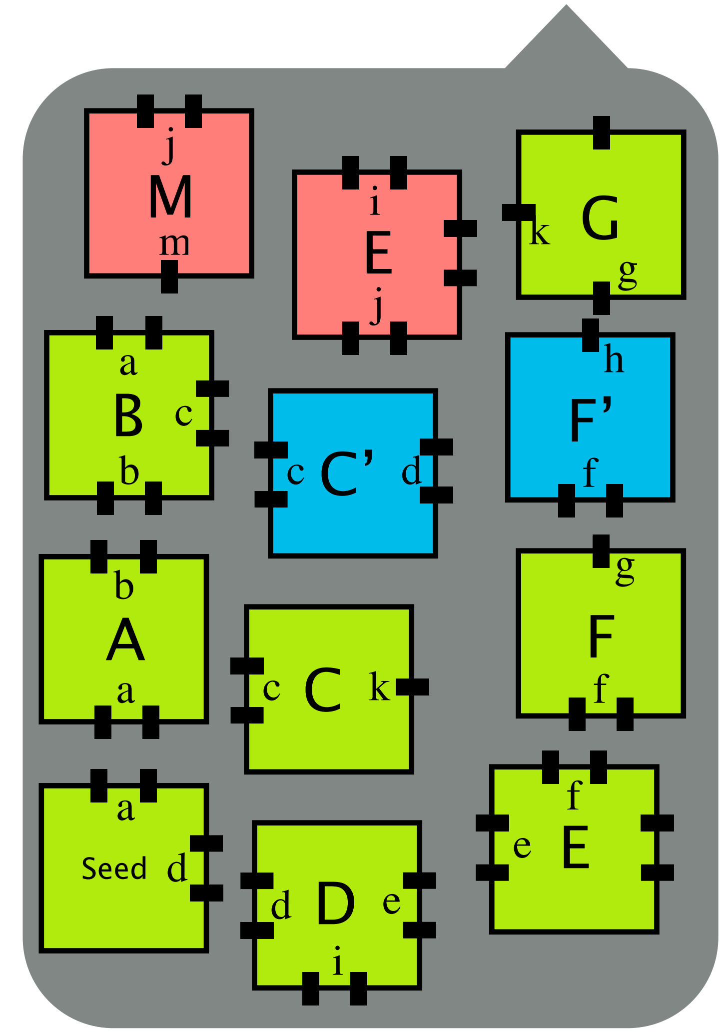




Mismatching glues are permitted



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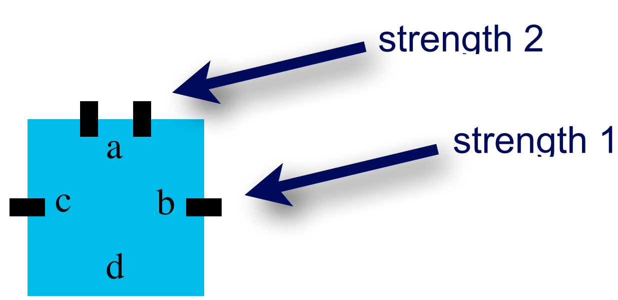

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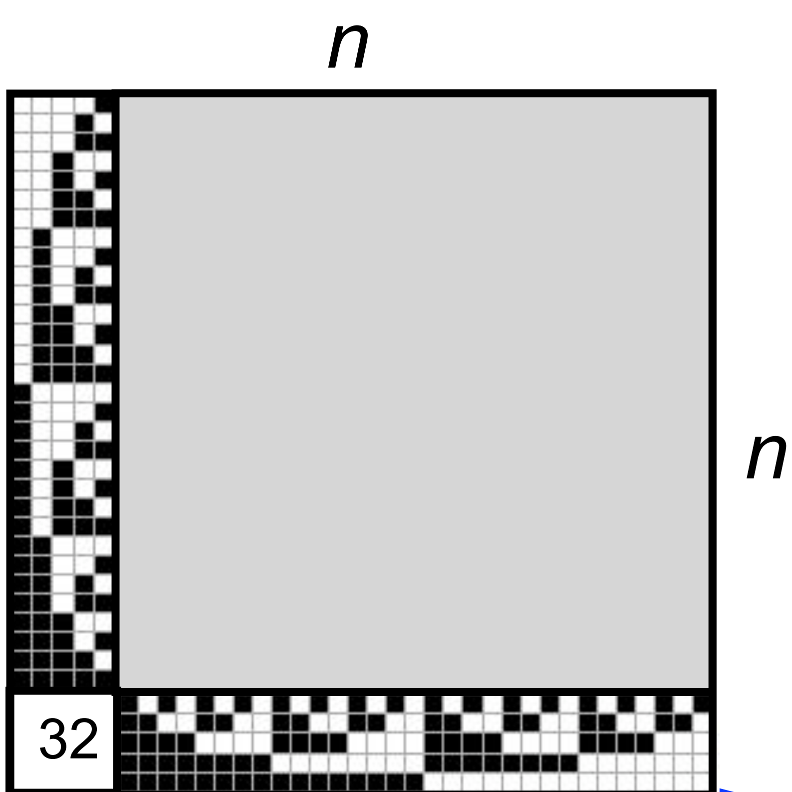
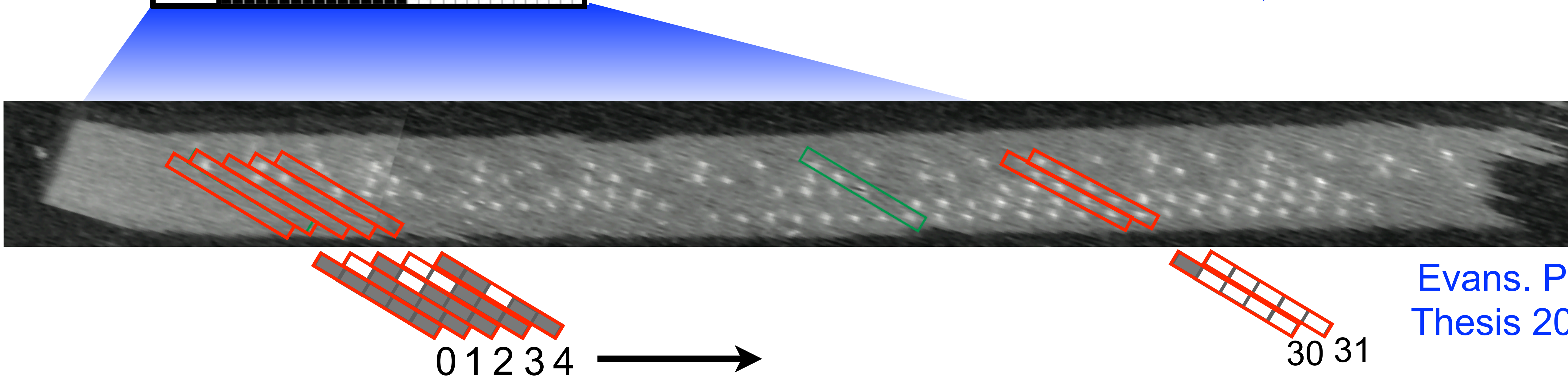
Computation with tile assembly: theory

- Turing universality
  Winfree, PhD Thesis. 1998

- Efficient assembly of simple shapes: n x n squares using $\Theta(\log n / \log \log n)$ tile types
  Adleman, Cheng, Goel, Huang STOC 2001
  Rothemund, Winfree. STOC 2000





Evans. PhD Thesis 2014

- Efficient assembly of scaled complicated connected shapes using a number of tile types roughly equal to the Kolmogorov complexity of the shape
  Soloveichik, Winfree. SICOMP 2007

Theory of algorithmic self-assembly

- Helps us understand the abilities and limitations of self-assembly
  - Also, its fun!
- aTAM is Turing universal: can "run" any algorithm

Winfree, PhD Thesis 1998.

- Efficiently assemble *n* x *n* squares and other simple shapes
  - Using only Θ(log *n*/log log *n*) tile types

Rothemund, Winfree. STOC 2000

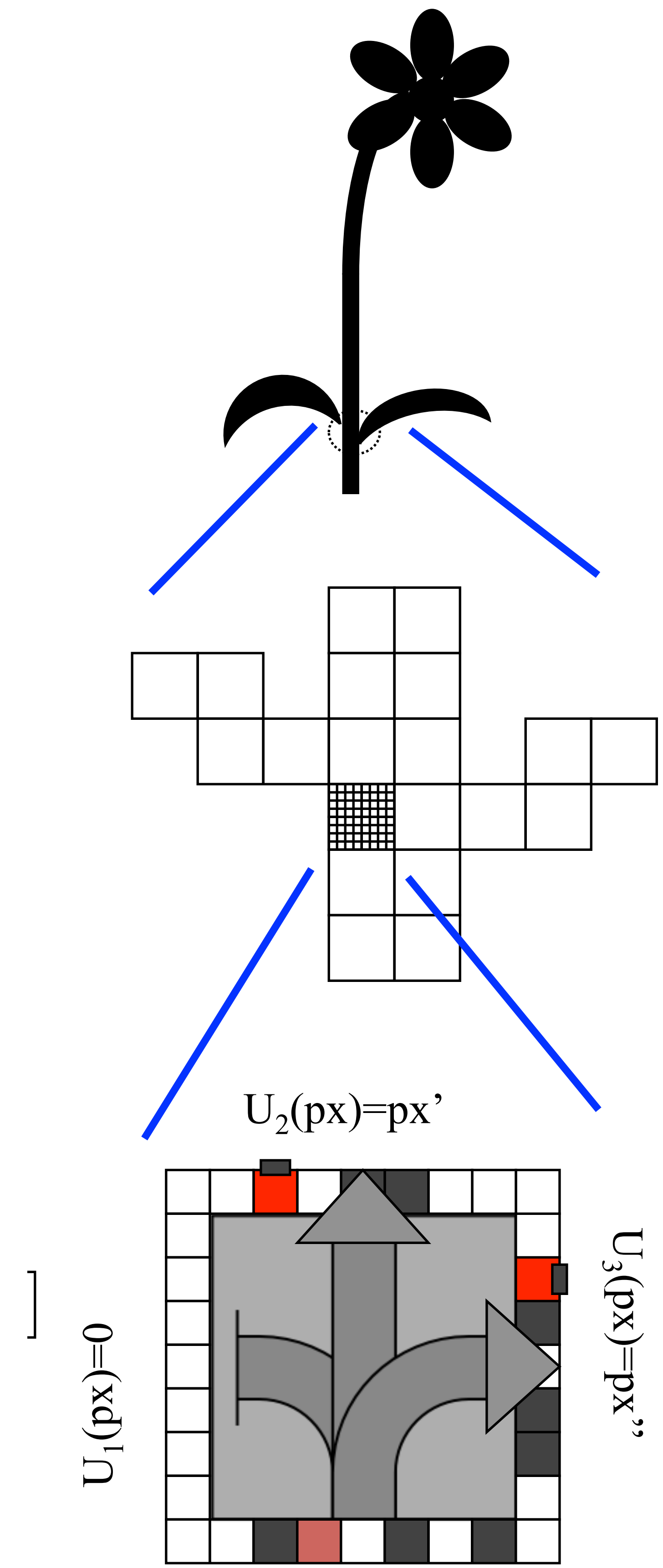
- Efficiently assemble arbitrary finite shapes
  - Number of tile types is roughly the Kolmogorov *complexity* of the shape

Soloveichik, Winfree. SICOMP 2007

- aTAM is intrinsically universal: there is one tile set that can simulate any tile assembly system
  - Shape complexity can be put into the seed

Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

- Thinking about these topics leads to a kind of "complexity theory" for self-assembly



Soloveichik & Winfree, SIAM J Comp, 2007

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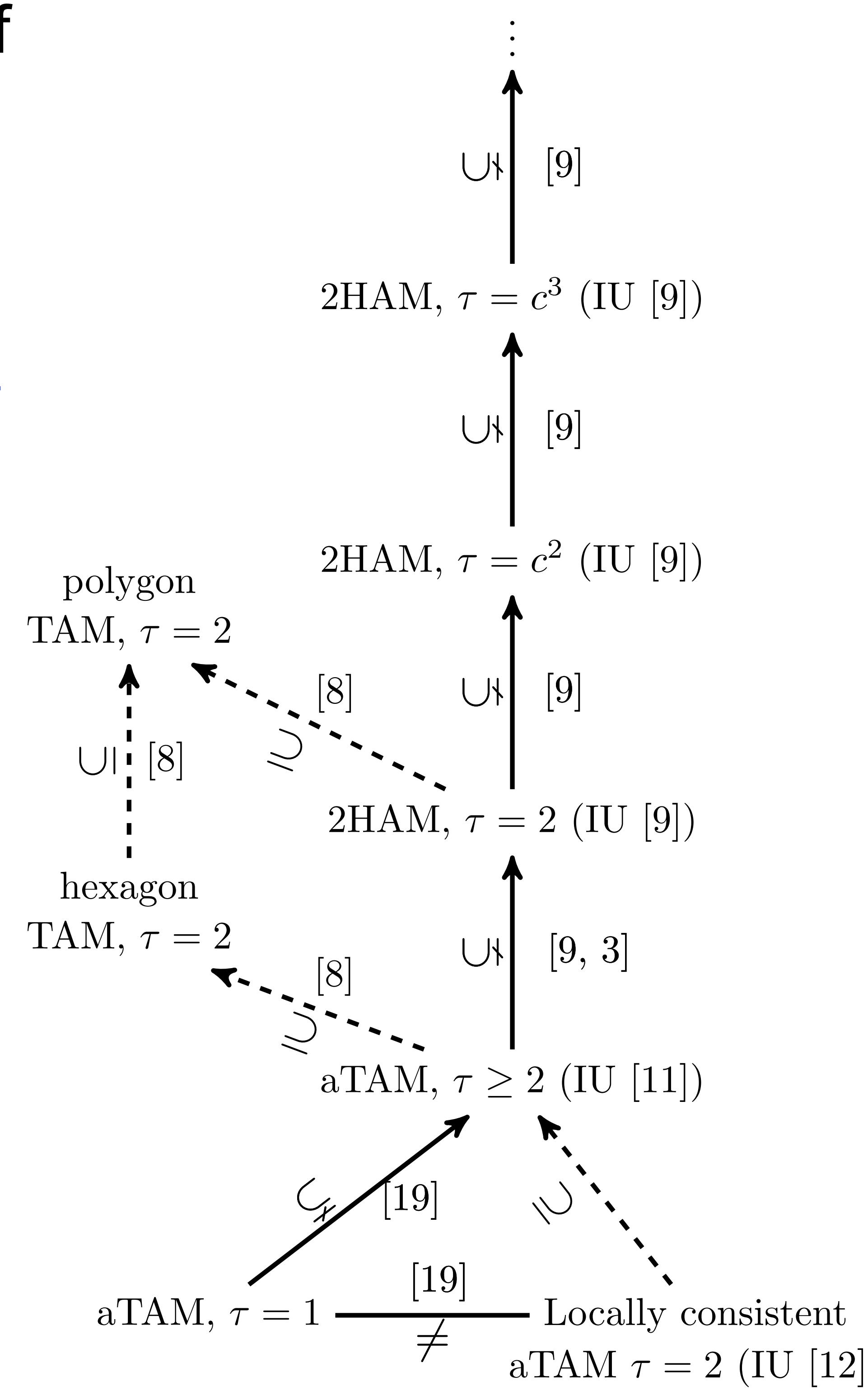
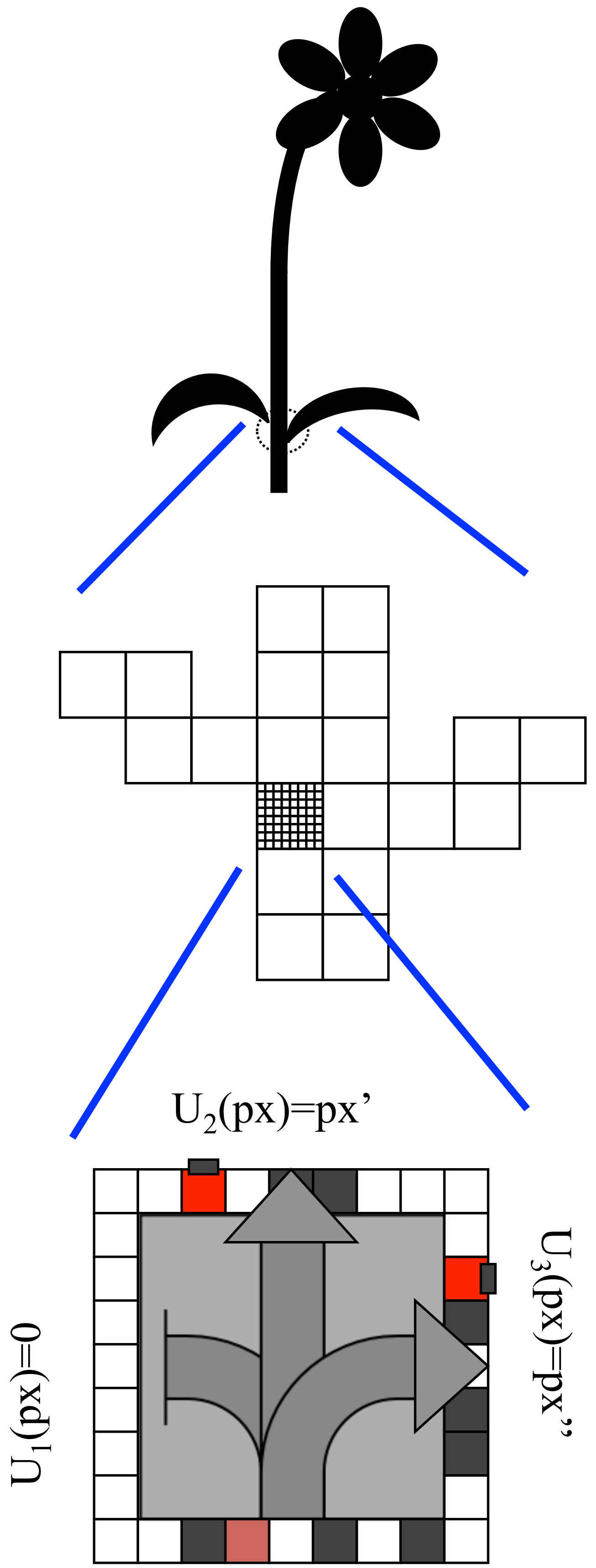


Woods. MCU. 2013

Computation with tile assembly: theory



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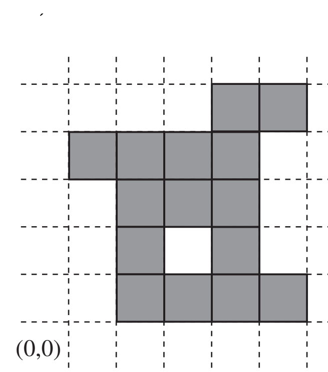

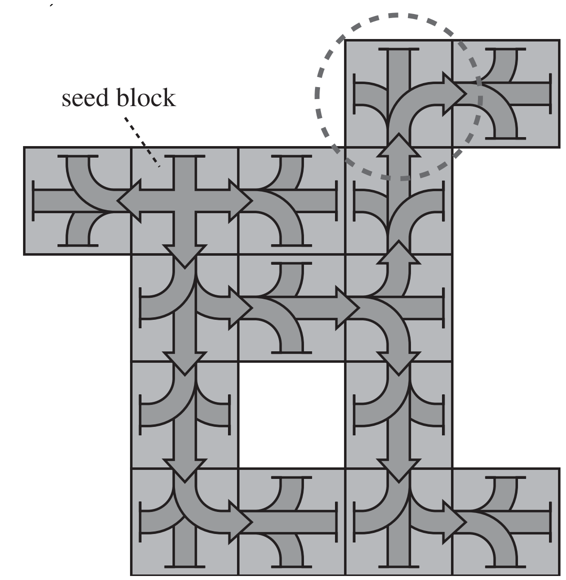

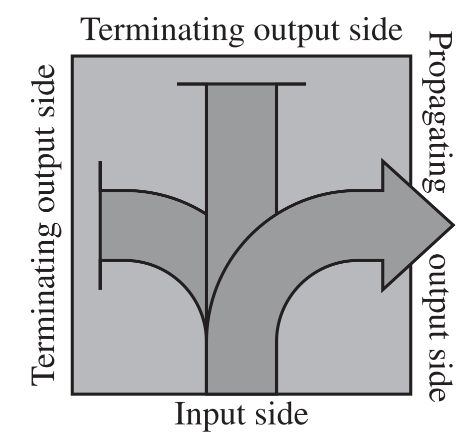

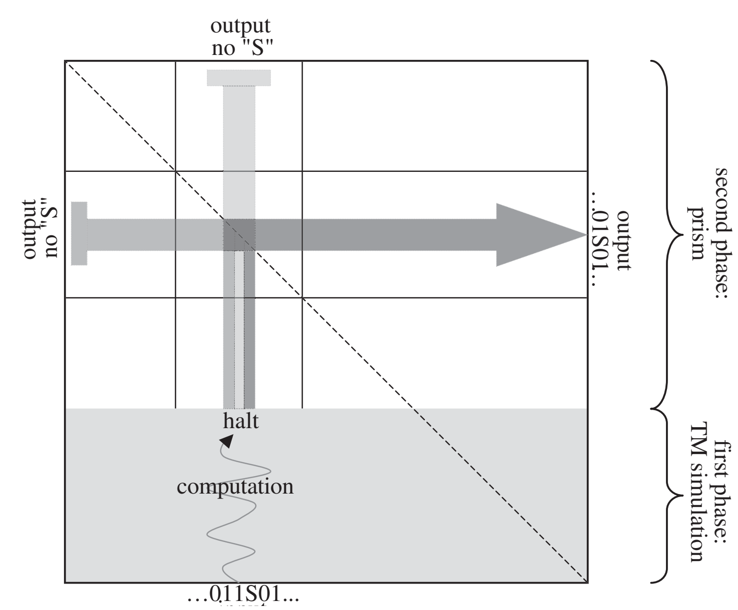

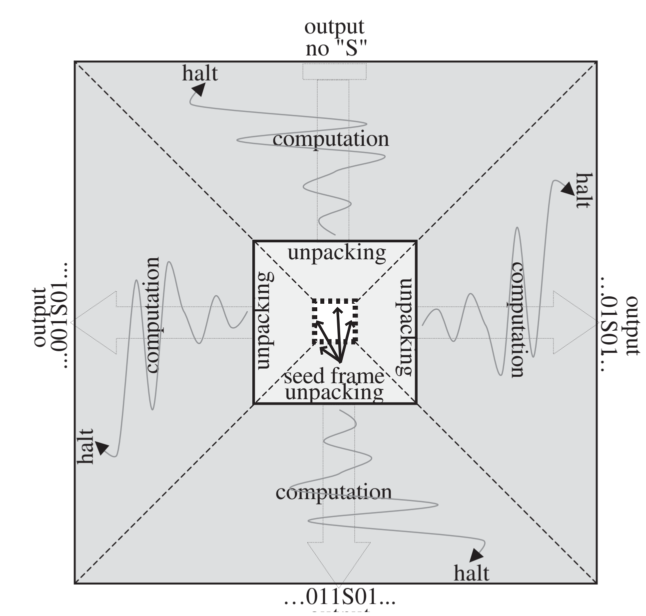

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Computation with tile assembly: theory

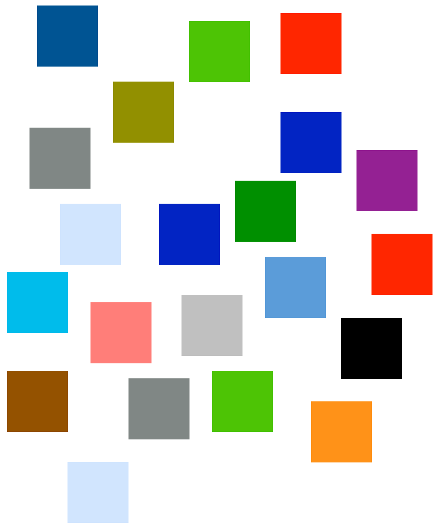
- Many other theoretical questions have been asked
- What questions would you ask?

- The goal is to understand the capabilities of these systems!
- Another goal is to motivate what we should build in the lab!

- Next slide: Let’s ask a question

Intrinsic universality

Is there a set of **intrinsically universal tiles:** a set of aTAM tiles U that can act like **any** other tile set?



One universal tile set
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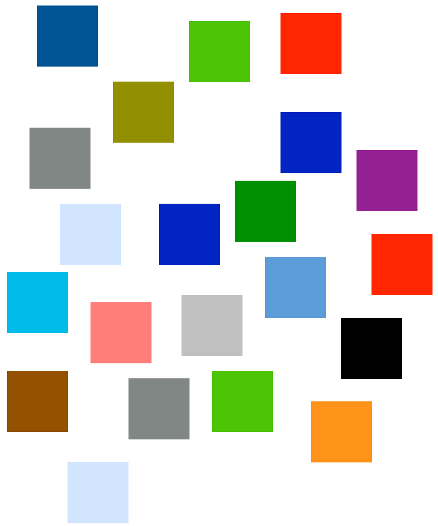

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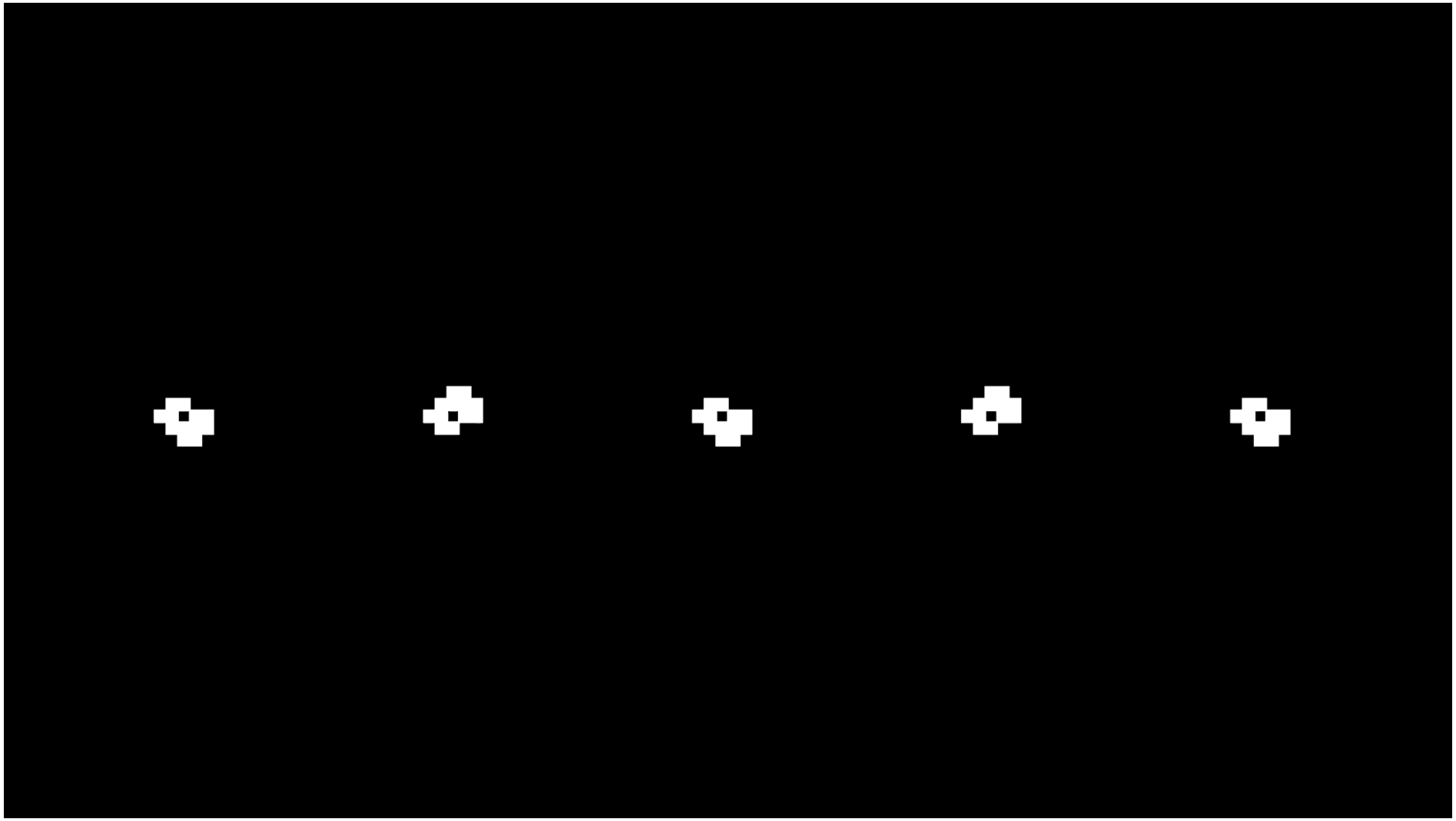
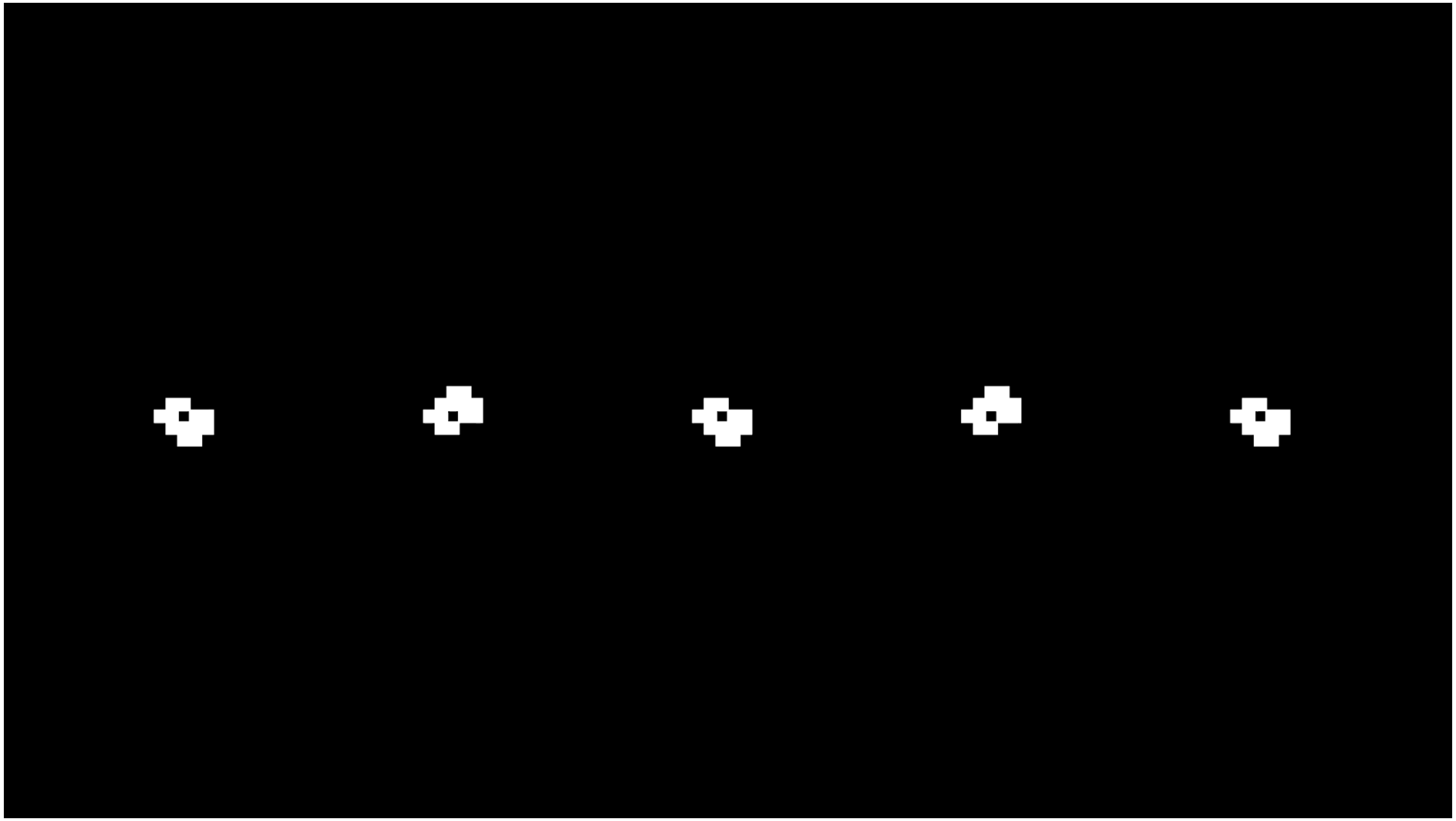
What does “act like” mean?



http://otcametapixel.blogspot.com/
http://www.youtube.com/watch?v=xP5-iIeKXE8



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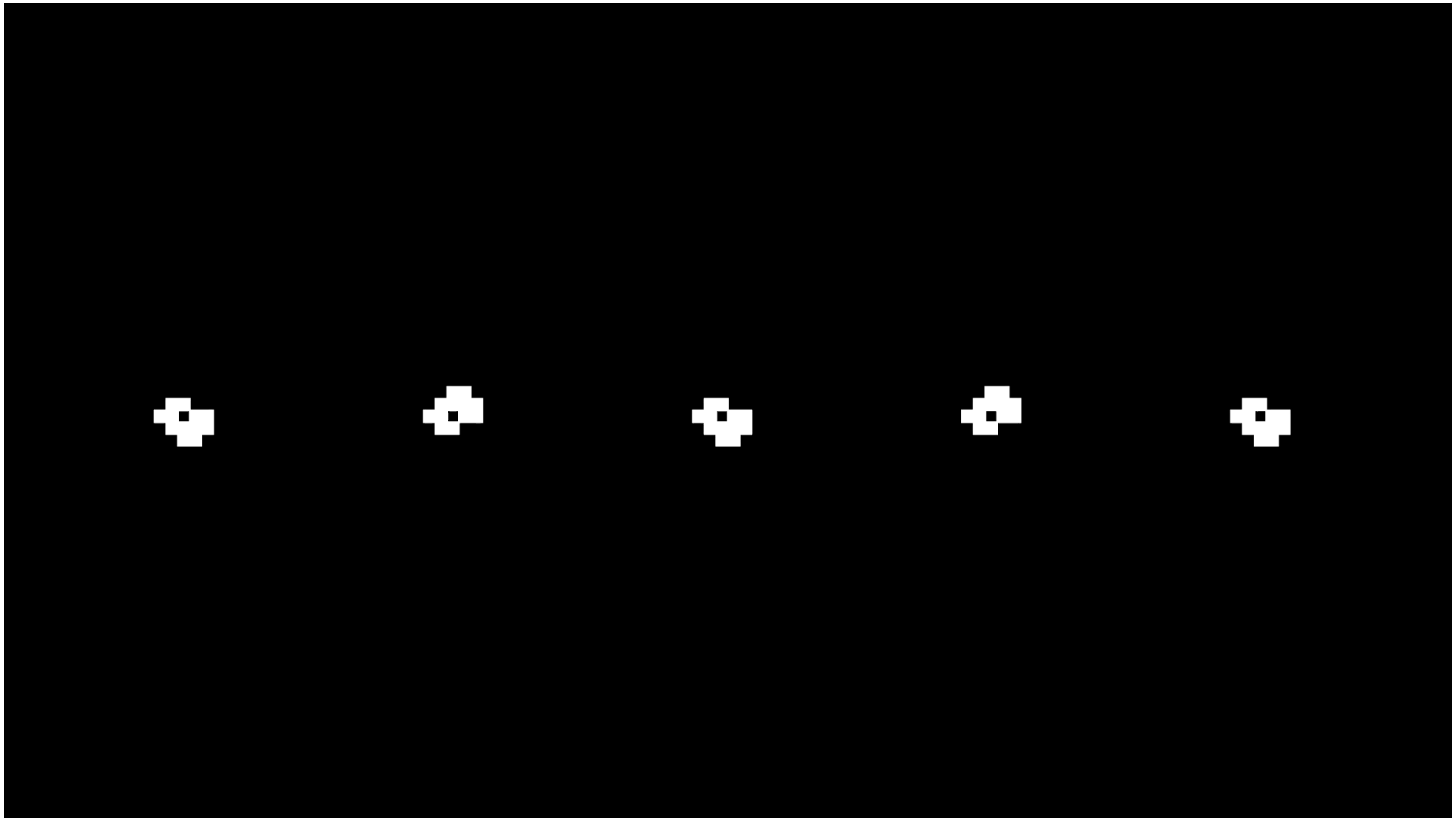
- Conway's Game of Life is an intrinsically universal cellular automaton



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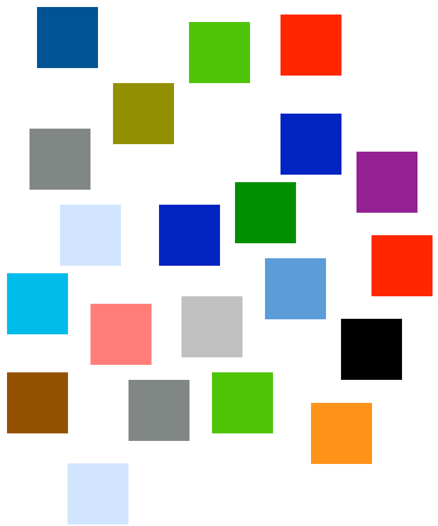
Durand, Roka, The game of life: universality revisited. 1999

Comparing tile assembly models

Is there a set of **intrinsically universal tiles** that can **simulate any tile set**?



- What is it that tile assembly systems do?
  - Make shapes and patterns
  - Carry out a crystal-like growth process (dynamics)
- Let define **simulate** using these criteria that are intrinsic to the model

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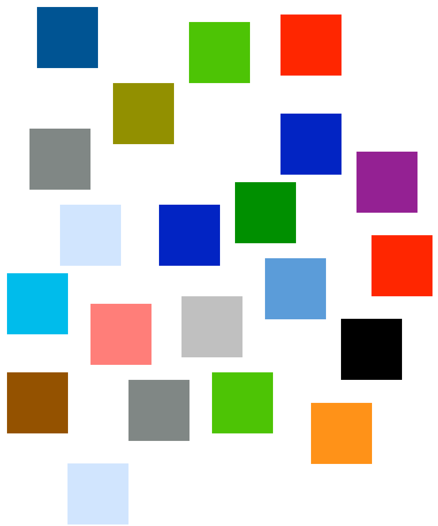

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Simulation

- For (any) simulated tile assembly system $\mathcal{T}$
    - $\mathcal{T}$ = (tileset T, seed assembly σ, temperature τ)
- Tile assembly system $\mathcal{U}$ simulates $\mathcal{T}$ if:
    - **Tiles** from $\mathcal{T}$ are represented by ***m* x *m* supertiles** in $\mathcal{U}$
    - Assemblies produced by $\mathcal{U}$ **represent exactly** assemblies produced by $\mathcal{T}$ (via a representation function R : Blocks of tiles from *U* -> tiles from T)
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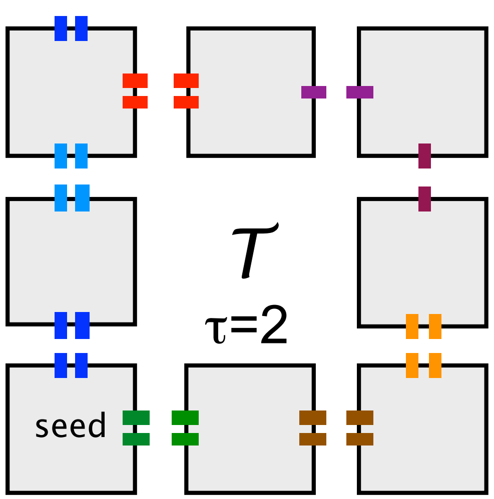
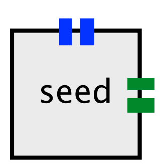
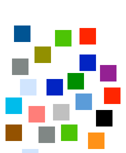


Simulated system



Simulator system

U



m x *m* seed assembly

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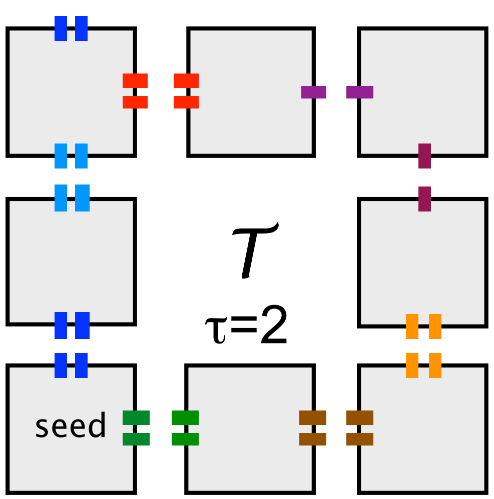

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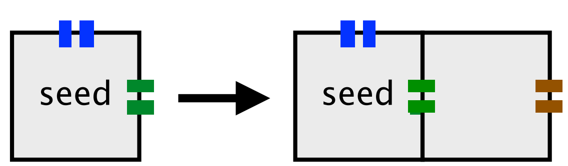

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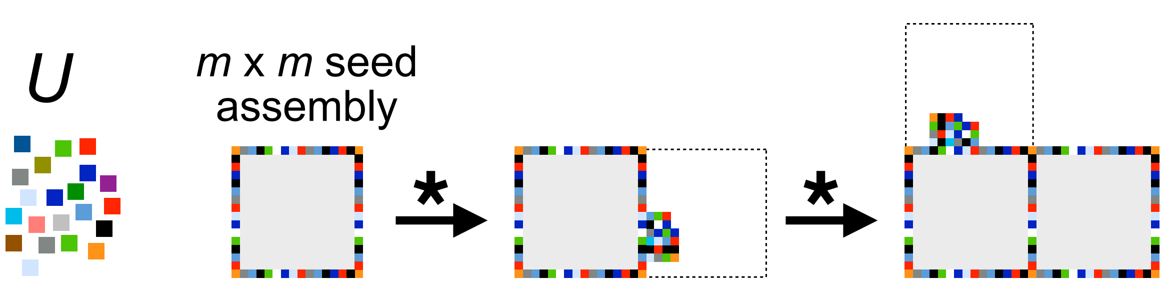

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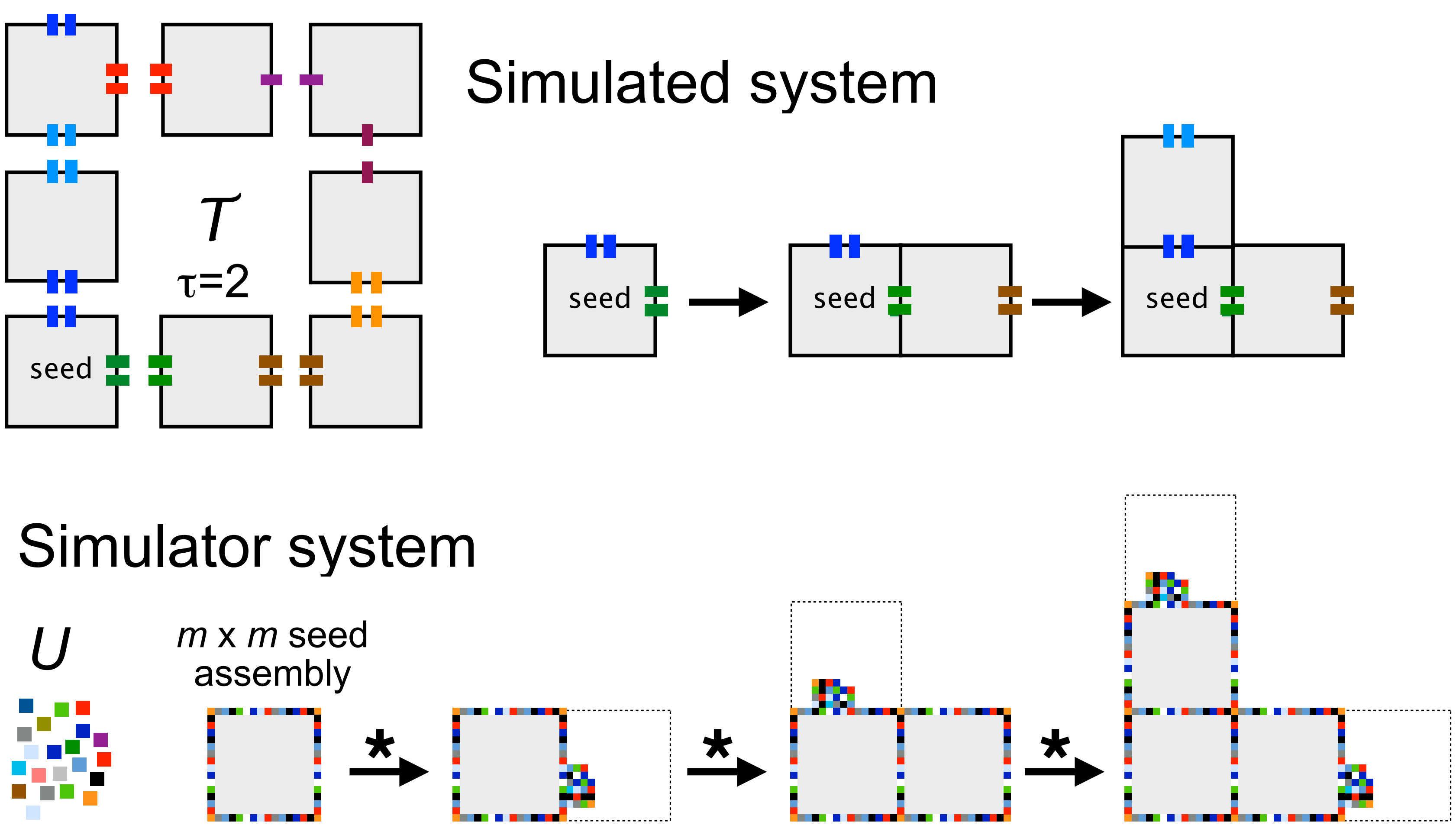

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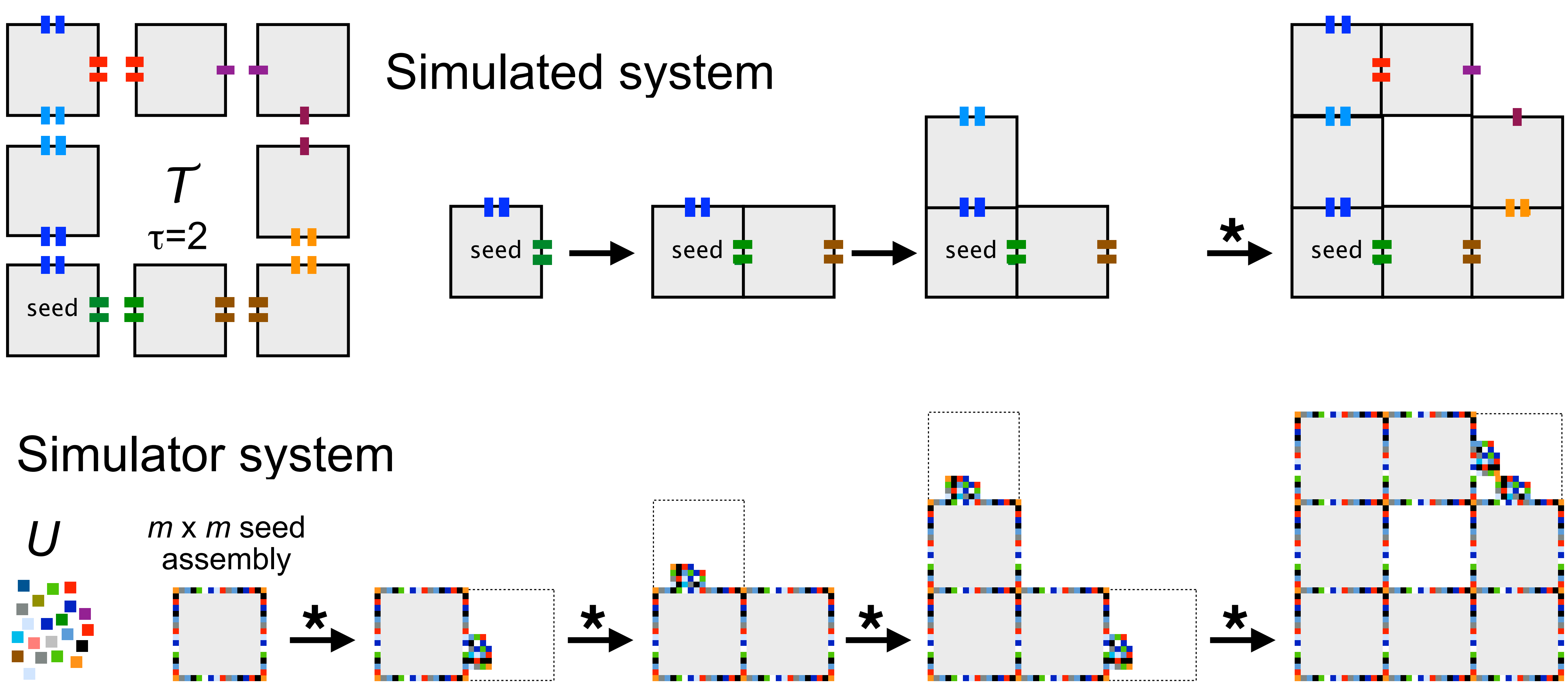

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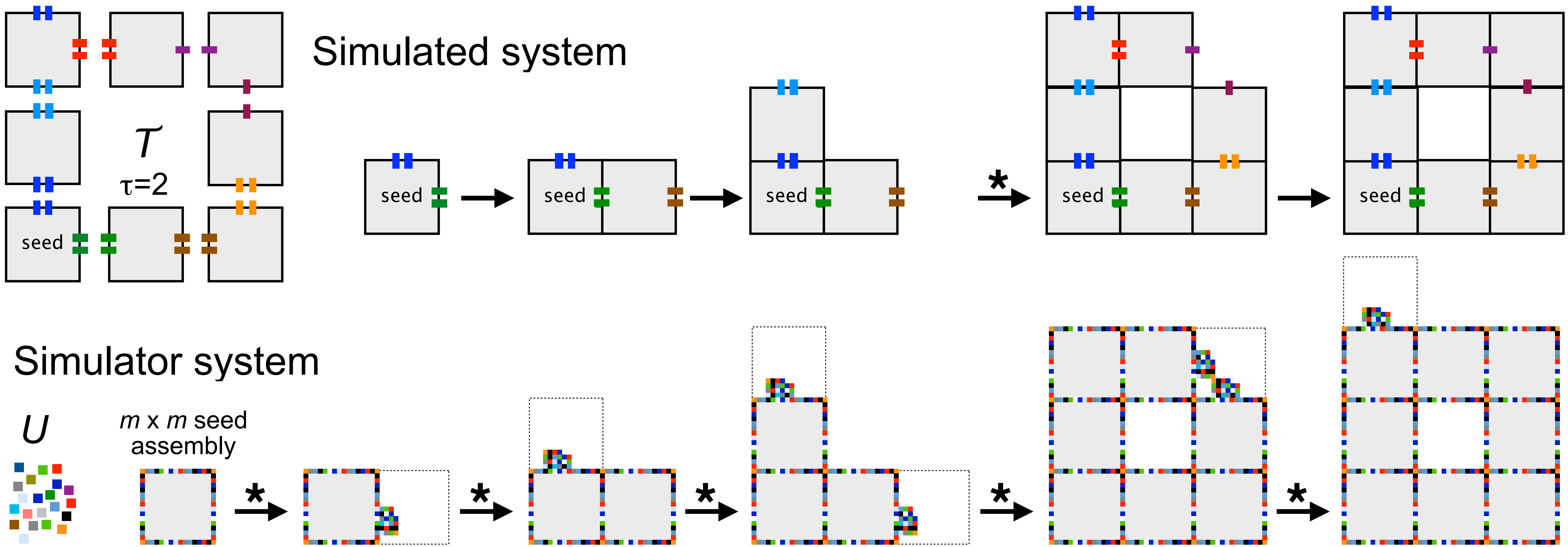
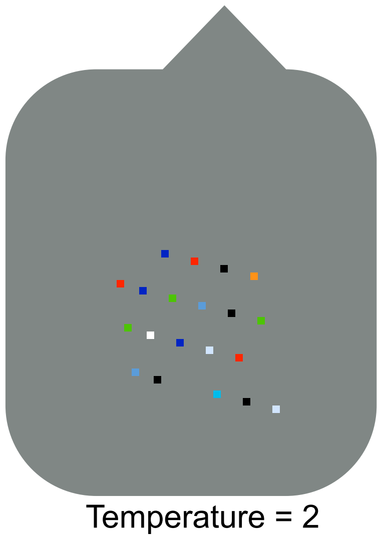
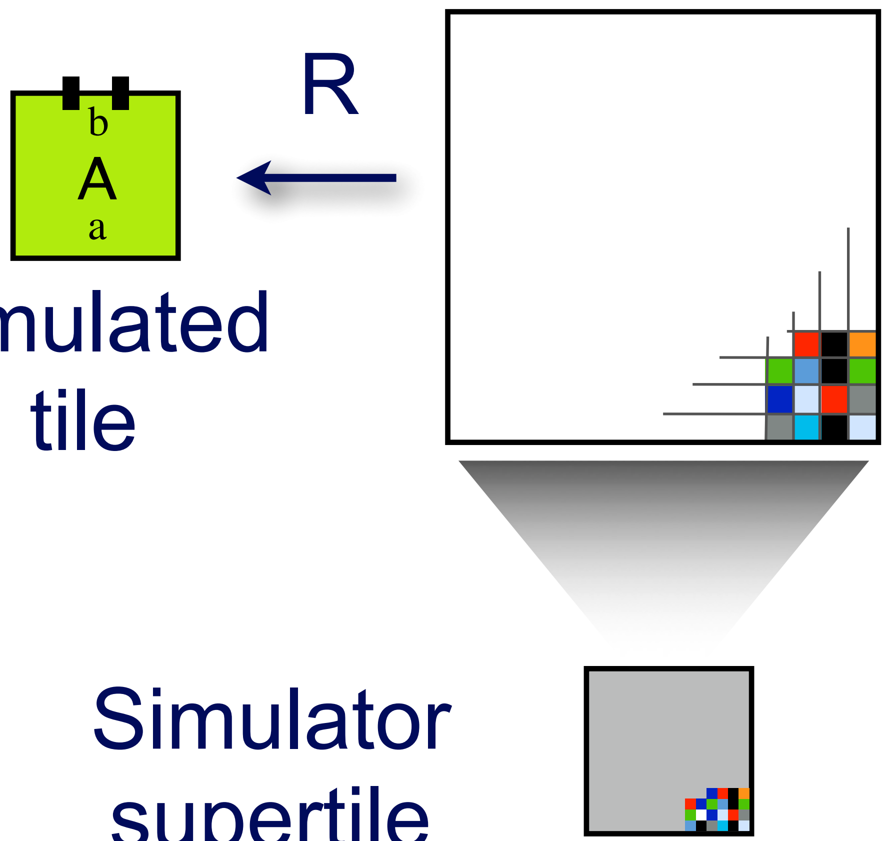


Simulation definition

Universal (simulator) tile set

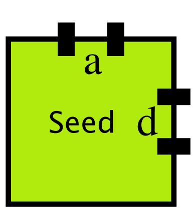




Simulated tile

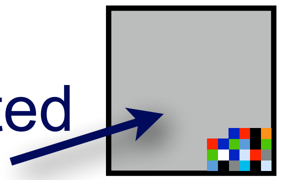
Simulator supertile

$\mathcal{T}$



Preassembled seed structure (encodes simulated TAS)

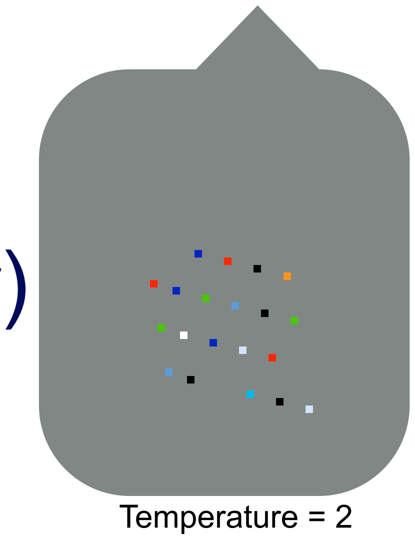
$\mathcal{U}$

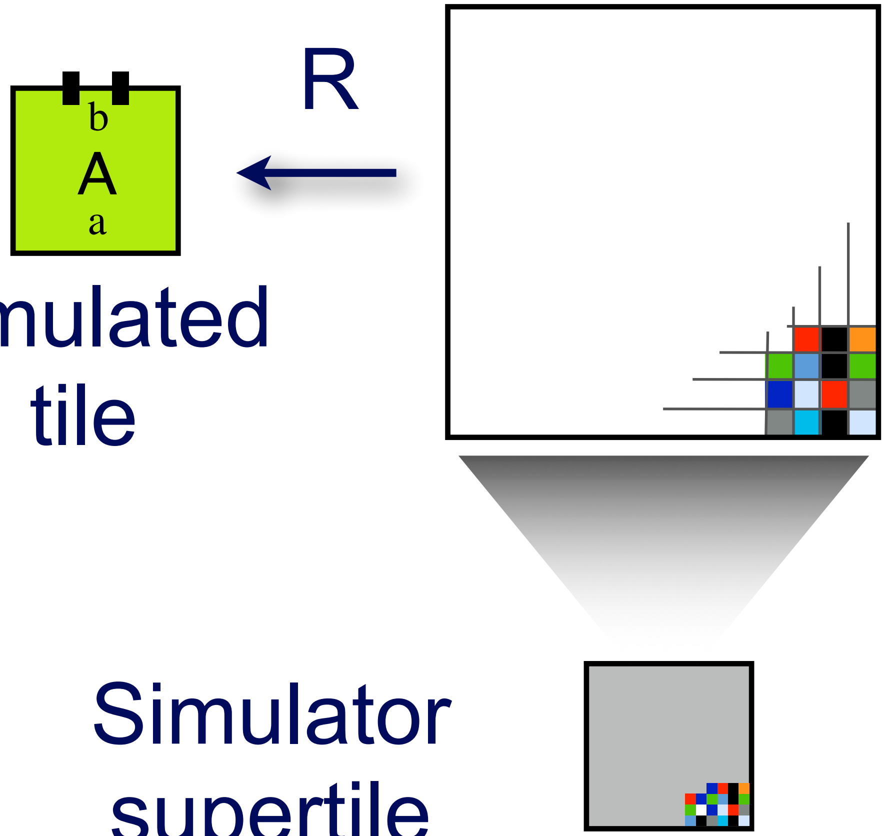


- Green tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation definition

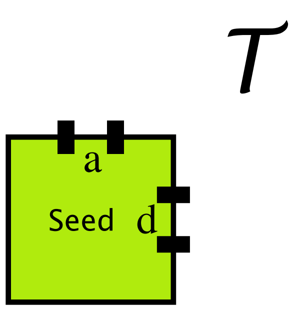
Universal (simulator) tile set



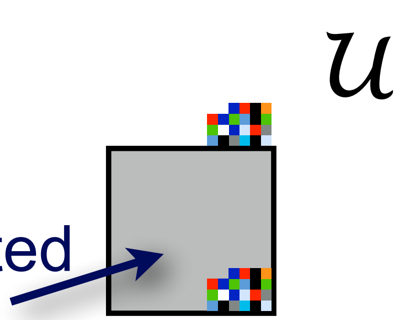


Simulated tile

Simulator supertile



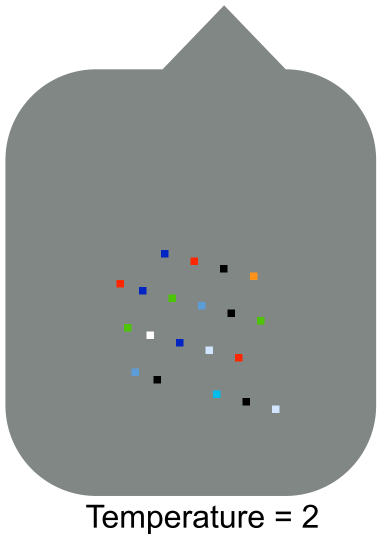
Preassembled seed structure (encodes simulated TAS)

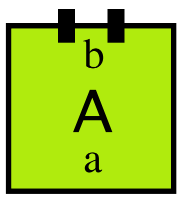


- Green tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

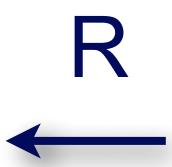
Simulation definition

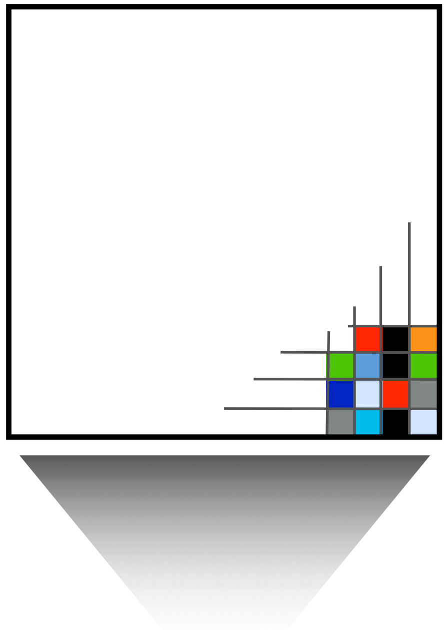
Universal (simulator) tile set



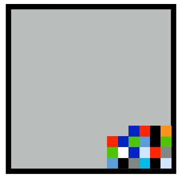


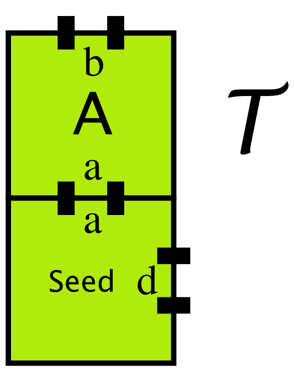
Simulated tile



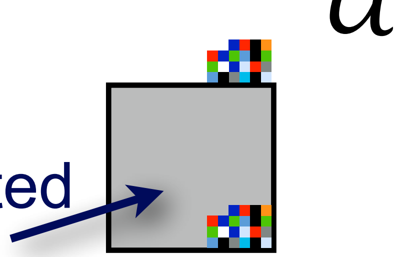


Simulator supertile





Preassembled seed structure (encodes simulated TAS)



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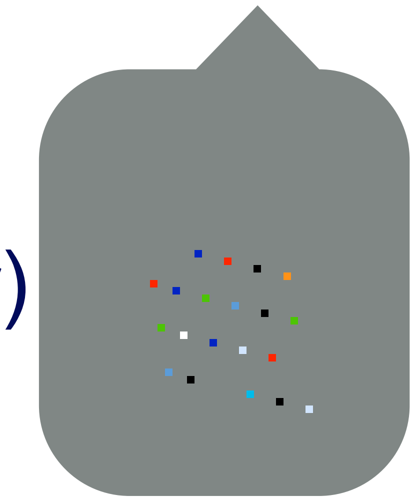
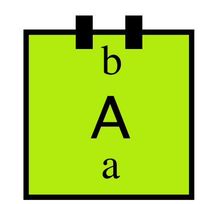
Simulation definition

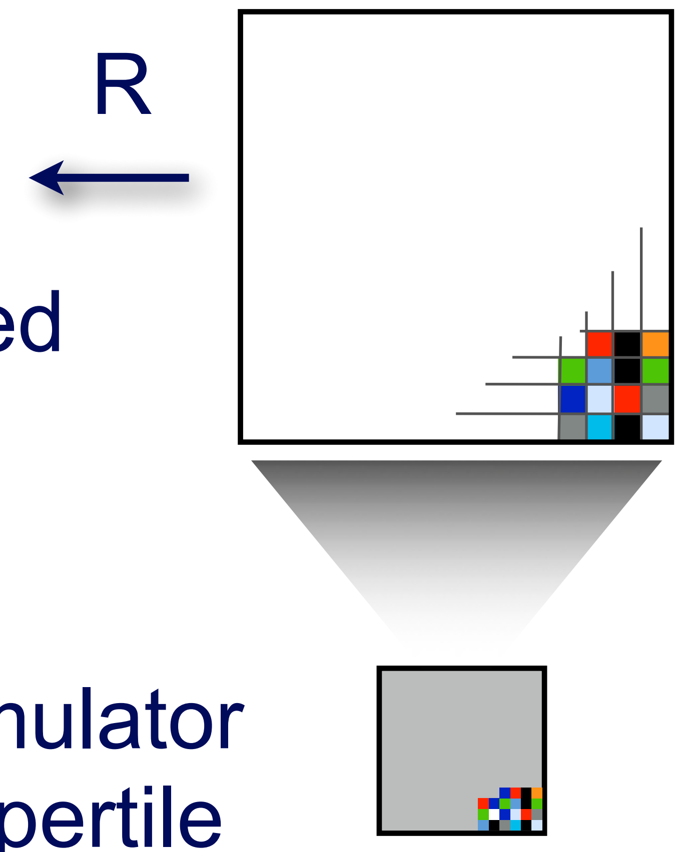
Universal (simulator) tile set



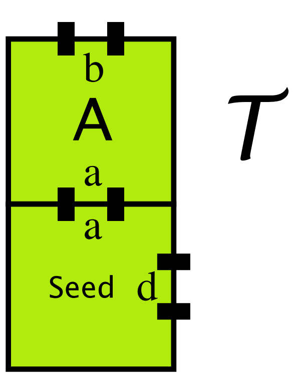
Temperature = 2



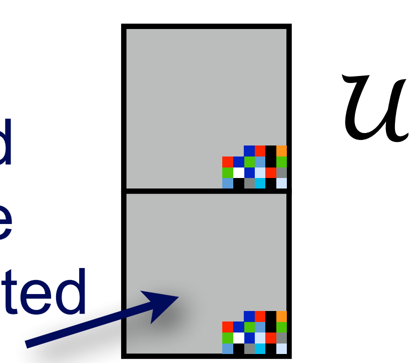
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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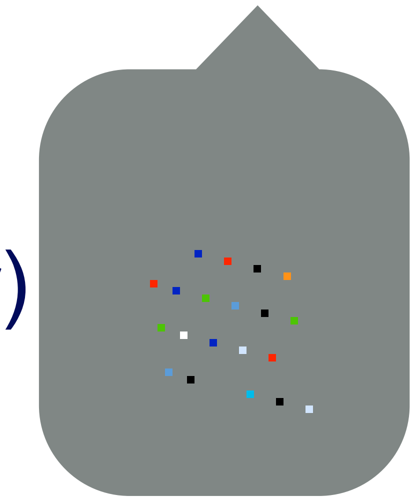
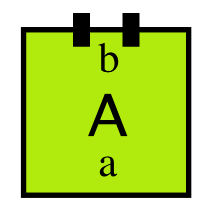
Simulation definition

Universal (simulator) tile set

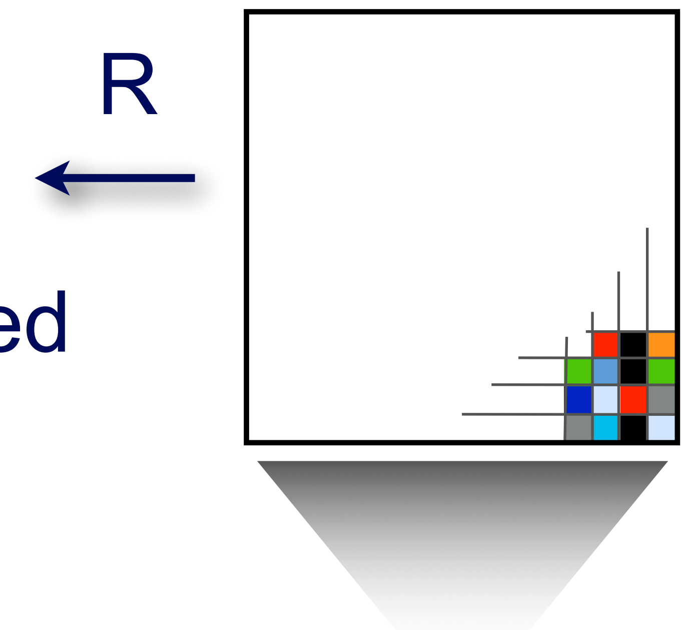


Temperature = 2

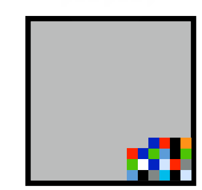


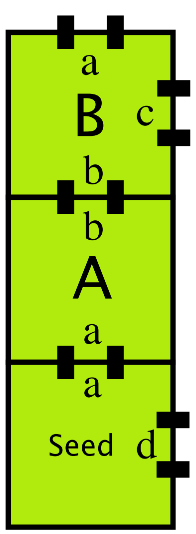
Simulated tile

R



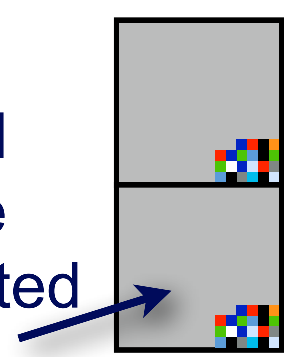
Simulator supertile





$\mathcal{T}$

Preassembled seed structure (encodes simulated TAS)



$\mathcal{U}$

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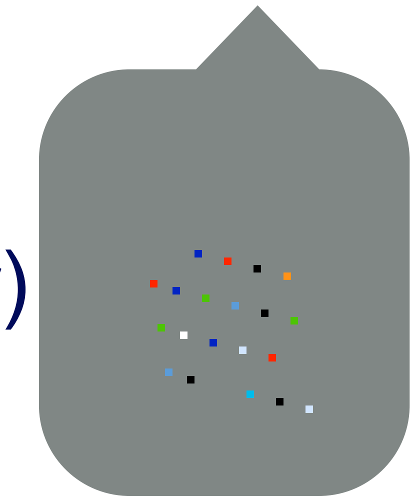
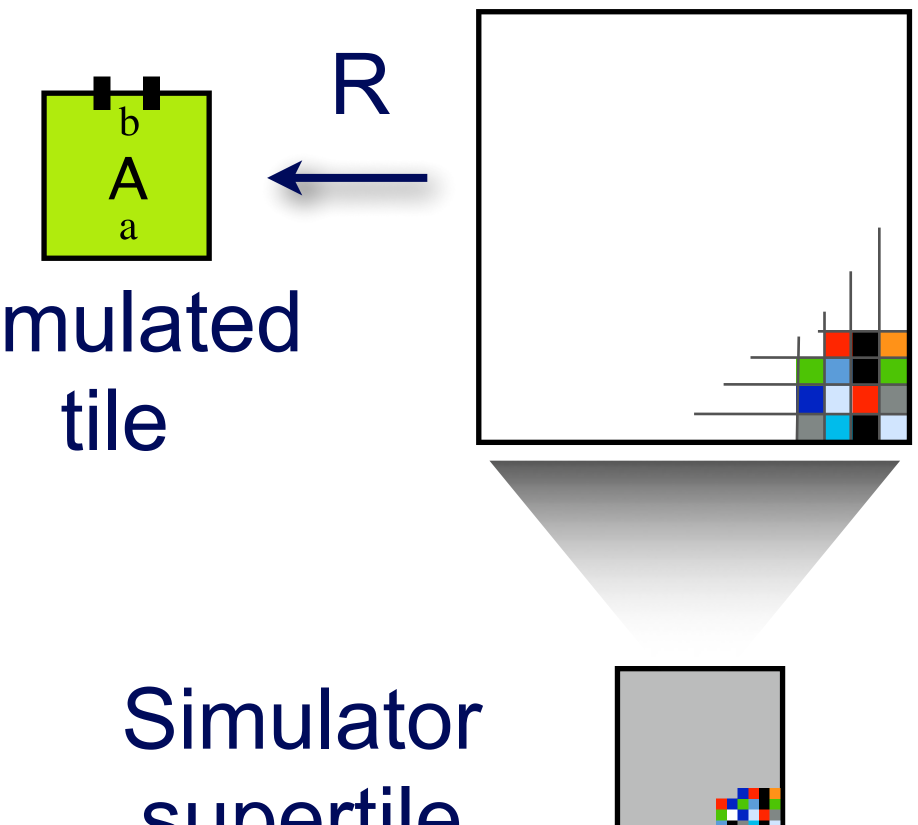
Simulation definition

Universal (simulator) tile set

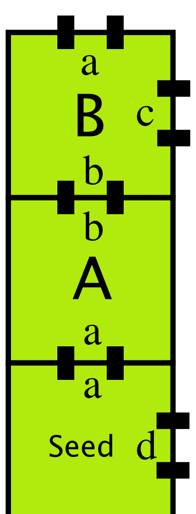


Temperature = 2



Simulated tile

Simulator supertile



$\mathcal{T}$

Preassembled seed structure (encodes simulated TAS)

$\mathcal{U}$

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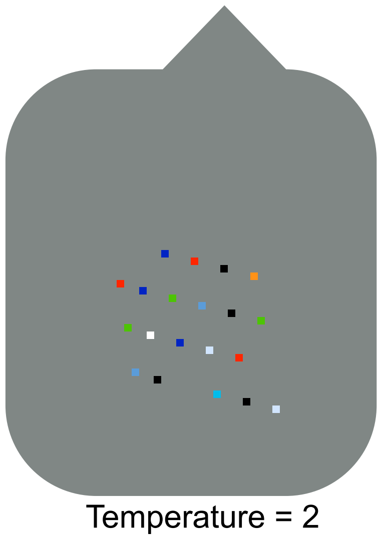
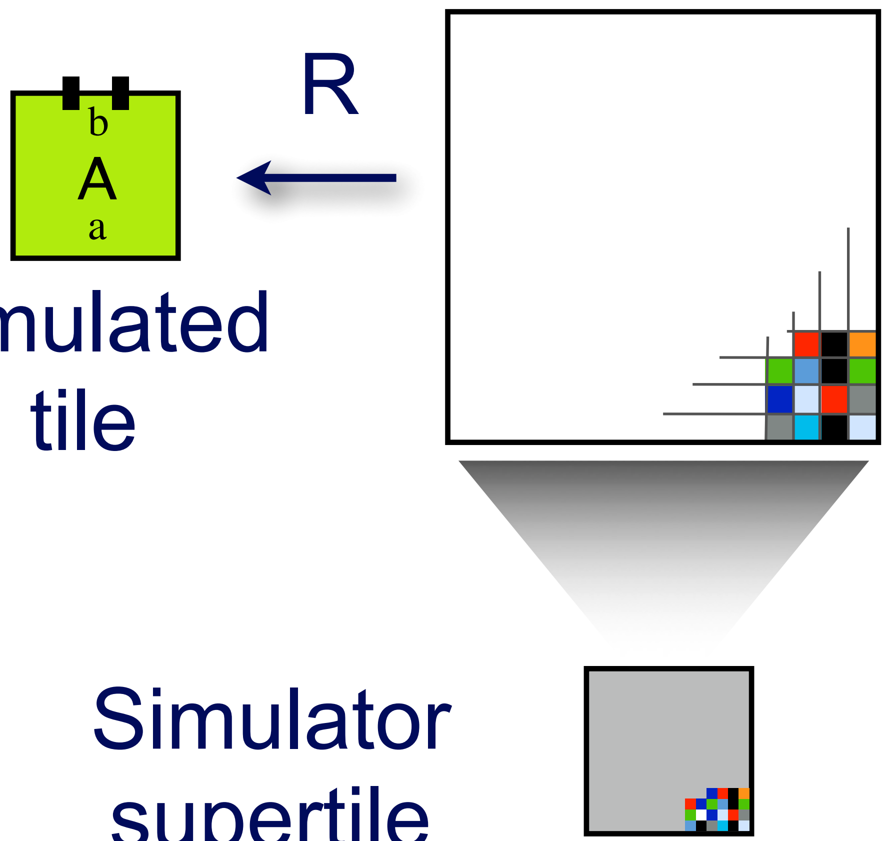
Simulation definition

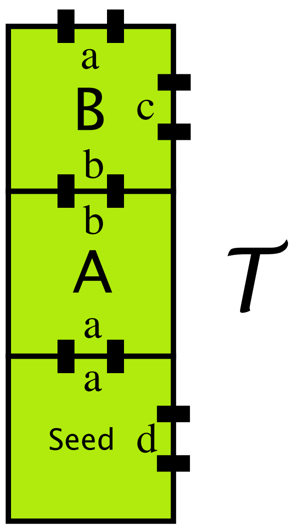
Universal (simulator) tile set



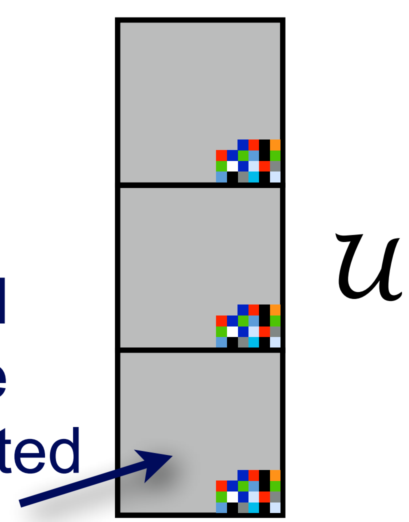


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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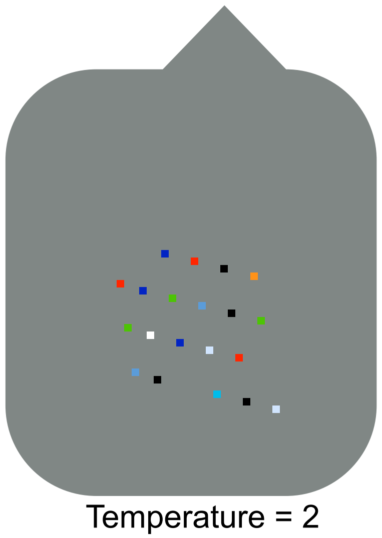
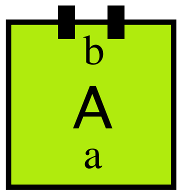
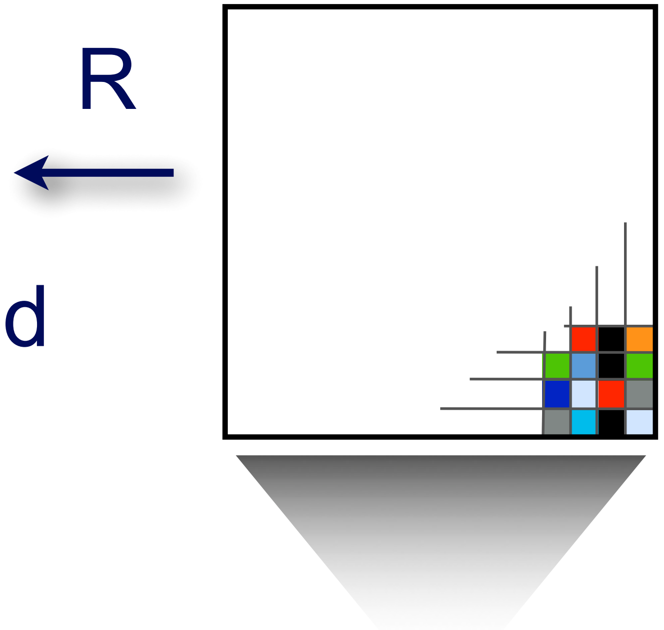
Simulation definition

Universal (simulator) tile set



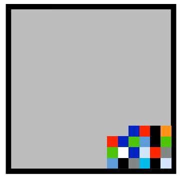


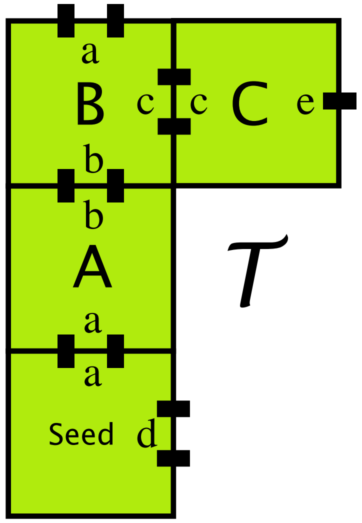
R



Simulated tile

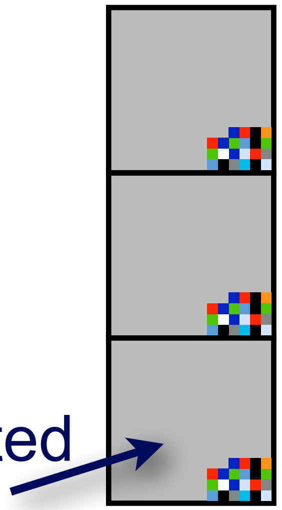
Simulator supertile





$\mathcal{T}$

Preassembled seed structure (encodes simulated TAS)



$\mathcal{U}$

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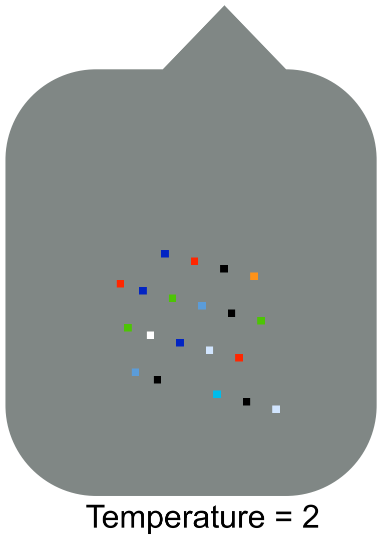
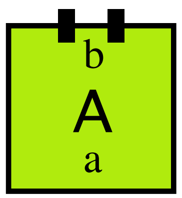
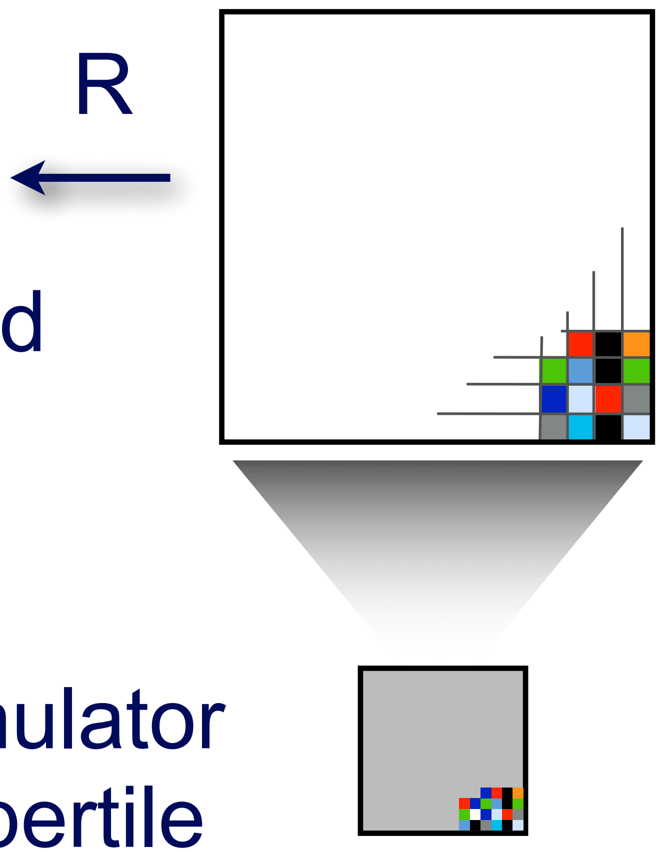
Simulation definition

Universal (simulator) tile set

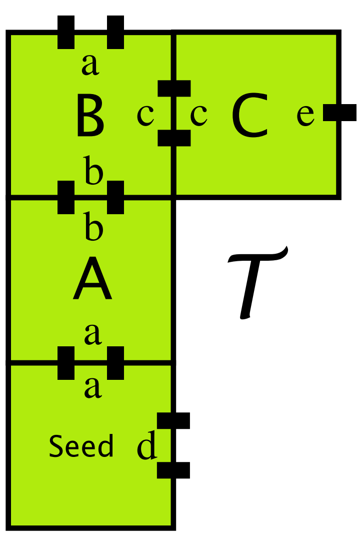




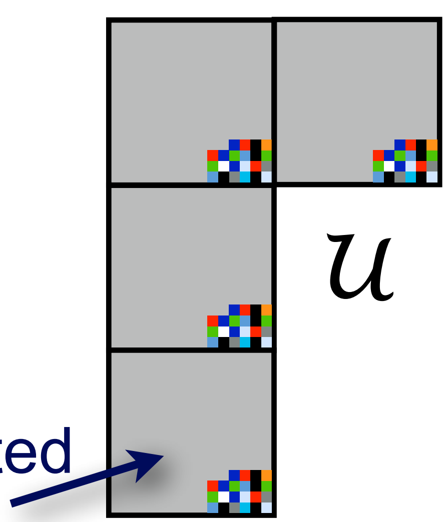
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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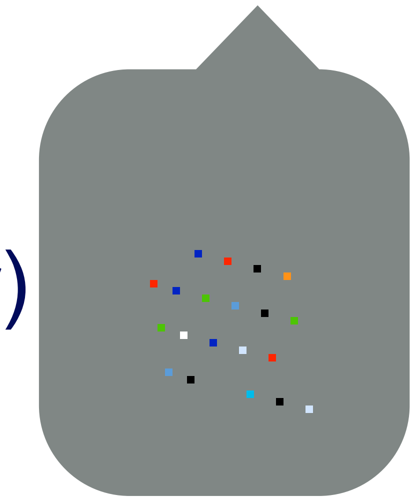
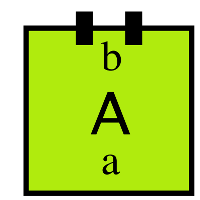
Simulation definition

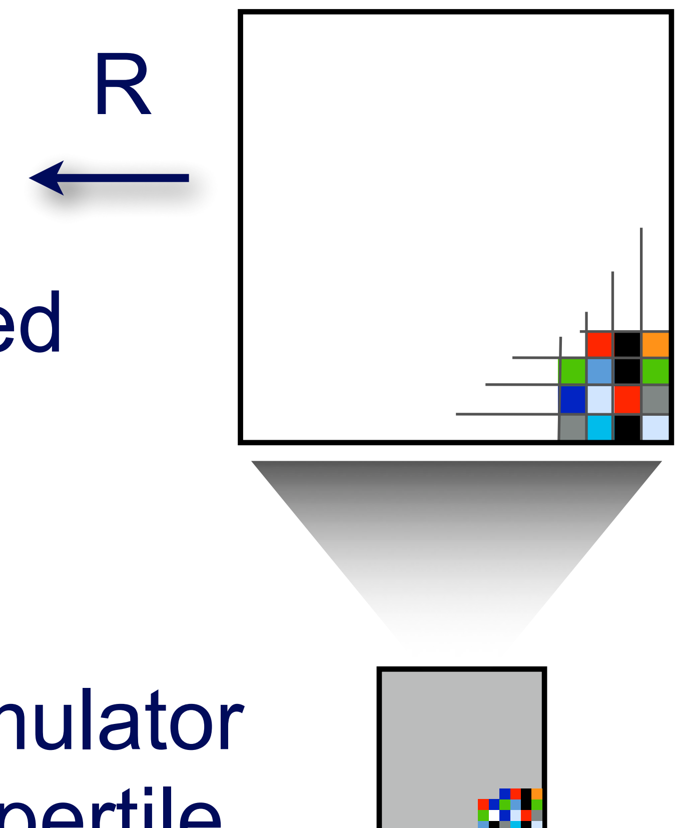
Universal (simulator) tile set



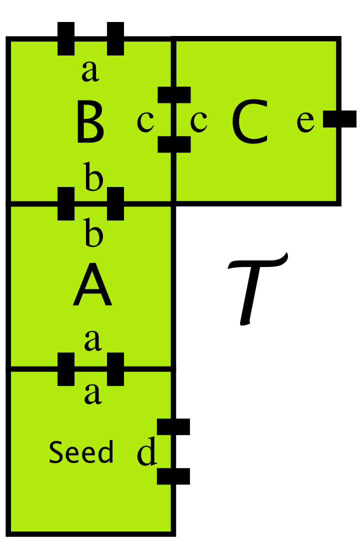
Temperature = 2



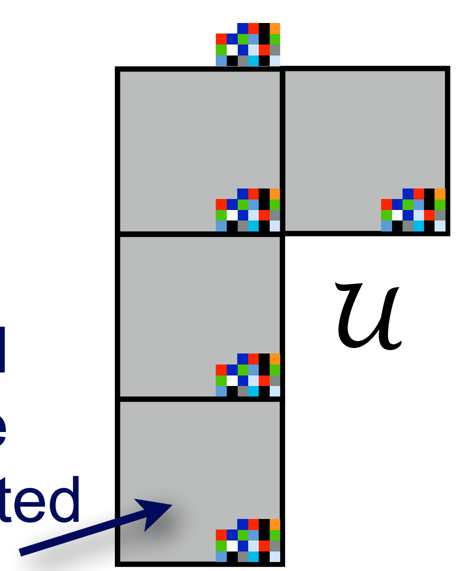
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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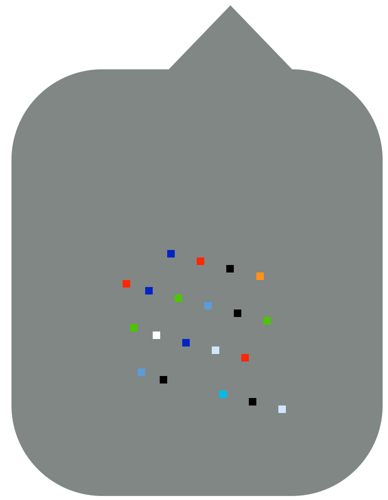
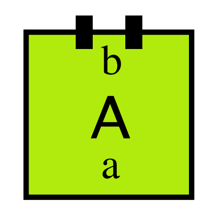
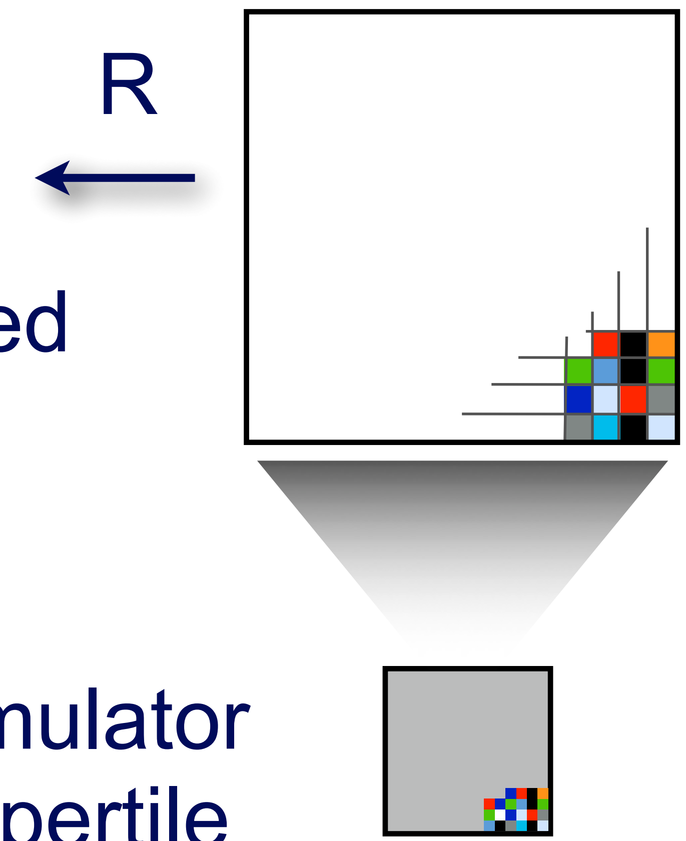


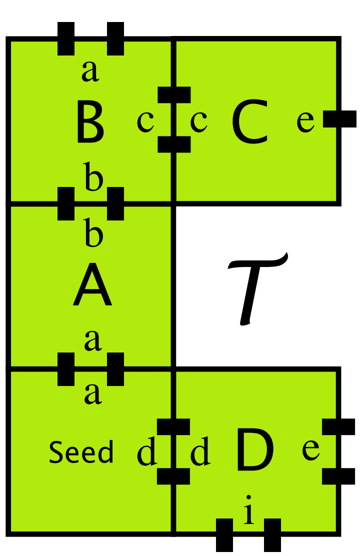
Temperature = 2



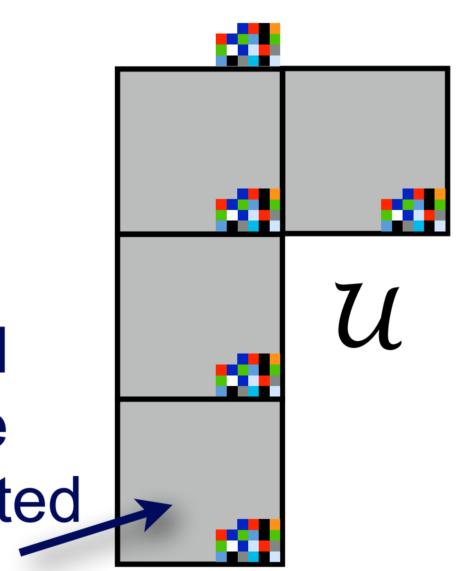


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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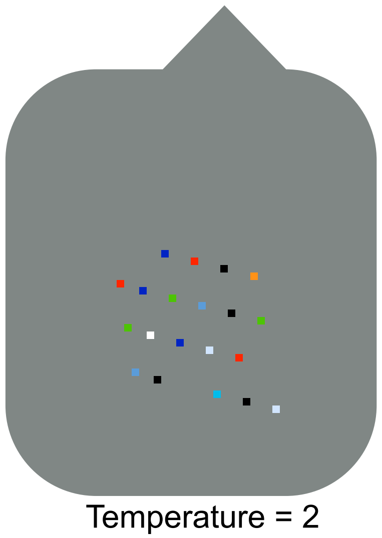
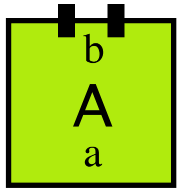
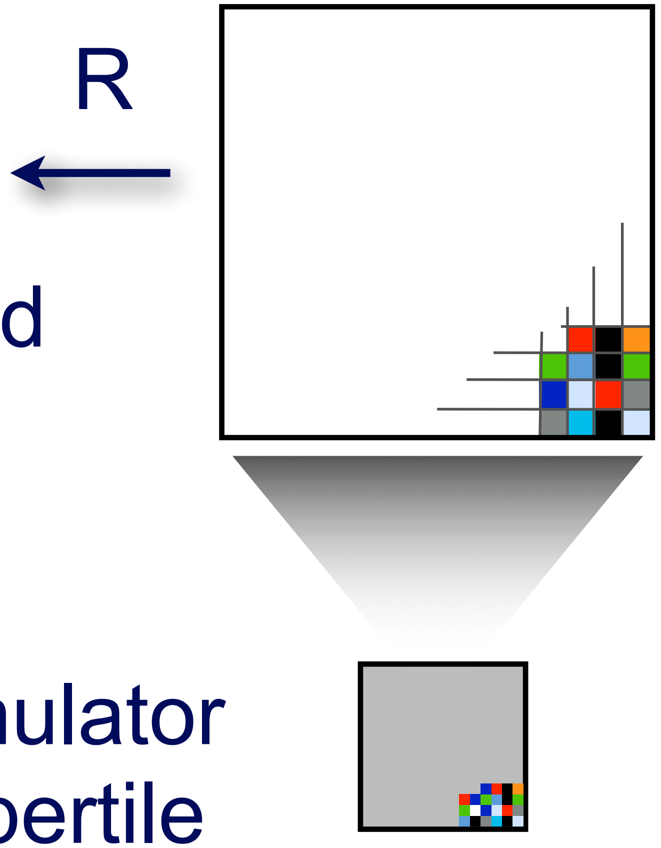
Simulation definition

Universal (simulator) tile set

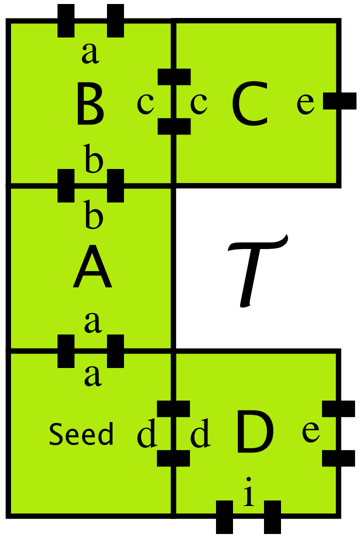




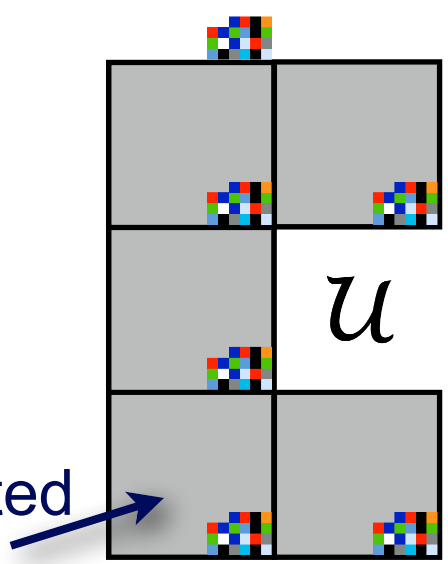
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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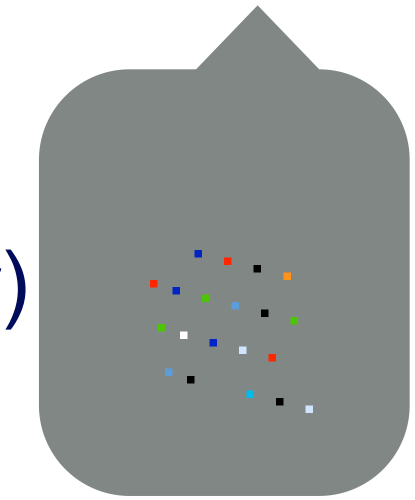
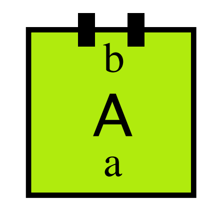
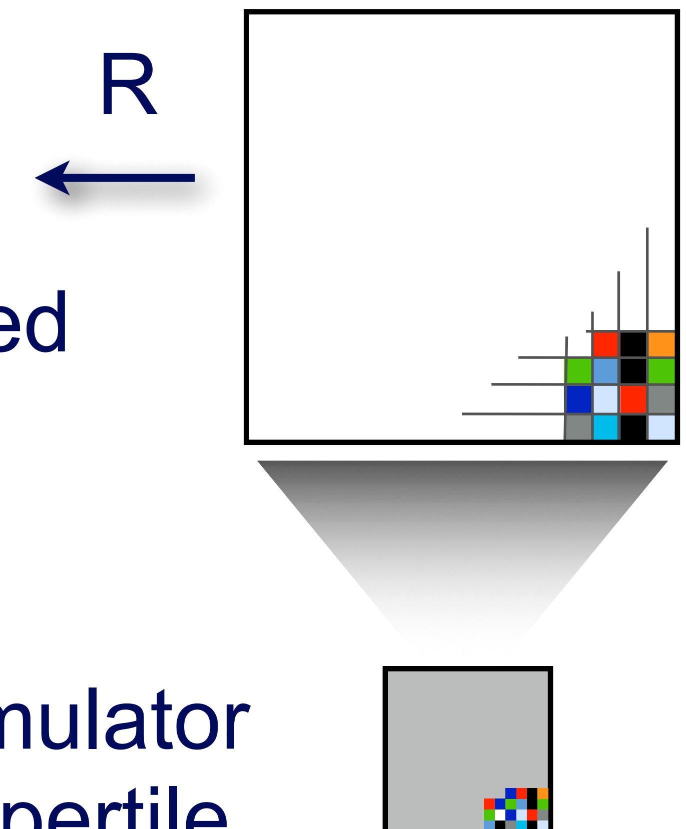


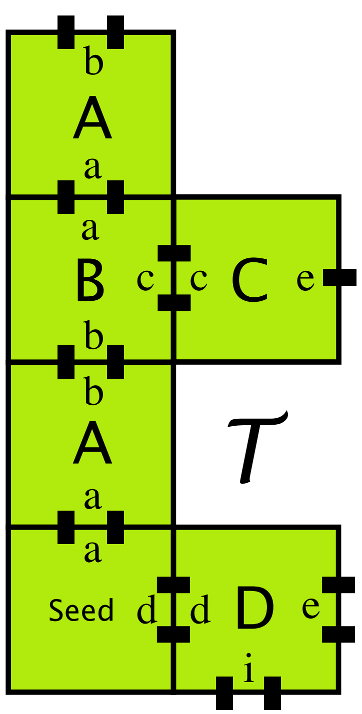
Temperature = 2



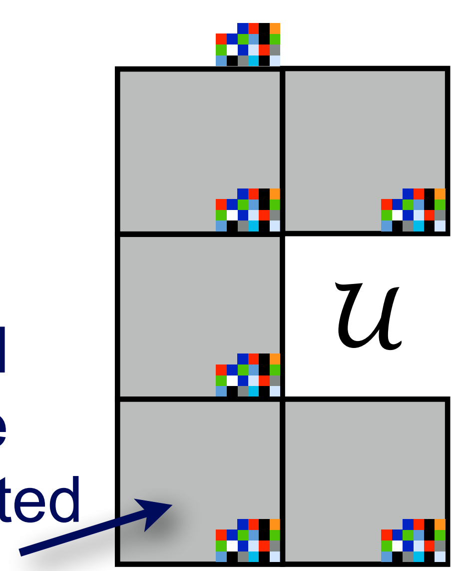


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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Simulation definition

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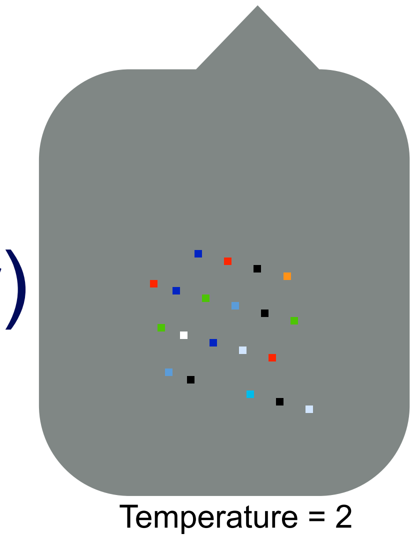
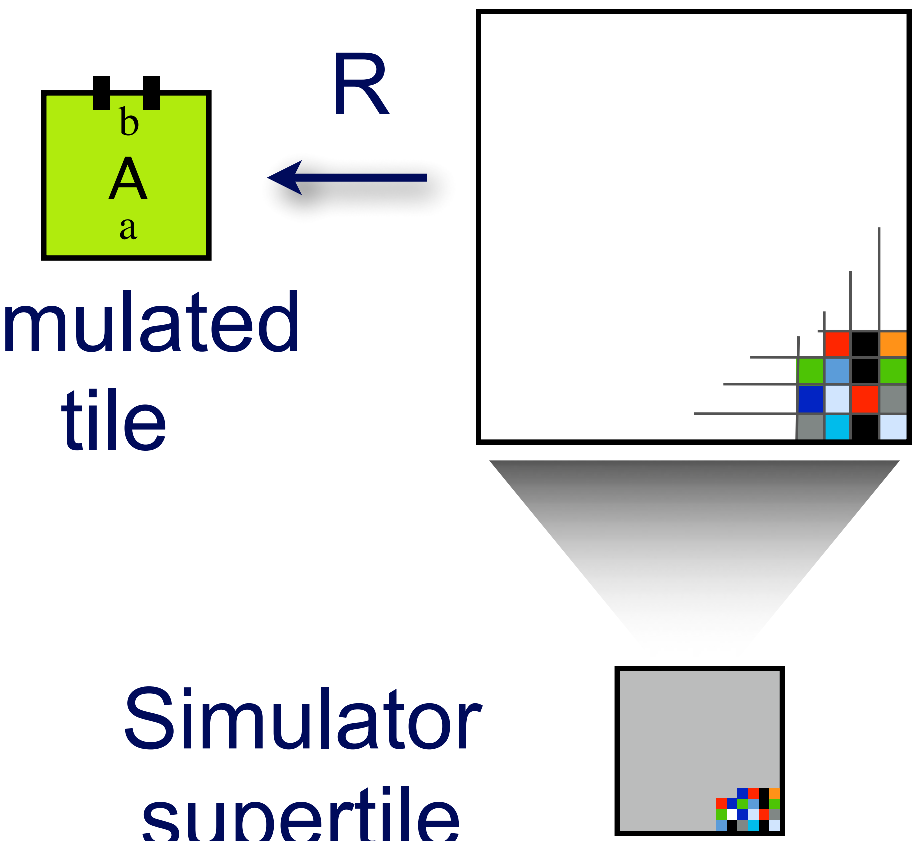
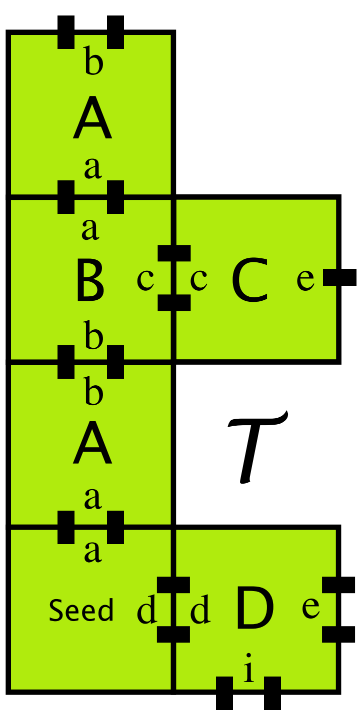




Simulated tile

Simulator supertile



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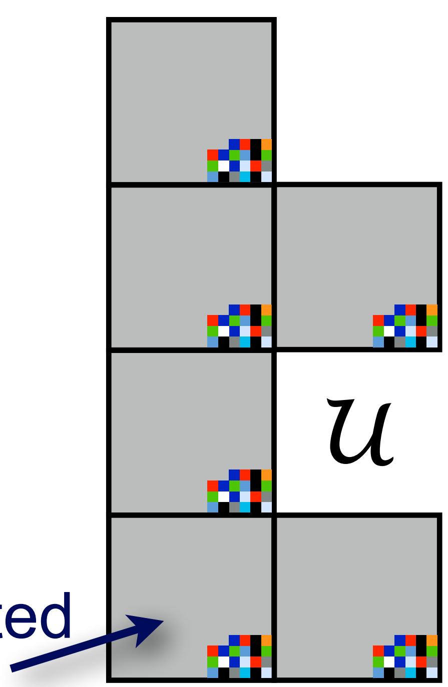

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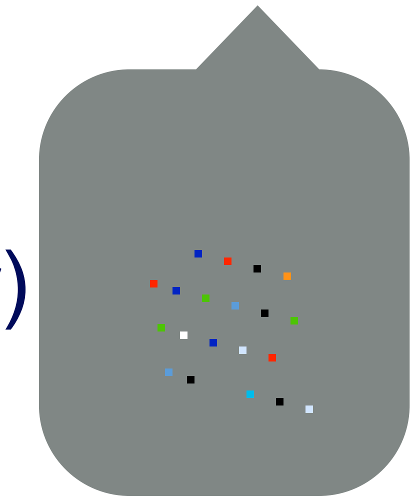
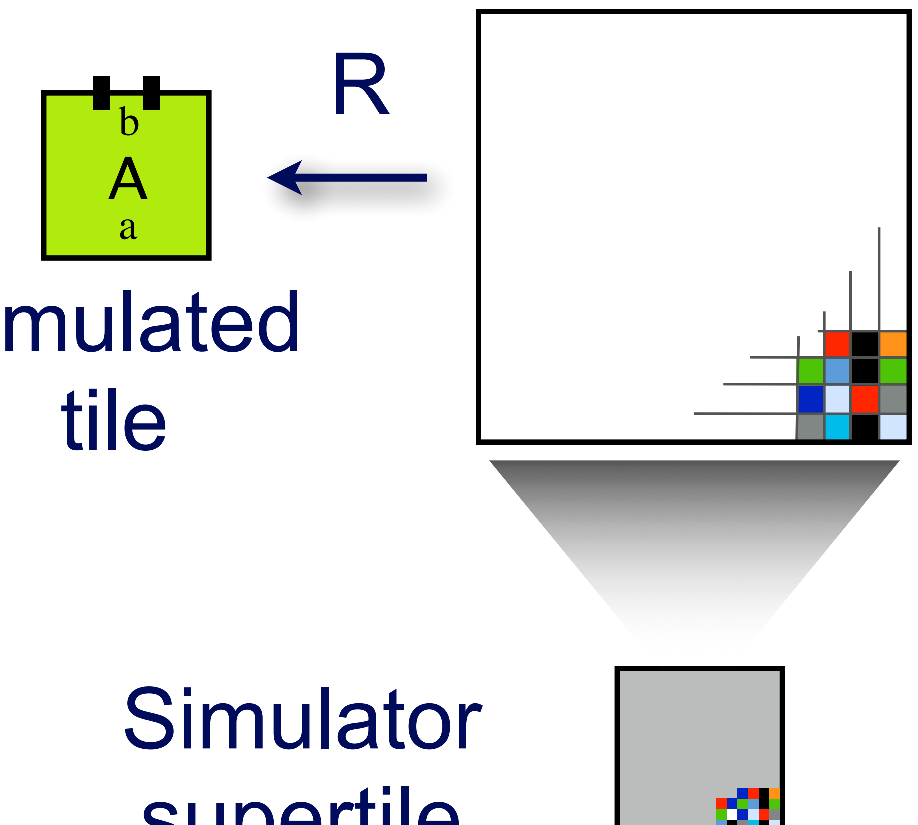
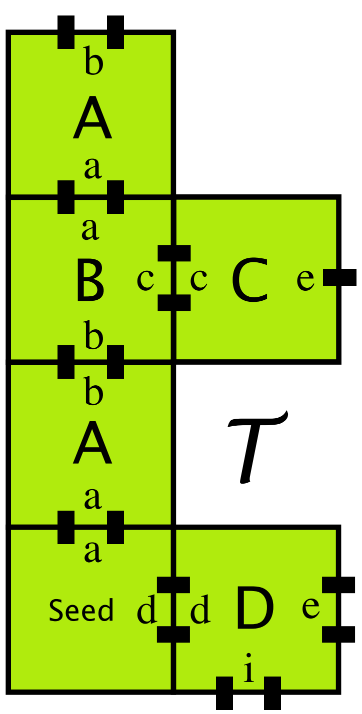


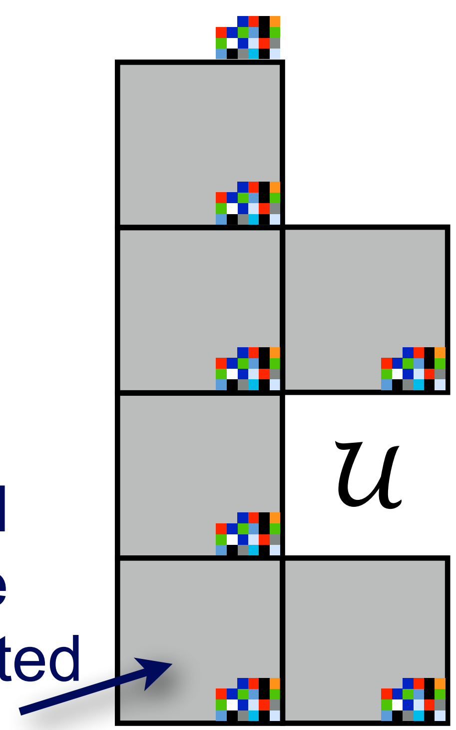


Simulated tile

Simulator supertile



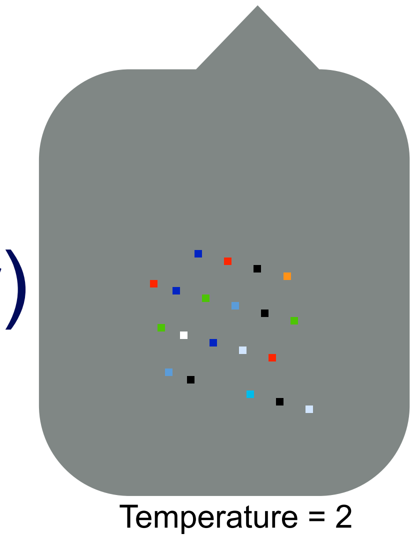
Preassembled seed structure (encodes simulated TAS)

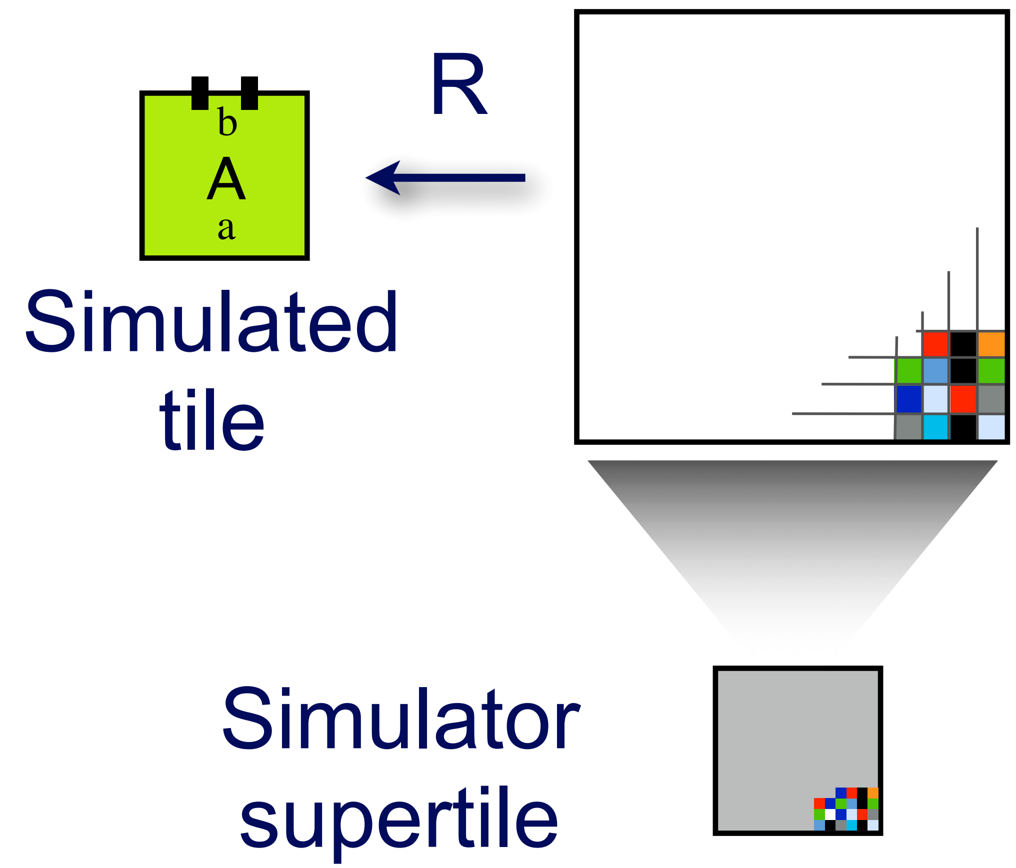


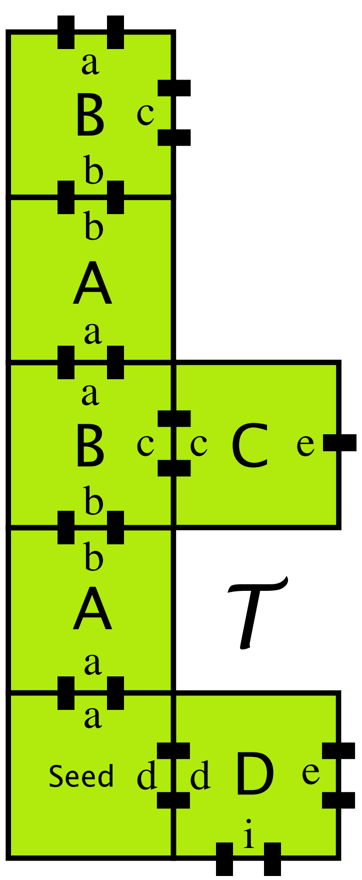
- Green tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation definition

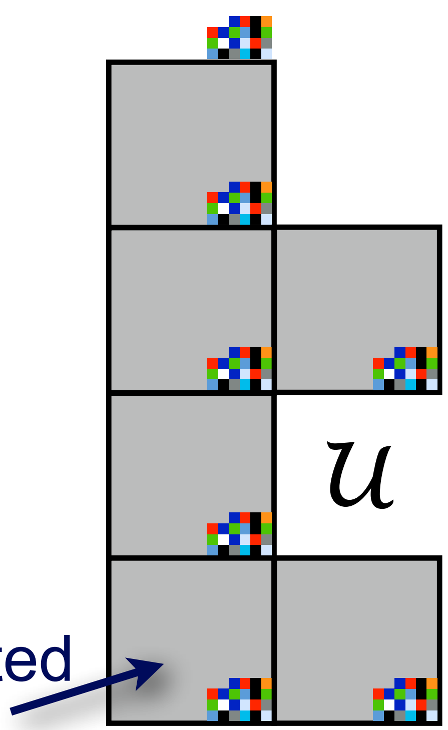
Universal (simulator) tile set







Preassembled seed structure (encodes simulated TAS)



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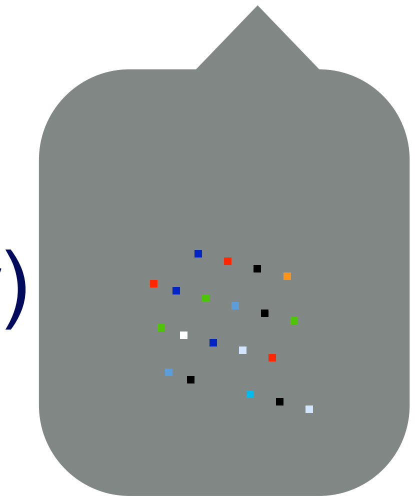
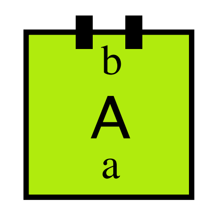
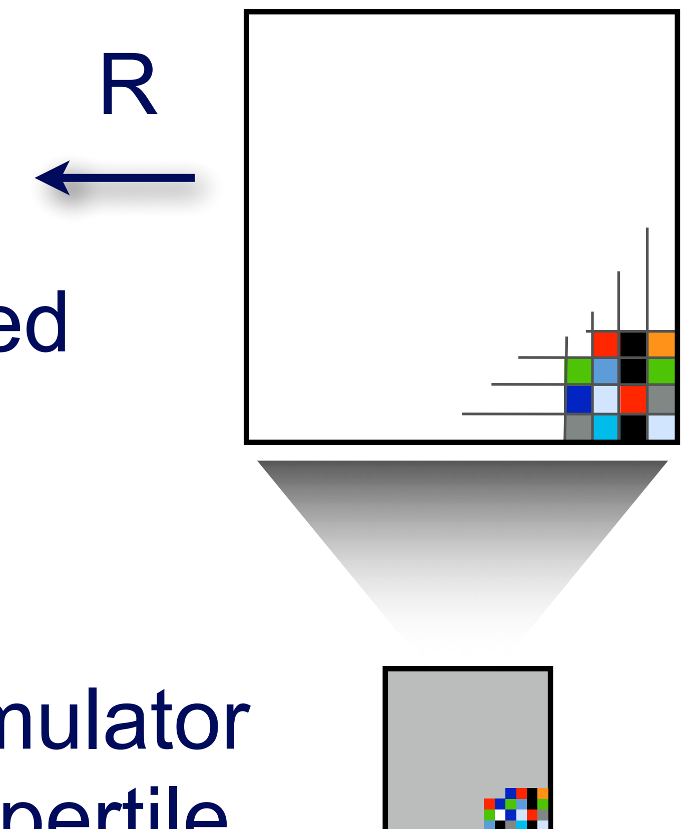
Simulation definition

Universal (simulator) tile set

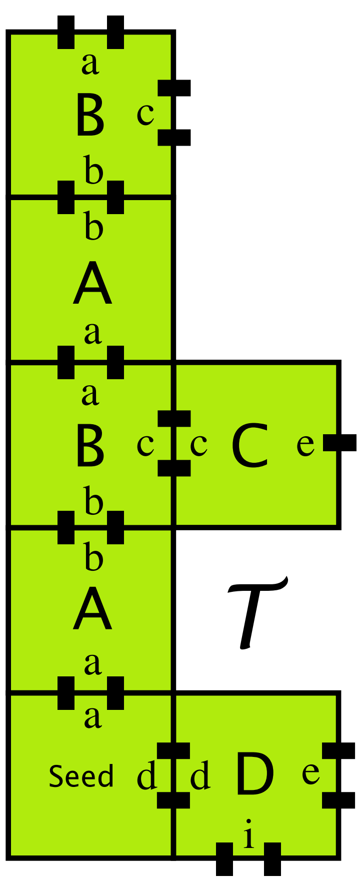




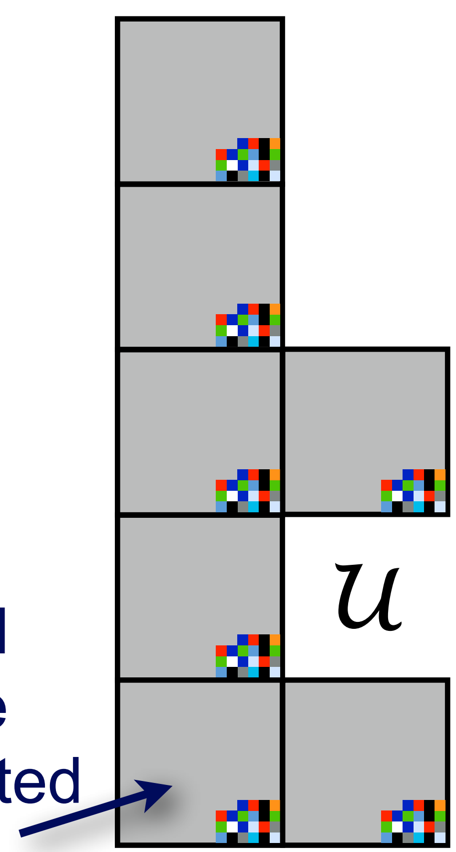
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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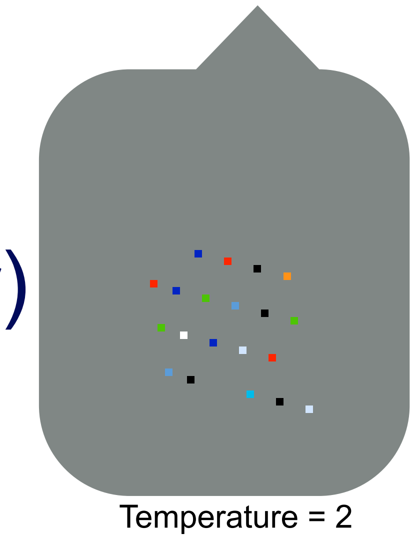
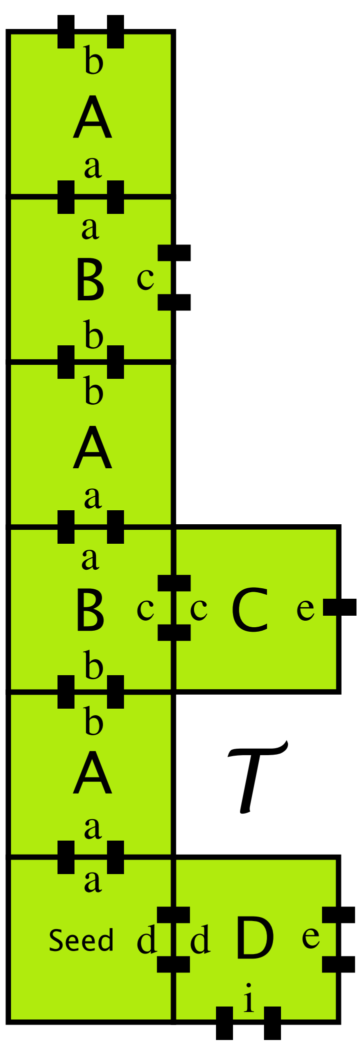
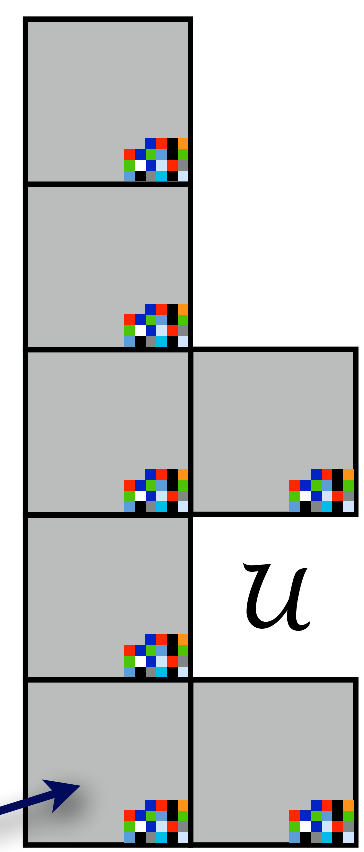
Simulation definition

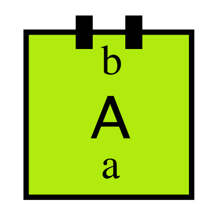
Universal (simulator) tile set





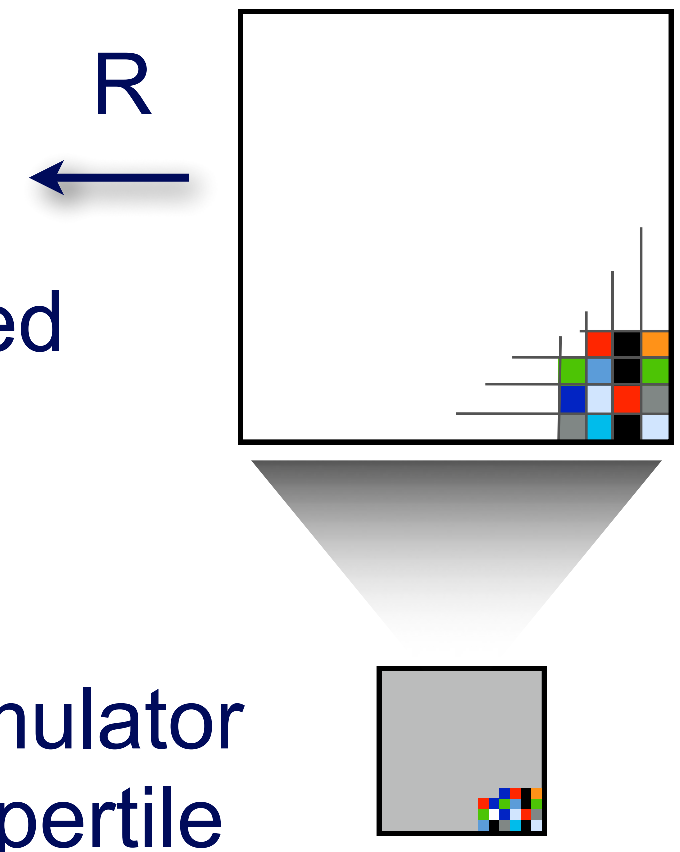
Preassembled seed structure (encodes simulated TAS)





Simulated tile

R



Simulator supertile

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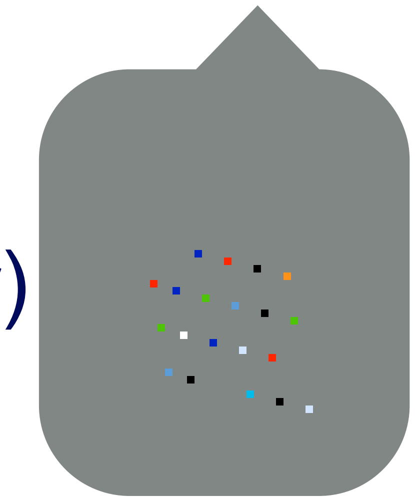
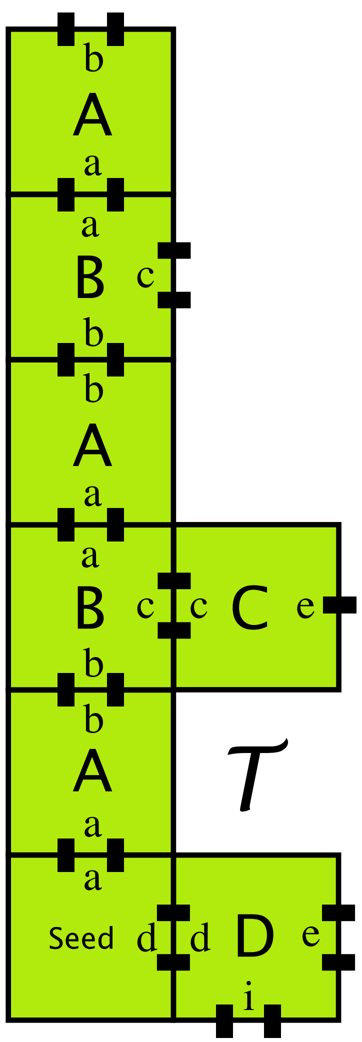
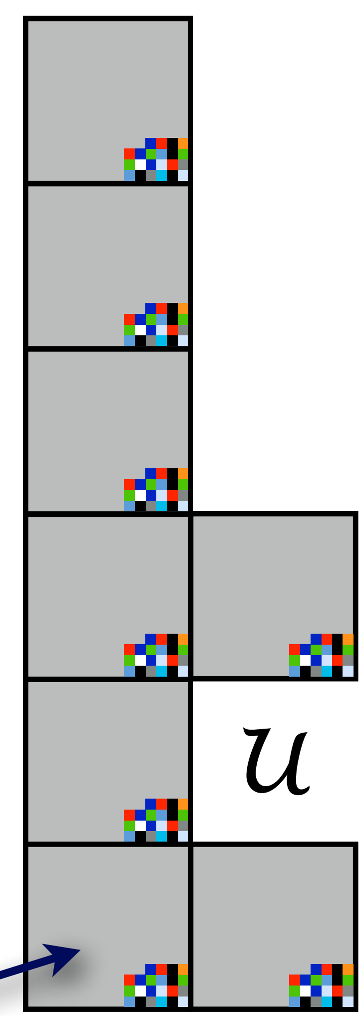
Simulation definition

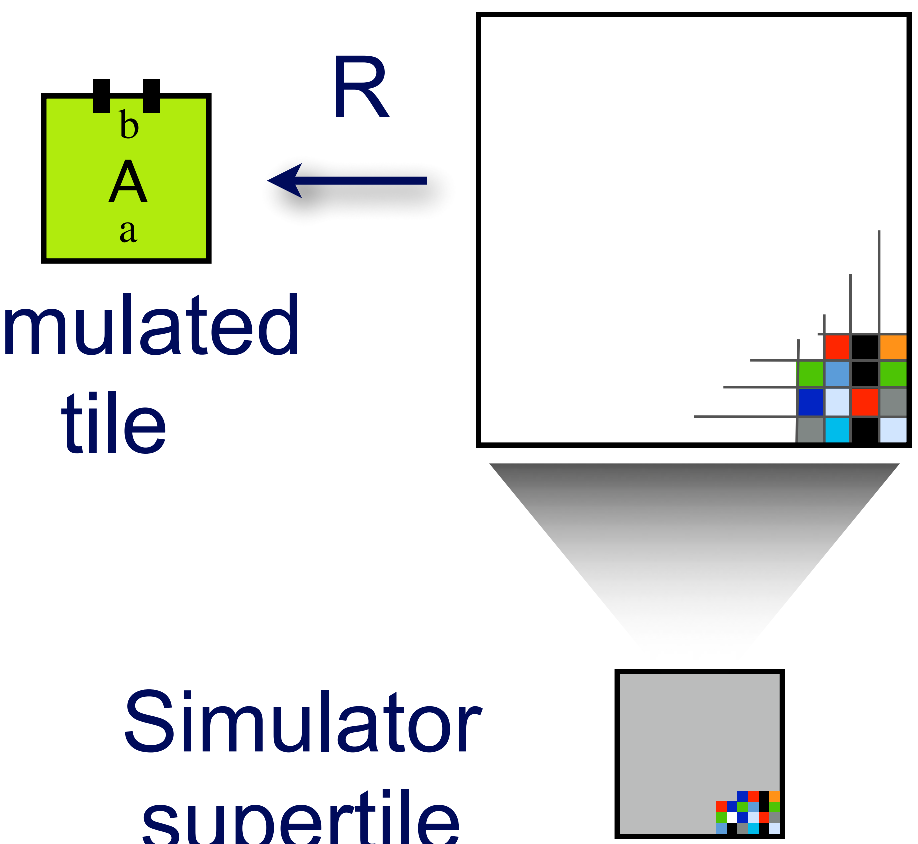
Universal (simulator) tile set





Preassembled seed structure (encodes simulated TAS)





Simulated tile

Simulator supertile

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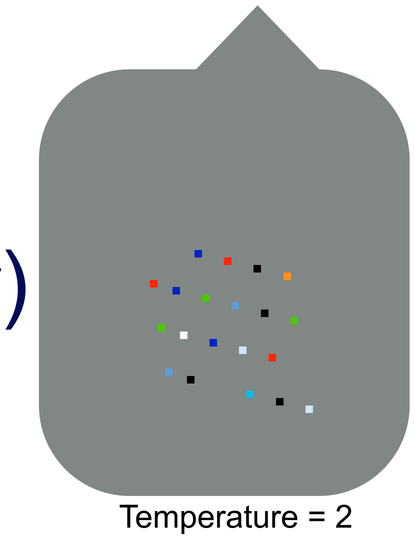
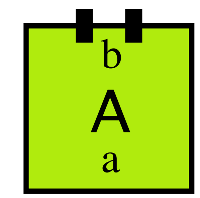
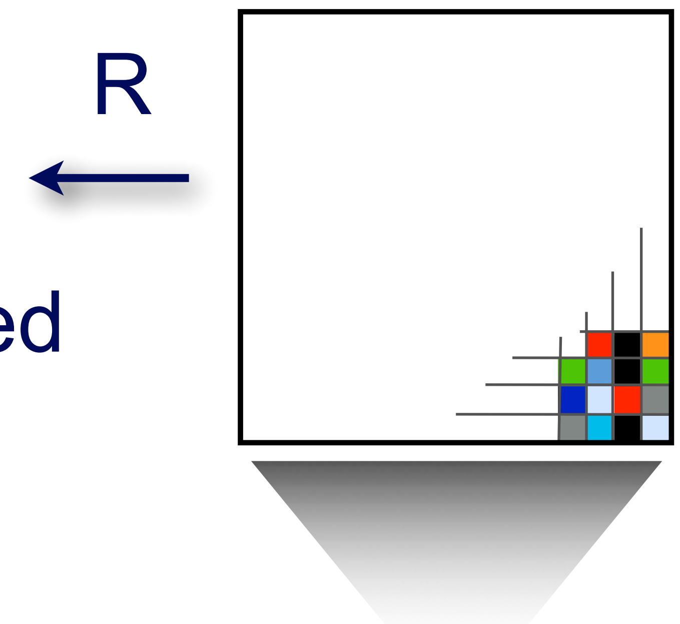
Simulation definition

Universal (simulator) tile set



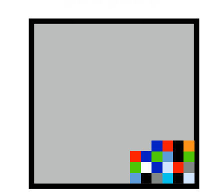


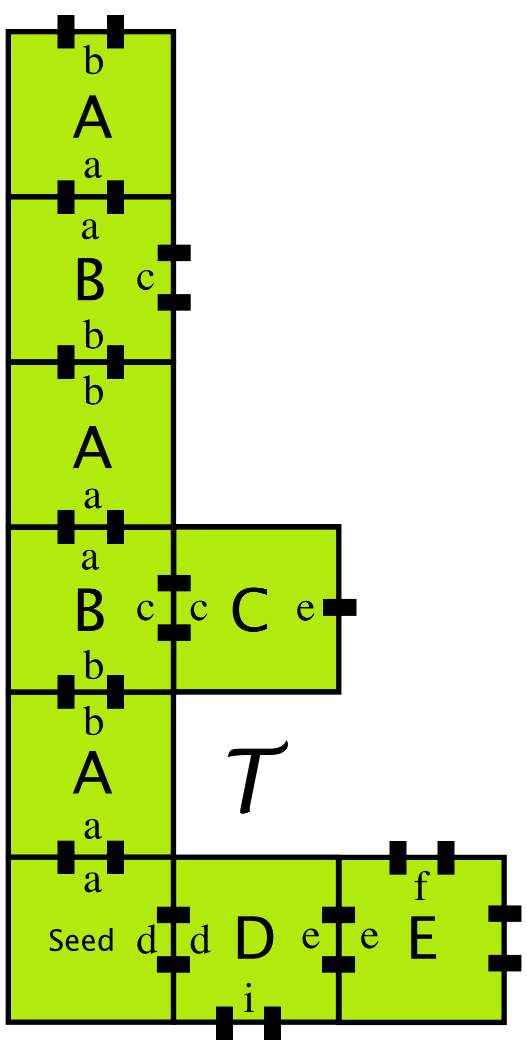
R



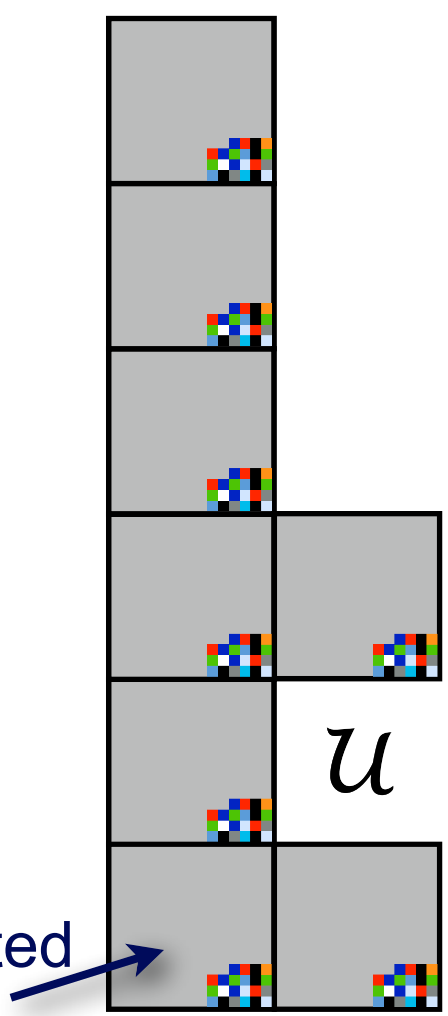
Simulated tile

Simulator supertile





Preassembled seed structure (encodes simulated TAS)



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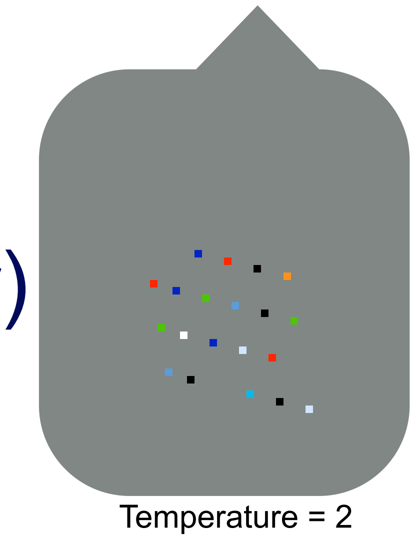
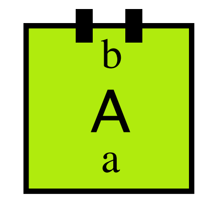
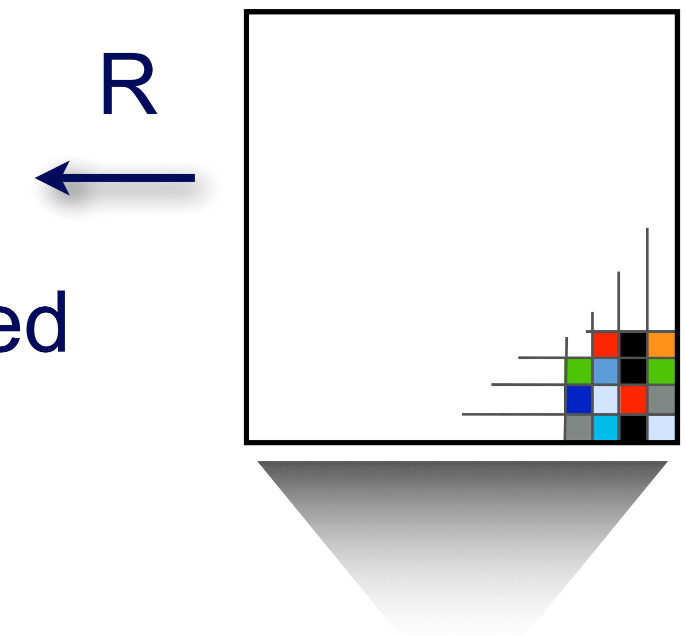
Simulation definition

Universal (simulator) tile set

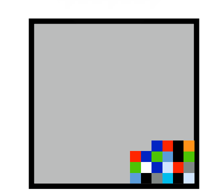


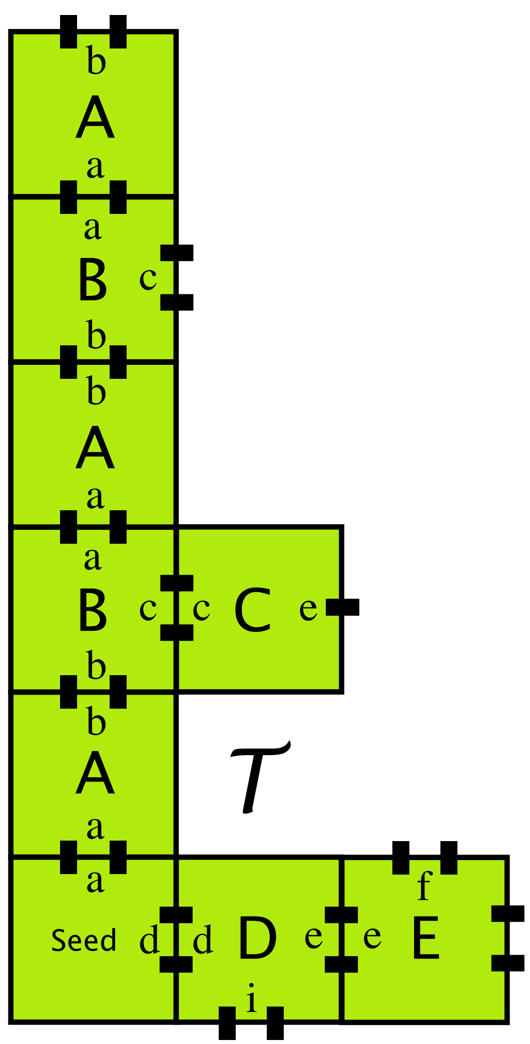




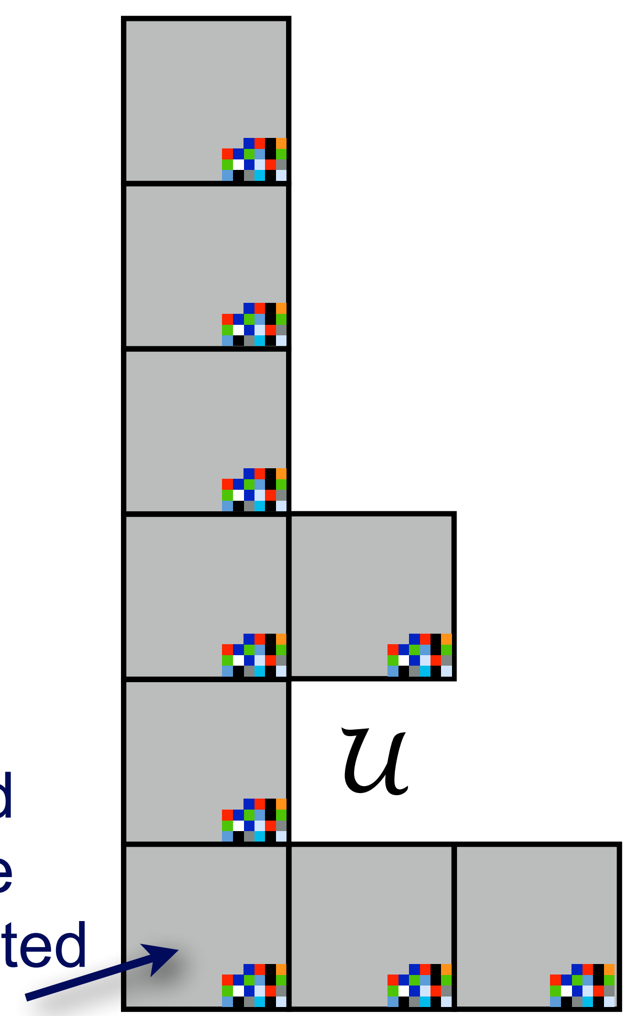
Simulated tile

Simulator supertile





Preassembled seed structure (encodes simulated TAS)



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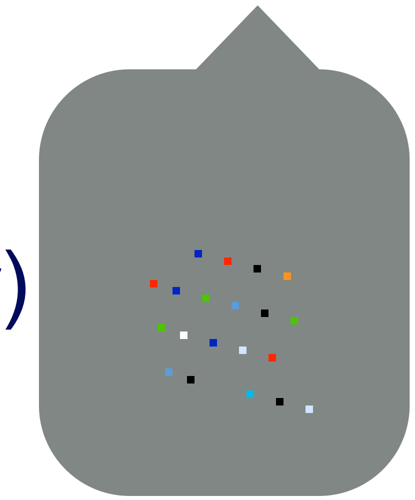
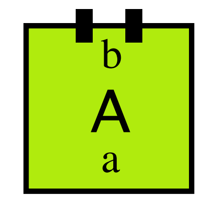
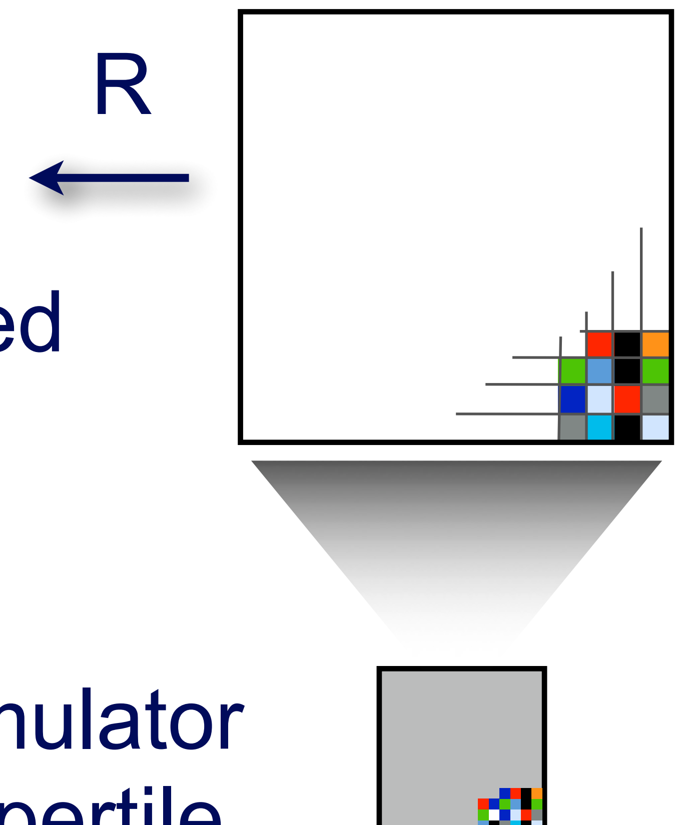
Simulation definition

Universal (simulator) tile set

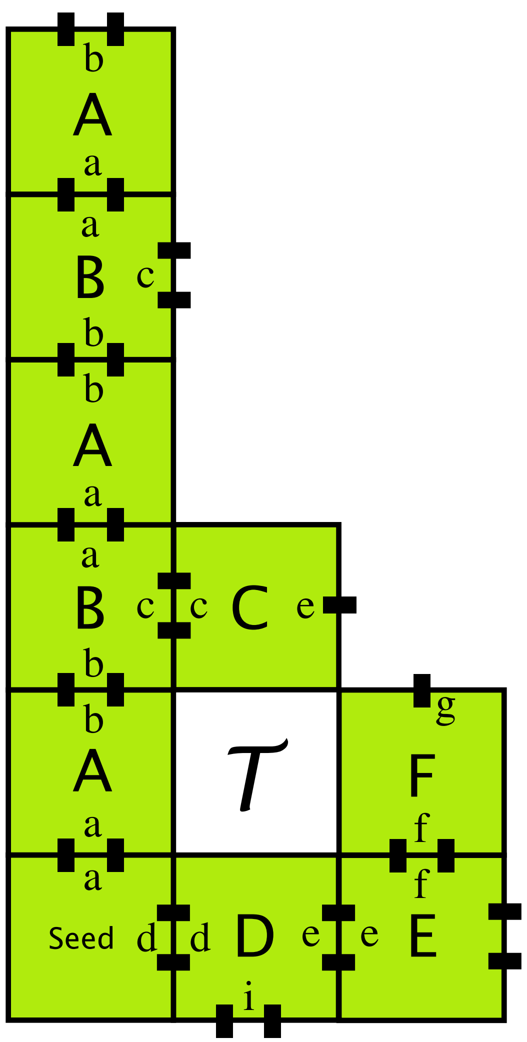




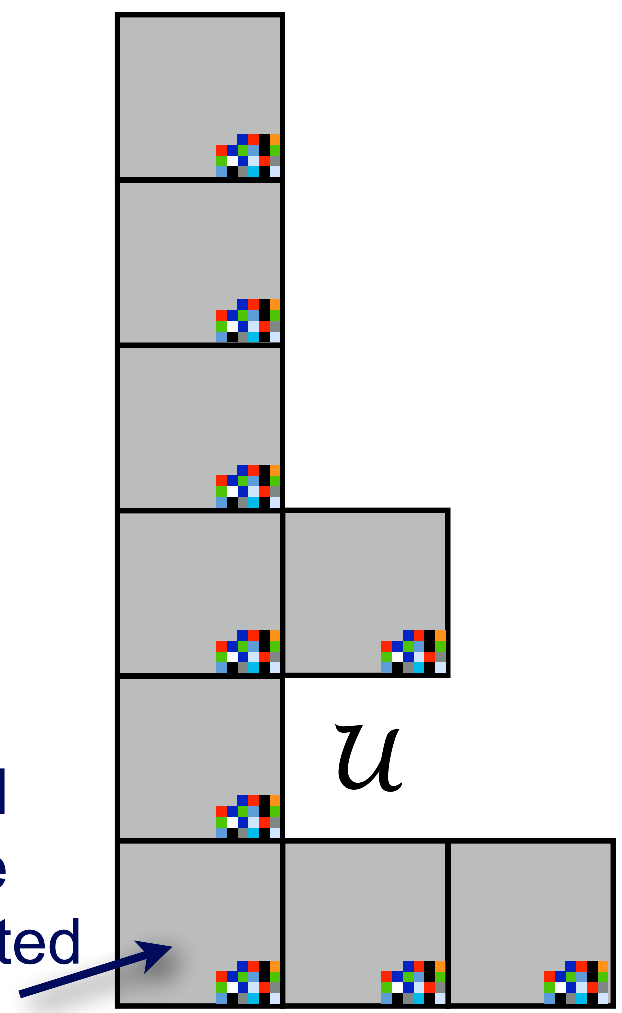


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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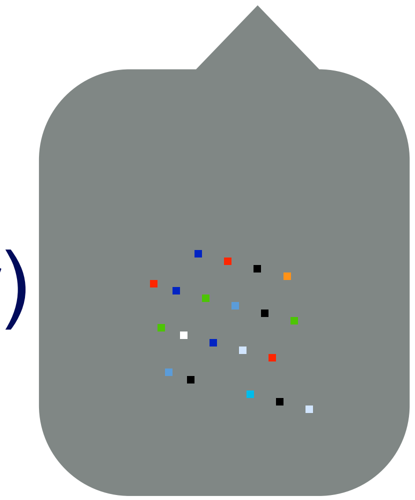
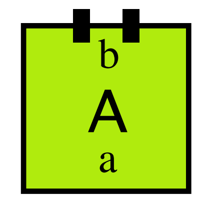
Simulation definition

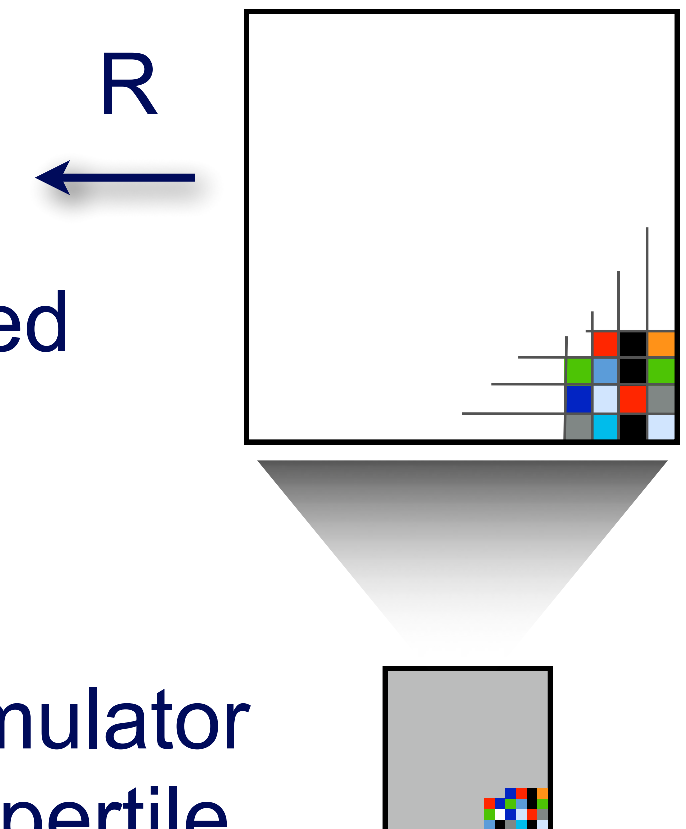
Universal
(simulator)
tile set



Temperature = 2

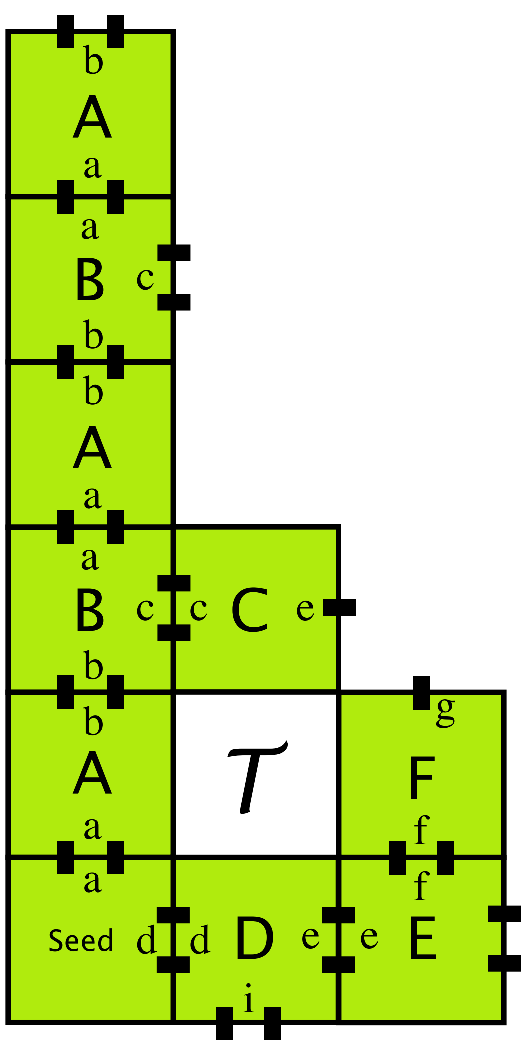


R

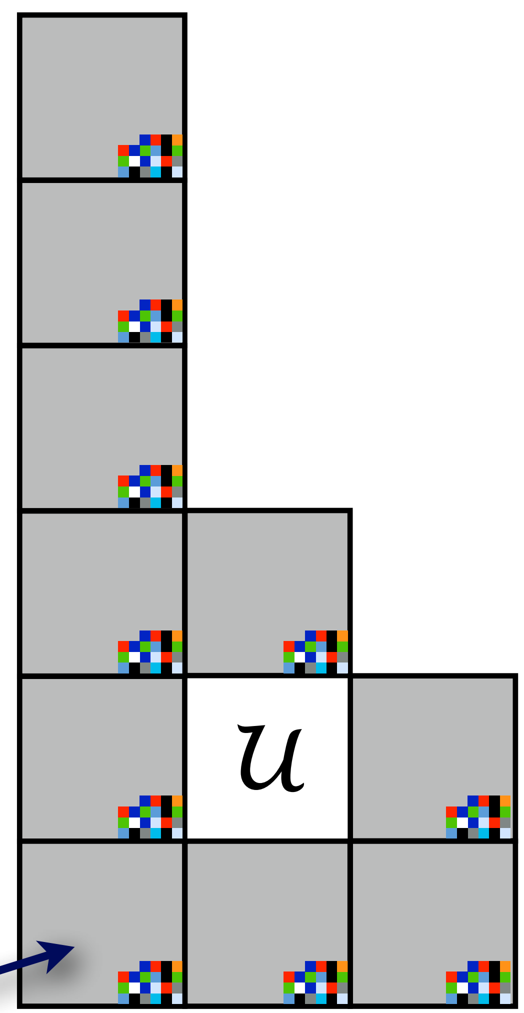


Simulated
tile

Simulator
supertile



Preassembled
seed structure
(encodes simulated
TAS)



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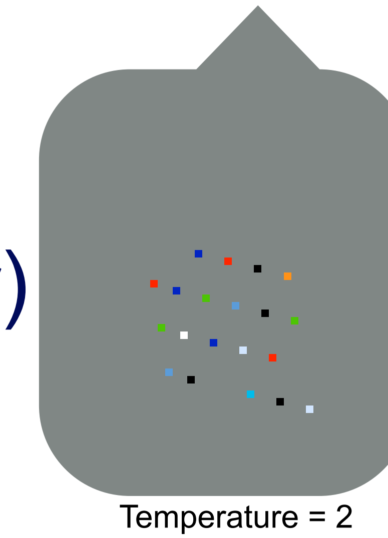
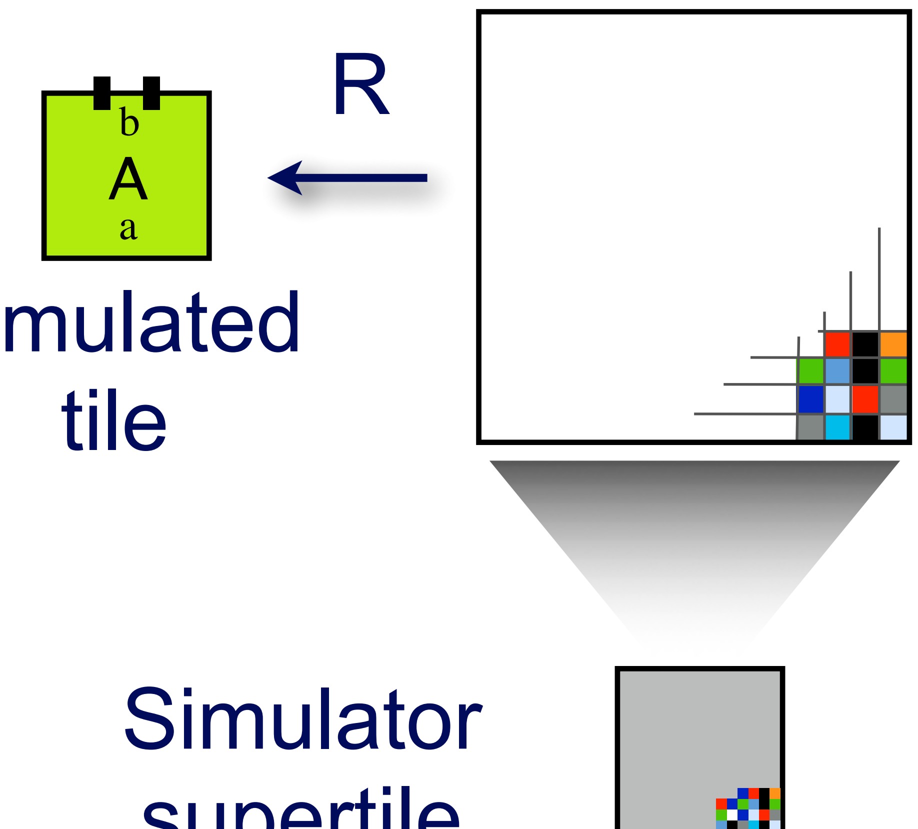
Simulation definition

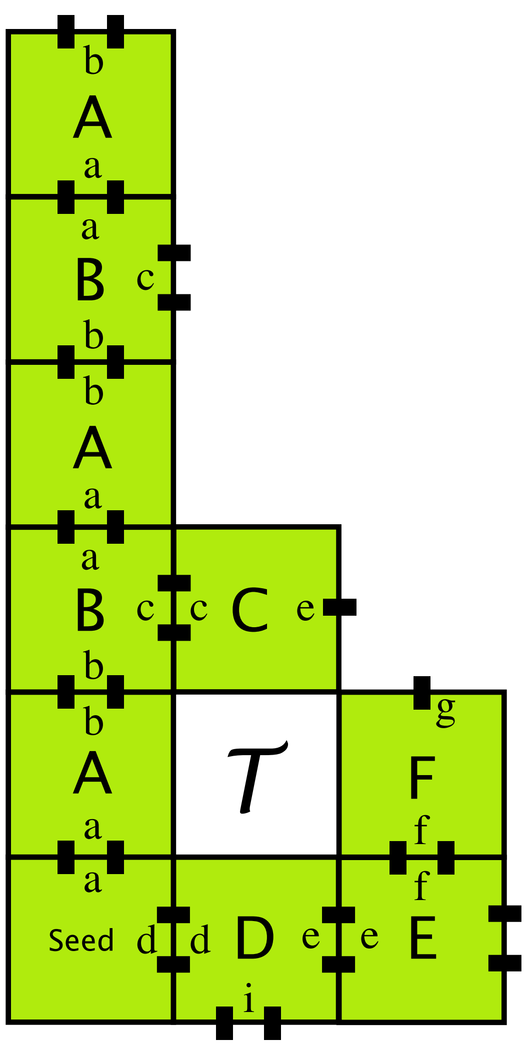
Universal (simulator) tile set



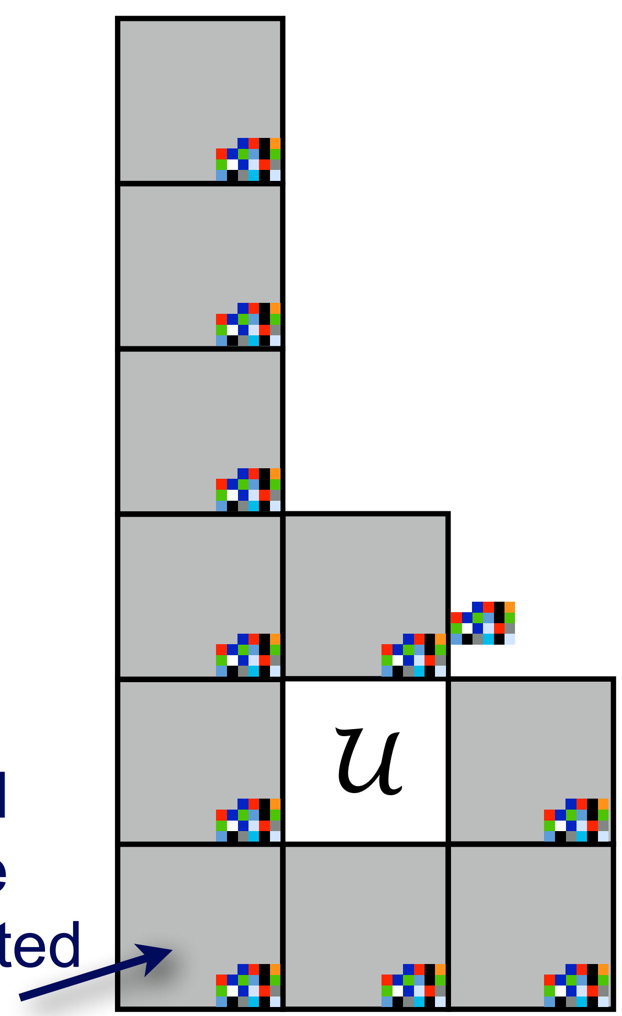


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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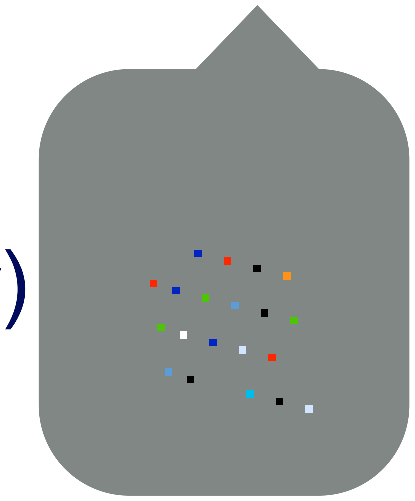
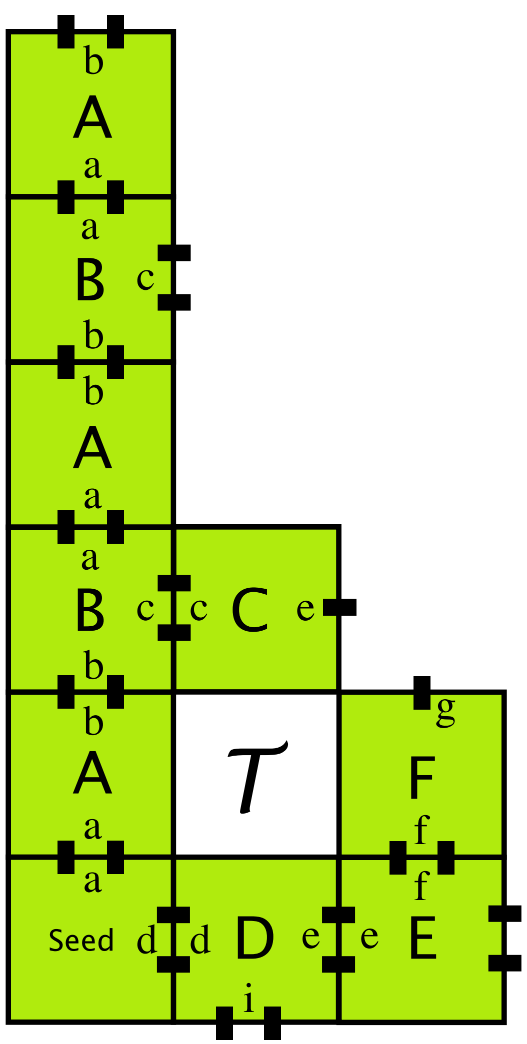
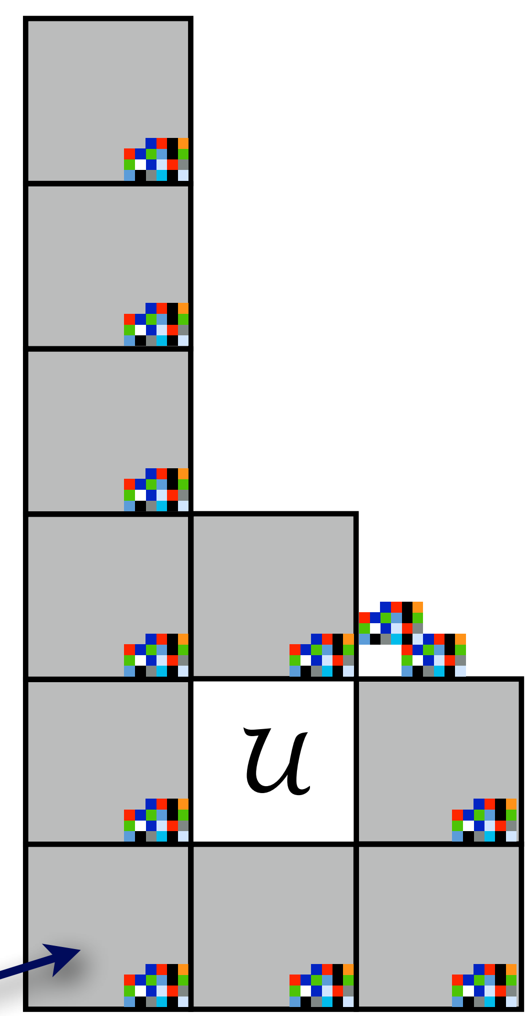
Simulation definition

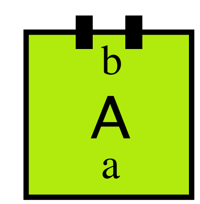
Universal (simulator) tile set





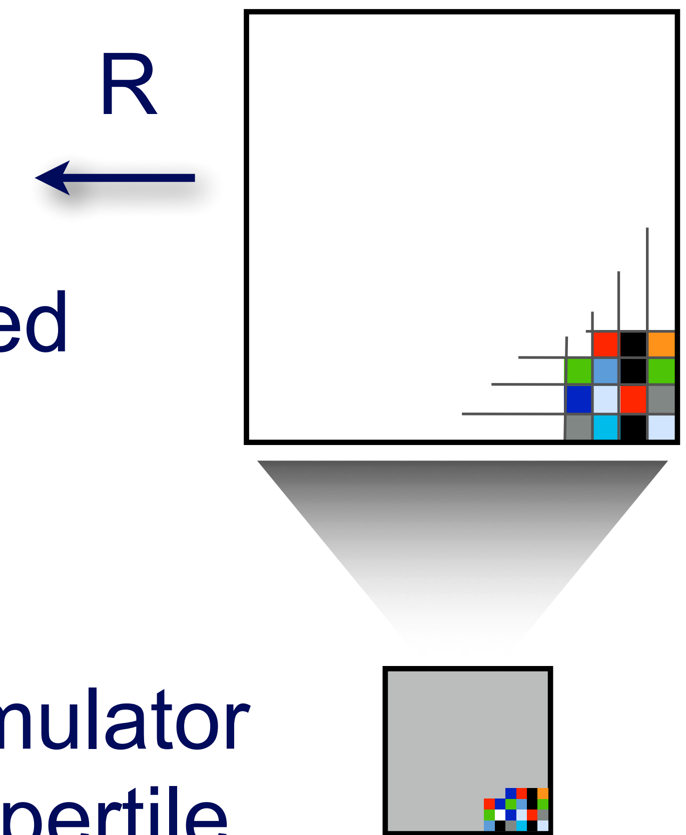
Preassembled seed structure (encodes simulated TAS)





Simulated tile

R



Simulator supertile

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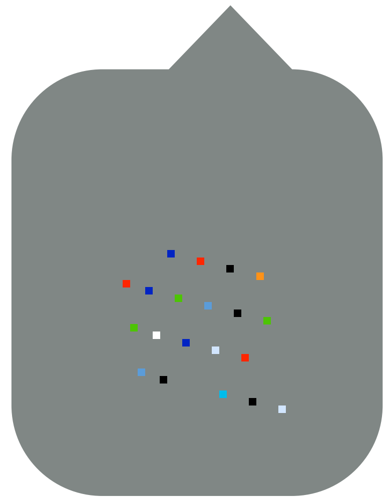
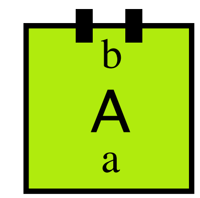
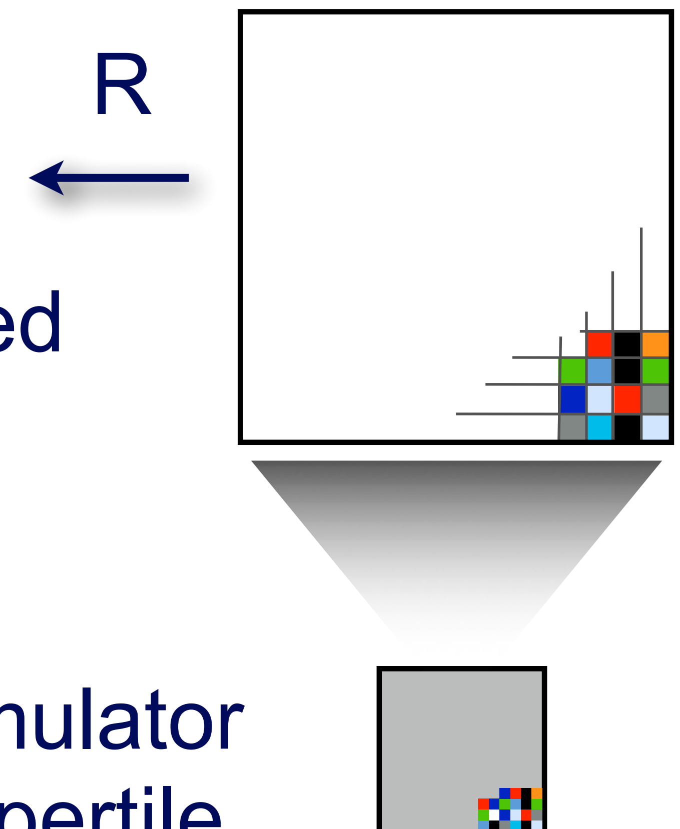
Simulation definition

Universal (simulator) tile set



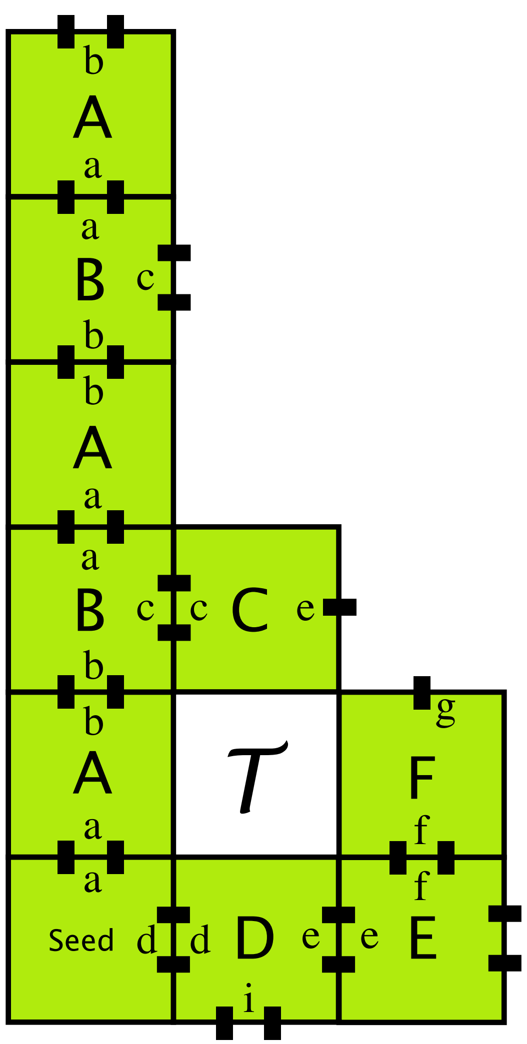
Temperature = 2



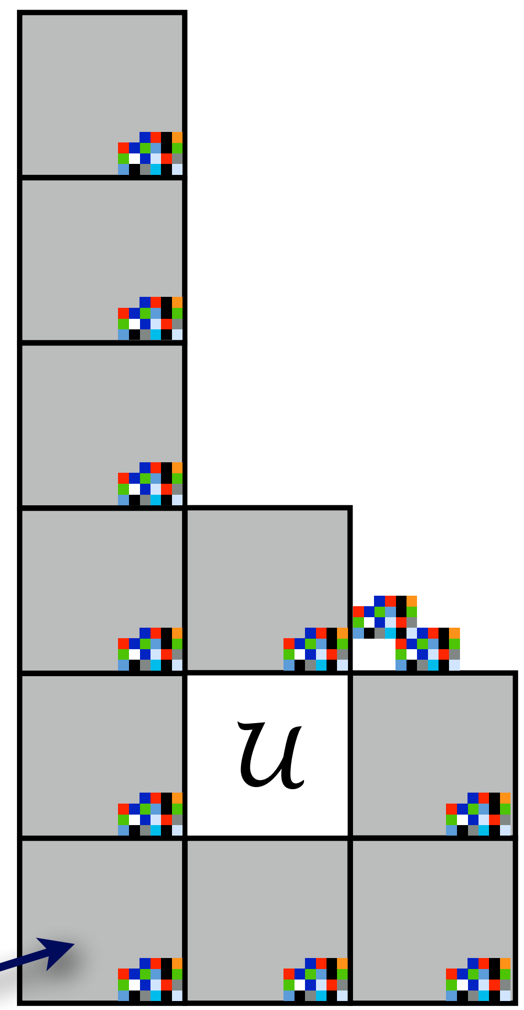


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



- Green tiles are simulated by supertiles
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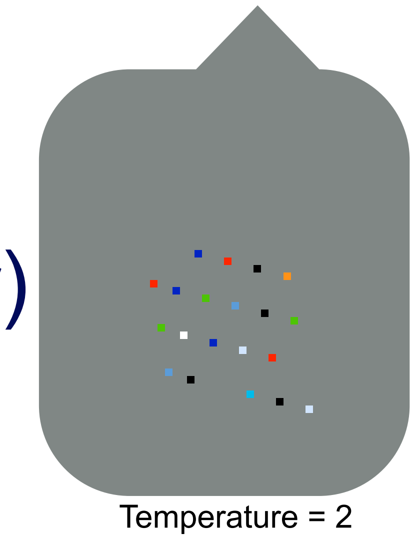
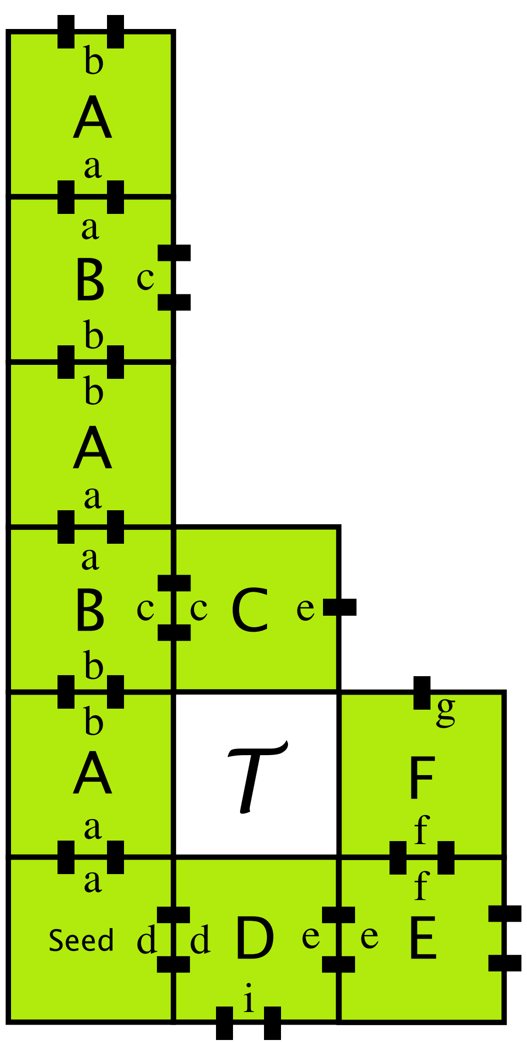
etc.

Simulation definition

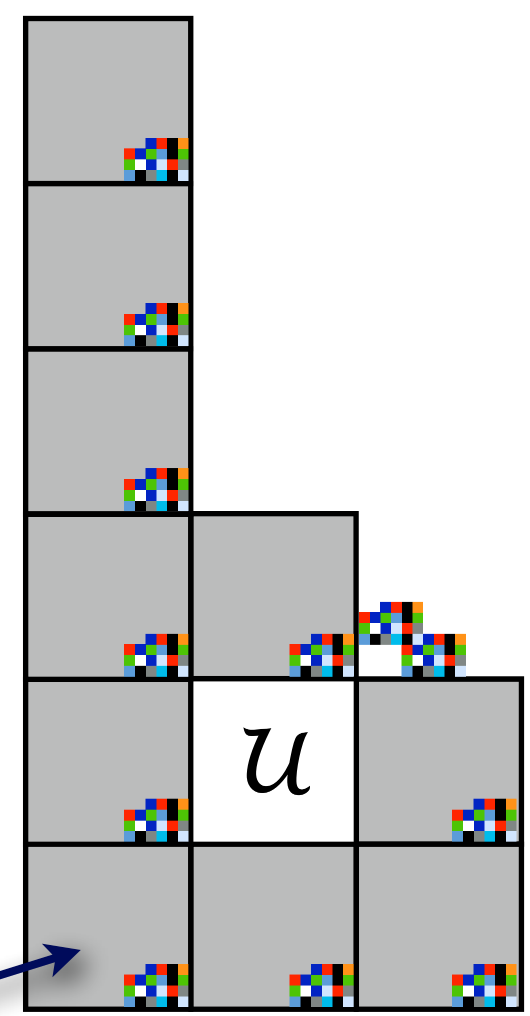
Ignoring *m* x *m* scaling, production & dynamics are equivalent in the simulated system and simulator

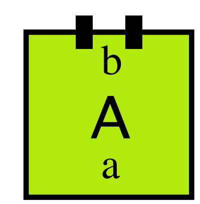
Universal (simulator) tile set





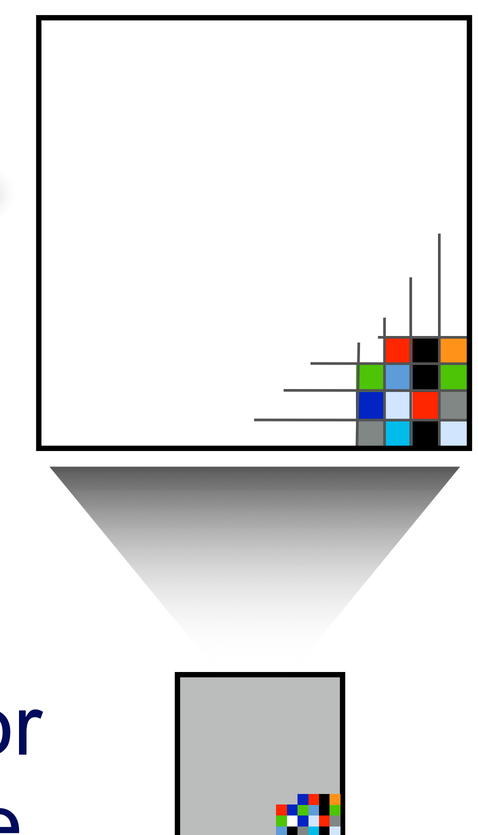
Preassembled seed structure (encodes simulated TAS)





R

Simulated tile



Simulator supertile

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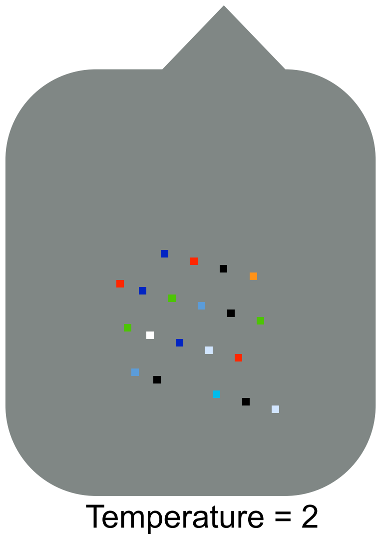
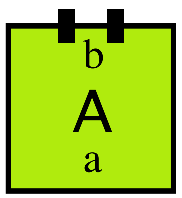
etc.

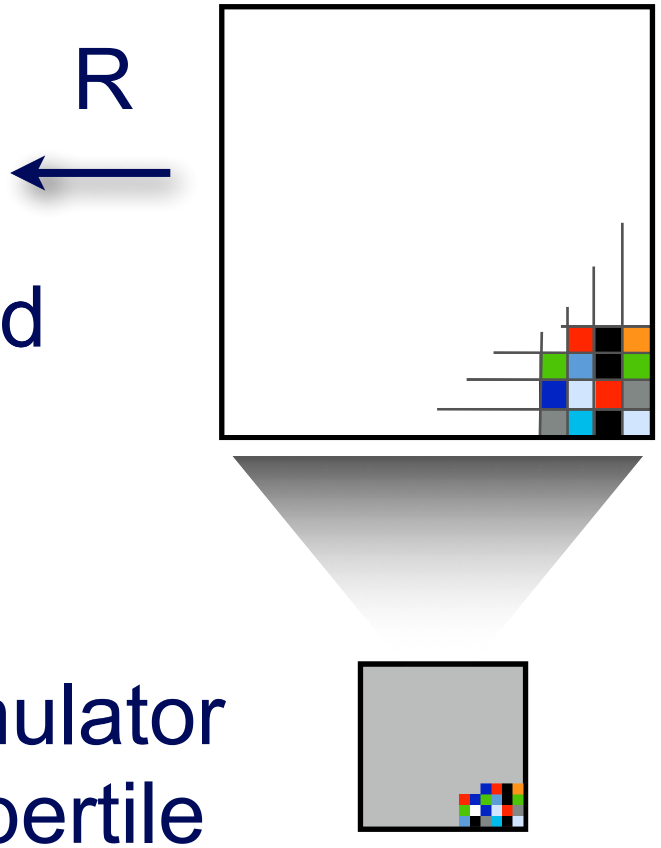
Simulation definition

Universal (simulator) tile set

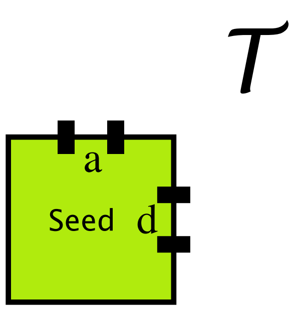




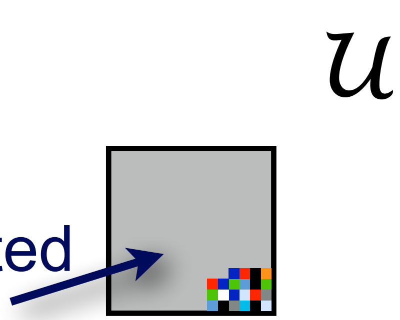
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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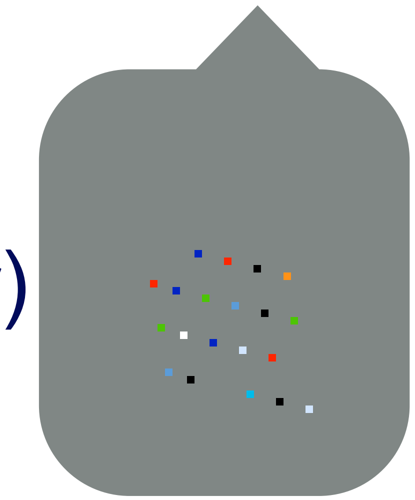
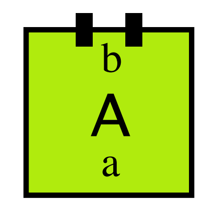
Simulation definition

Universal
(simulator)
tile set

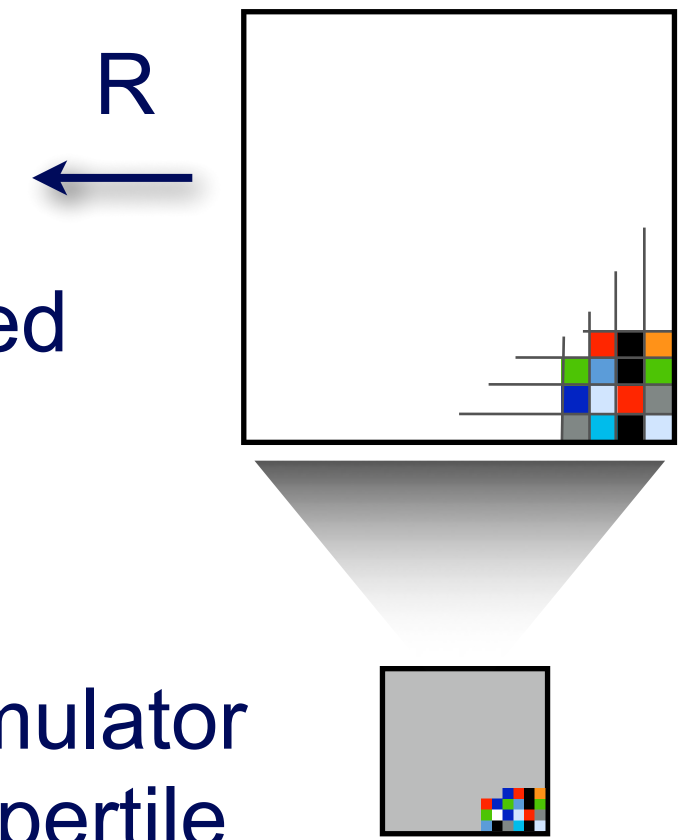


Temperature = 2



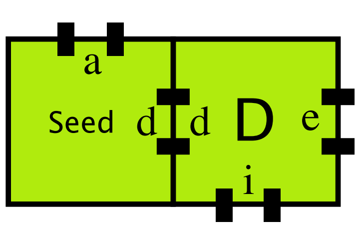
Simulated
tile

R



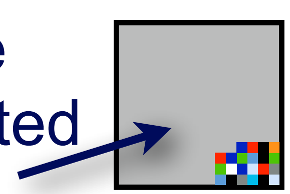
Simulator
supertile

$\mathcal{T}$



Preassembled
seed structure
(encodes simulated
TAS)

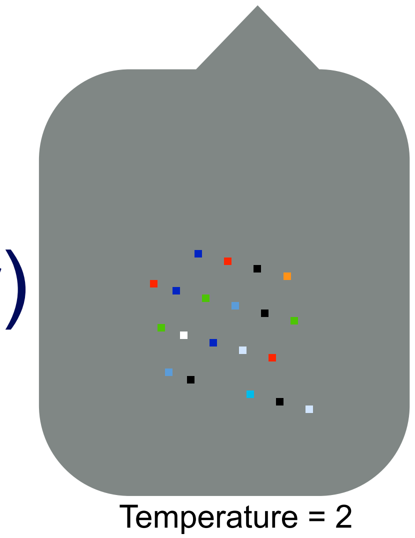
$\mathcal{U}$

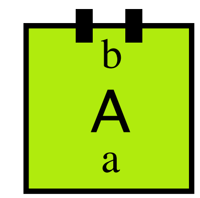


- Green tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation definition

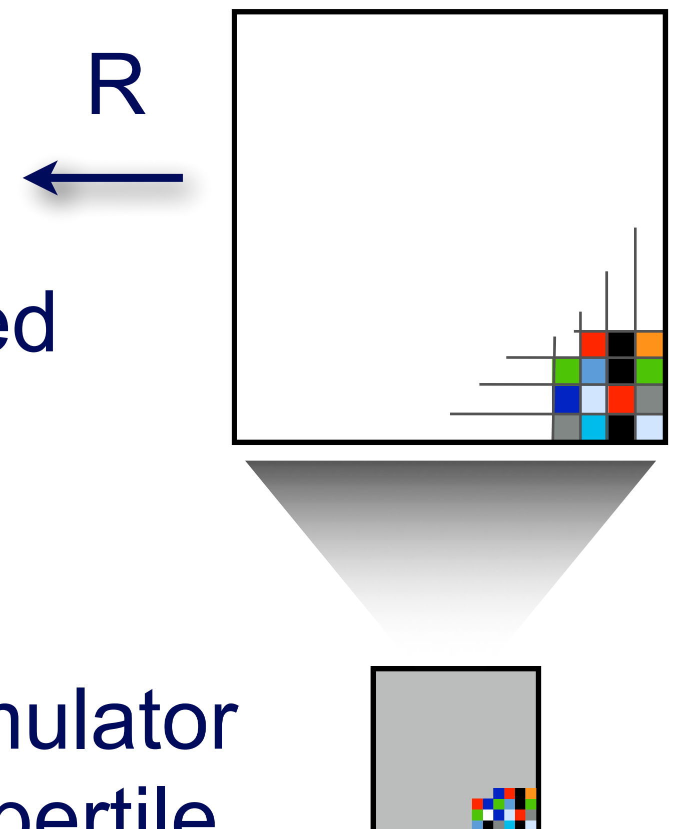
Universal (simulator) tile set



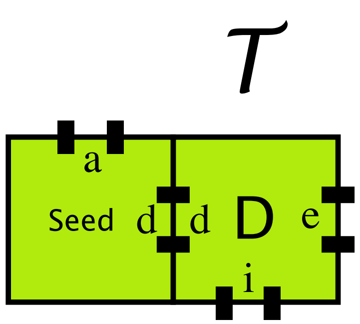


Simulated tile

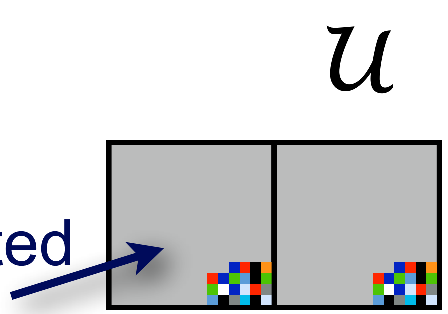
R



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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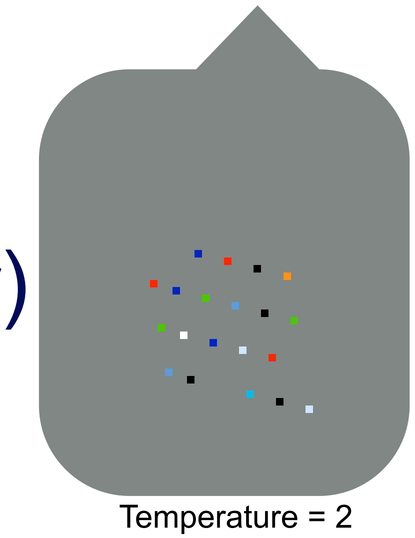
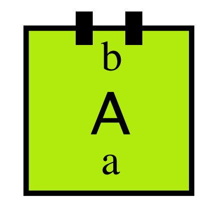
Simulation definition

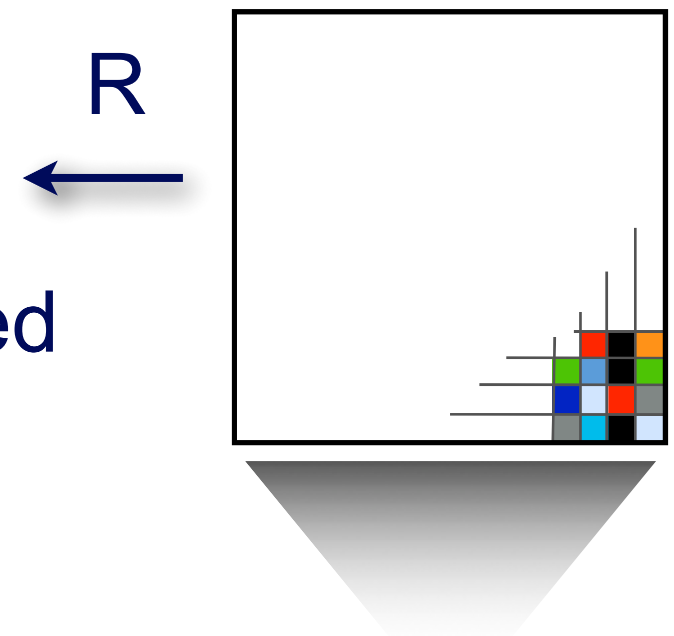
Universal (simulator) tile set



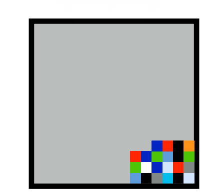


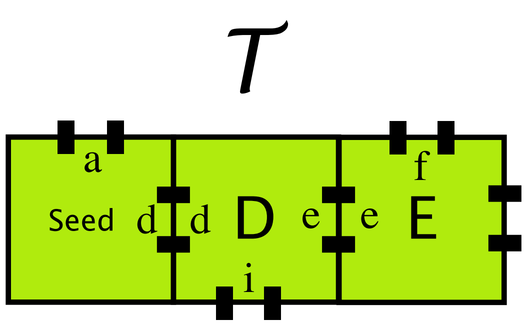
Simulated tile

R

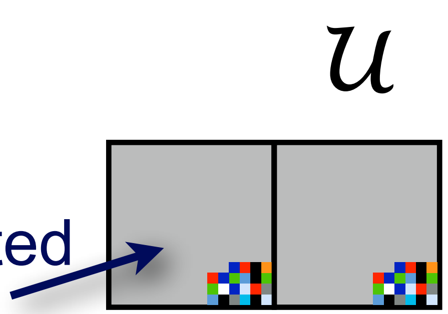


Simulator supertile





Preassembled seed structure (encodes simulated TAS)



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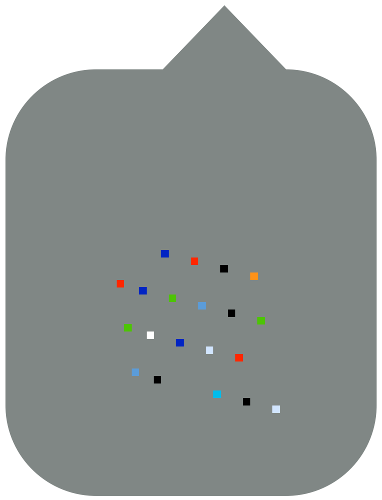
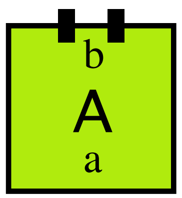
Simulation definition

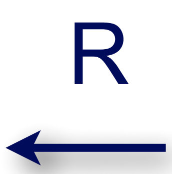
Universal (simulator) tile set

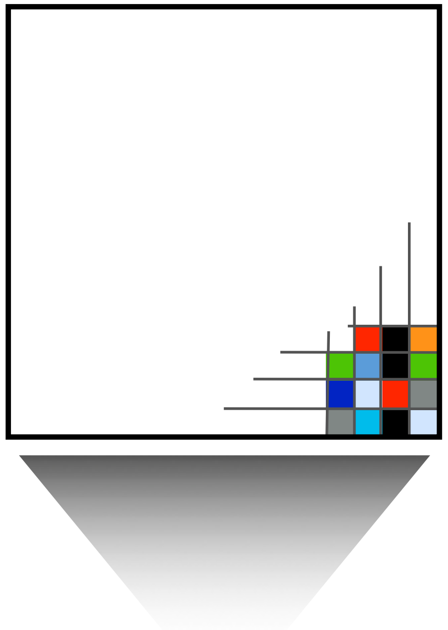


Temperature = 2

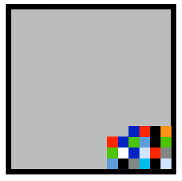


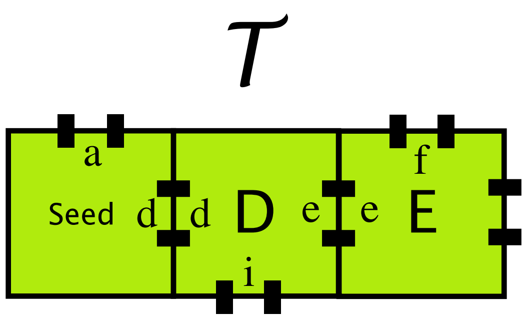
Simulated tile



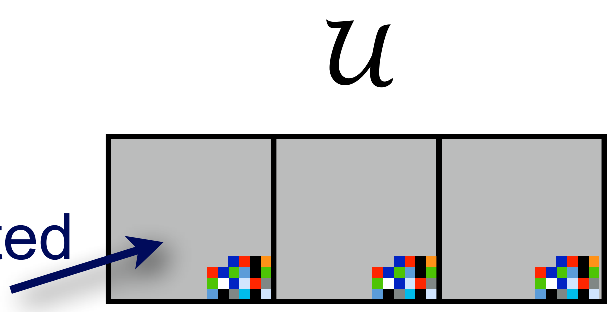


Simulator supertile





Preassembled seed structure (encodes simulated TAS)



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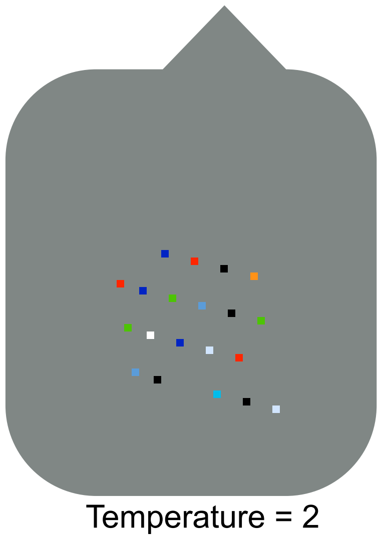
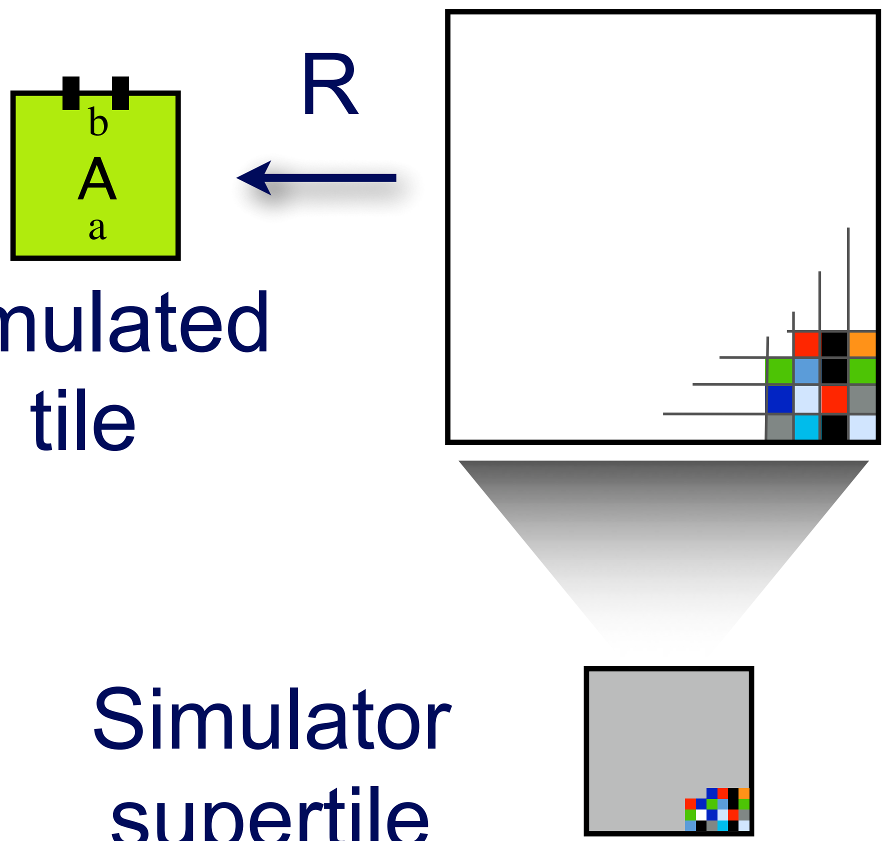
Simulation definition

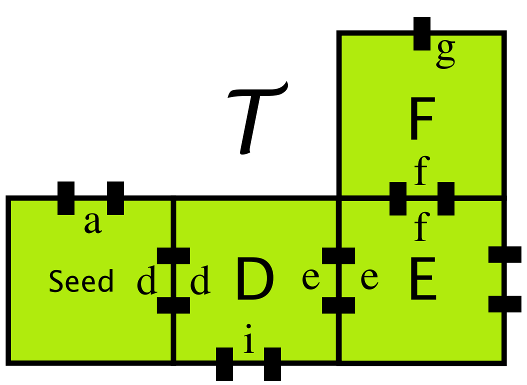
Universal (simulator) tile set



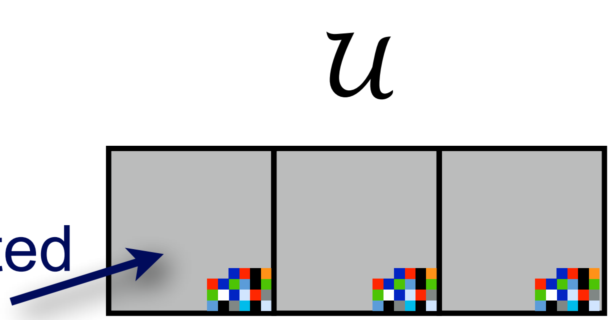


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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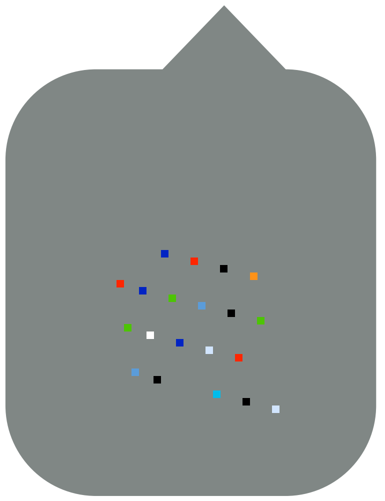
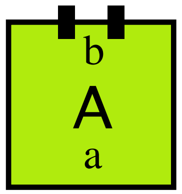
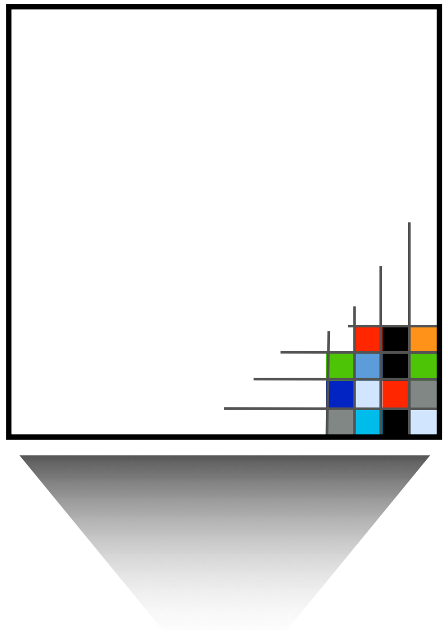
Simulation definition

Universal (simulator) tile set

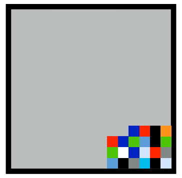




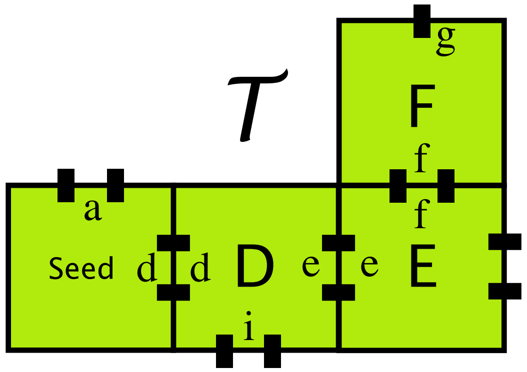
R



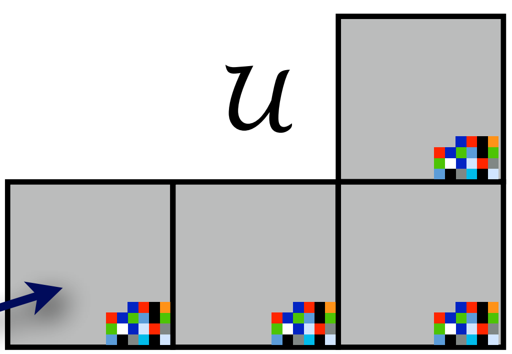
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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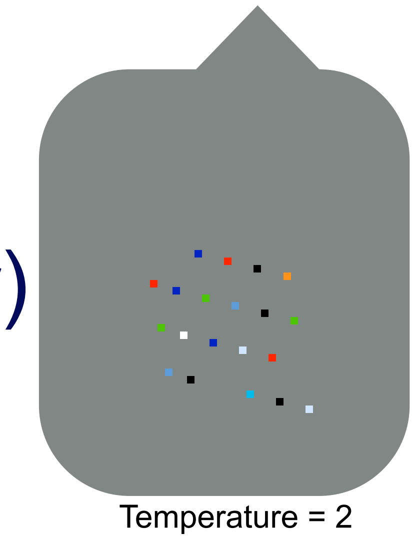
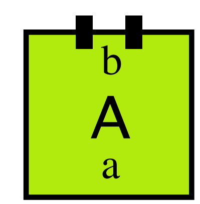
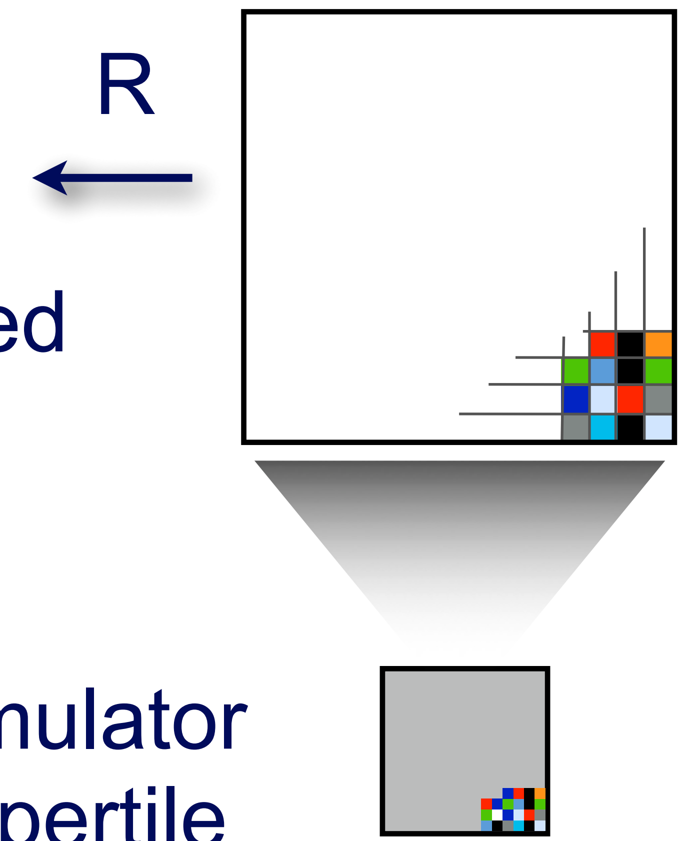
Simulation definition

Universal (simulator) tile set

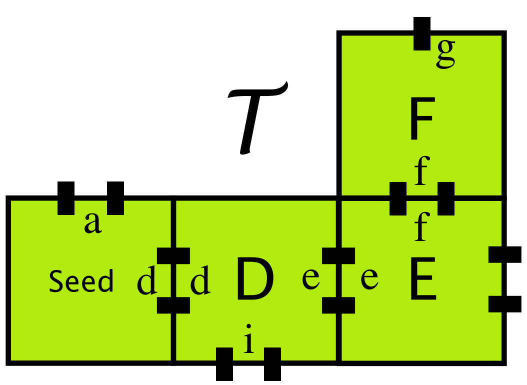




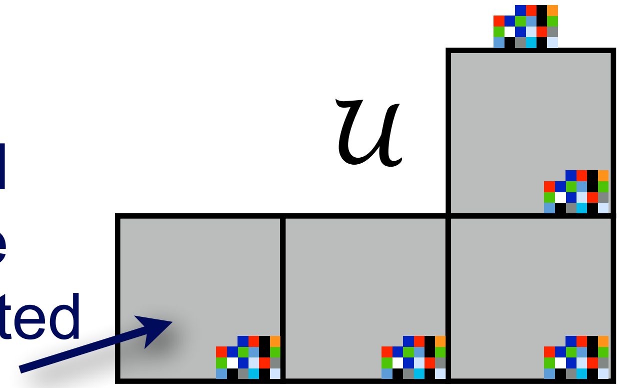
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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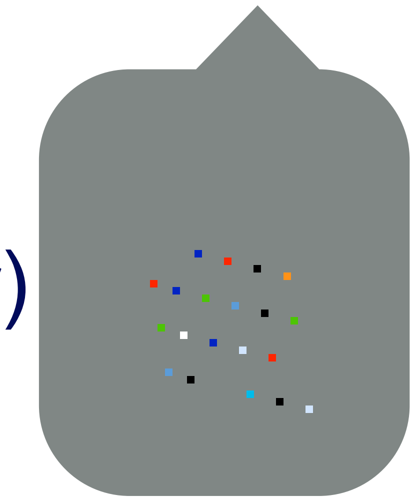
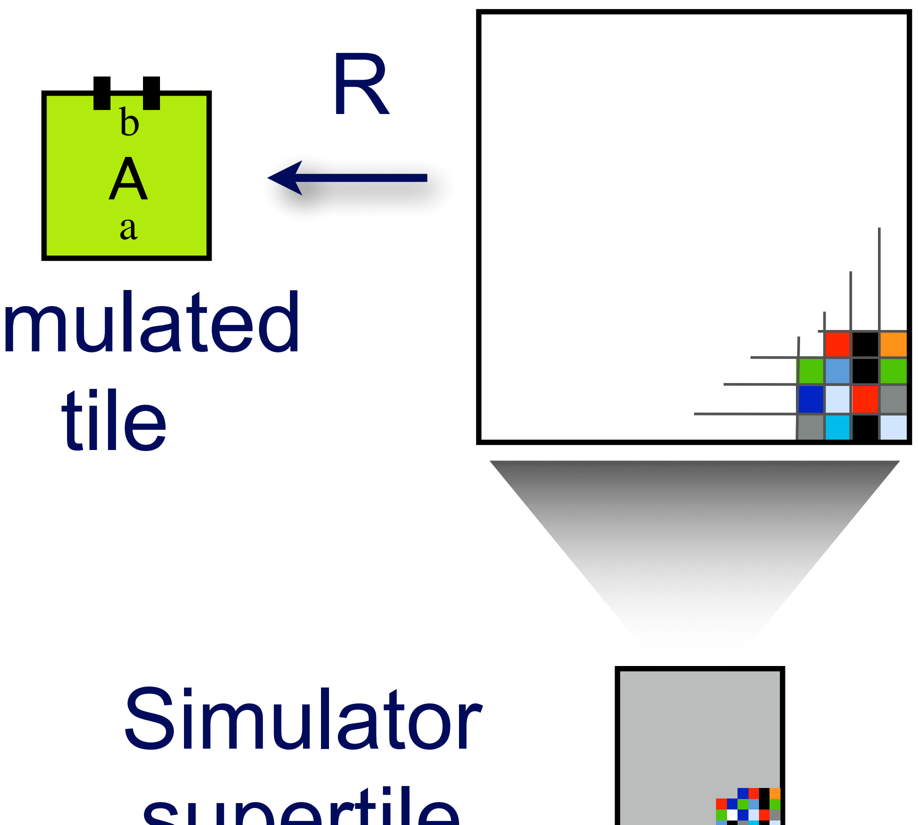
Simulation definition

Universal (simulator) tile set

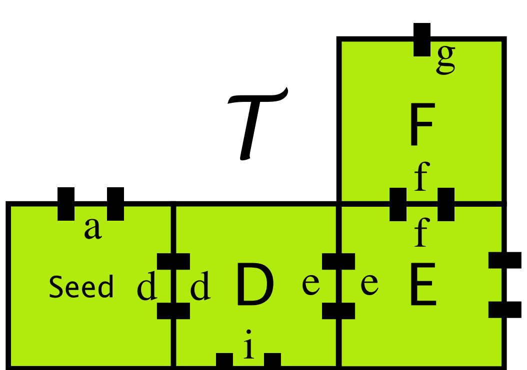


Temperature = 2

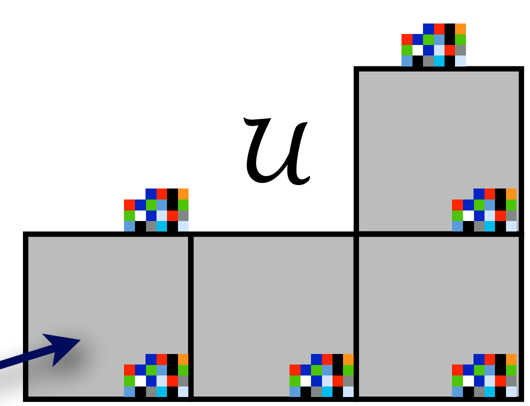


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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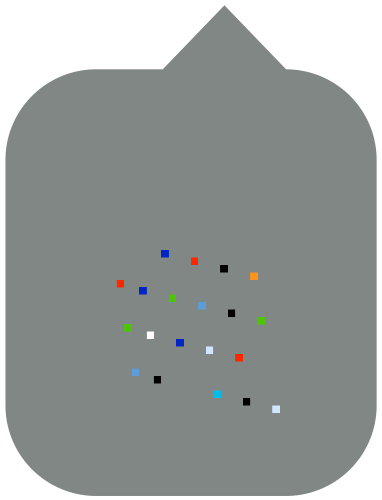
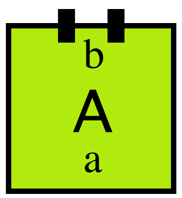
Simulation definition

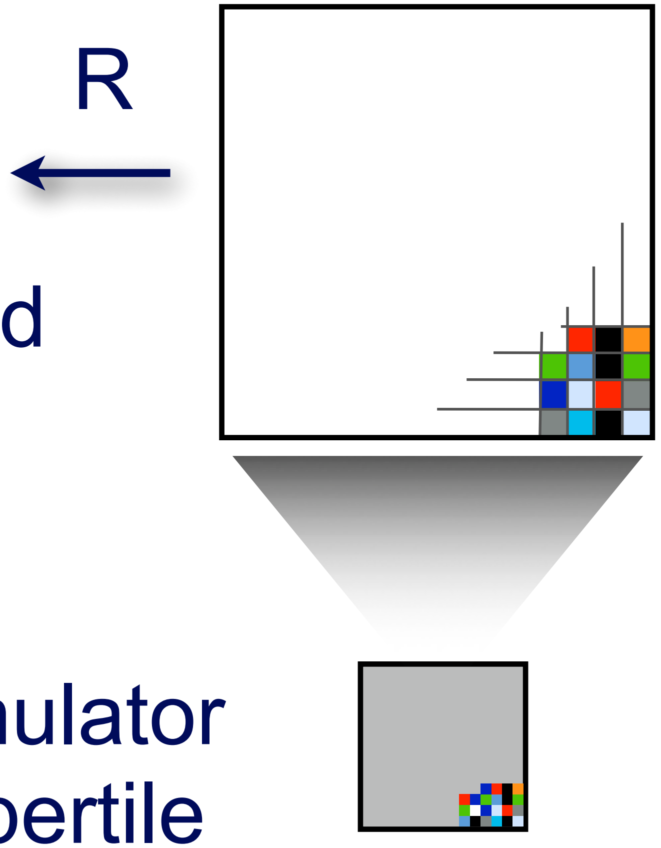
Universal (simulator) tile set



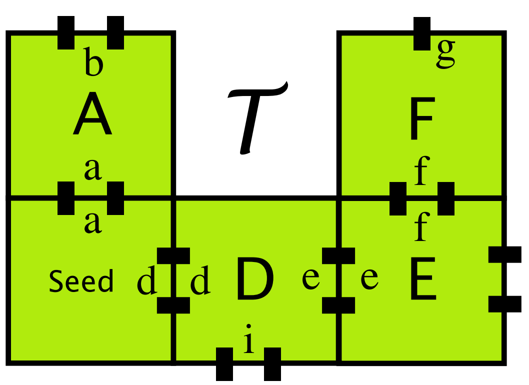
Temperature = 2



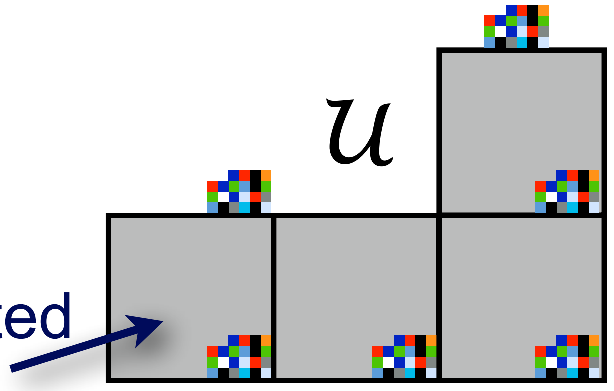
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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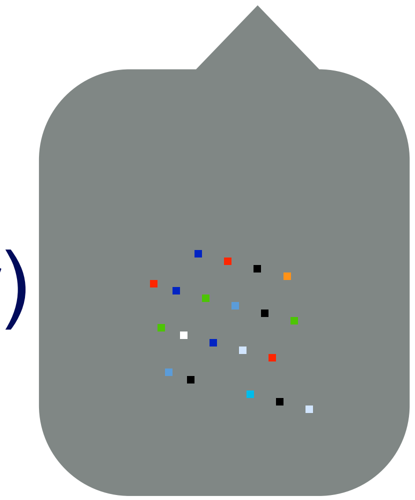
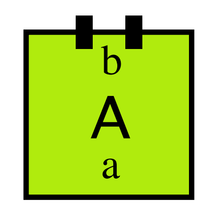
Simulation definition

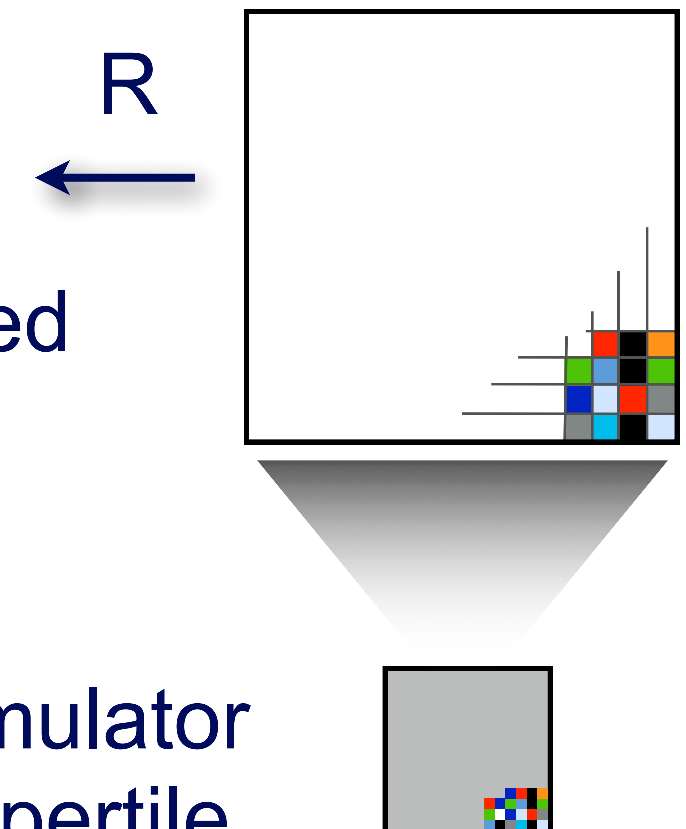
Universal (simulator) tile set



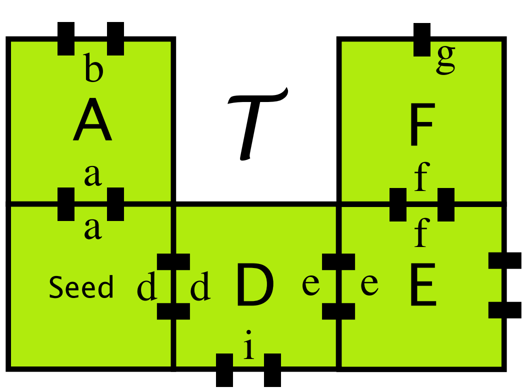
R



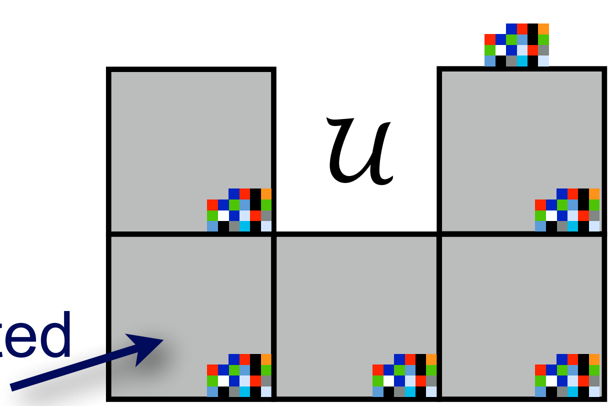
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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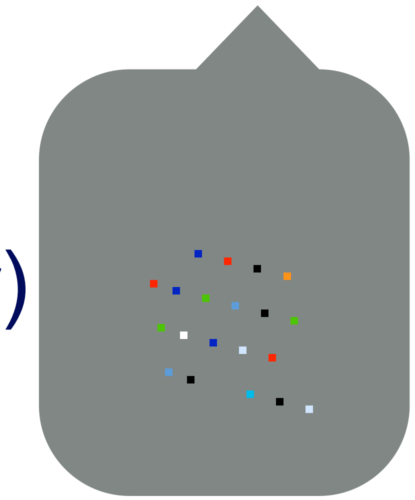
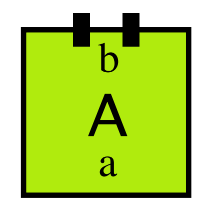
Simulation definition

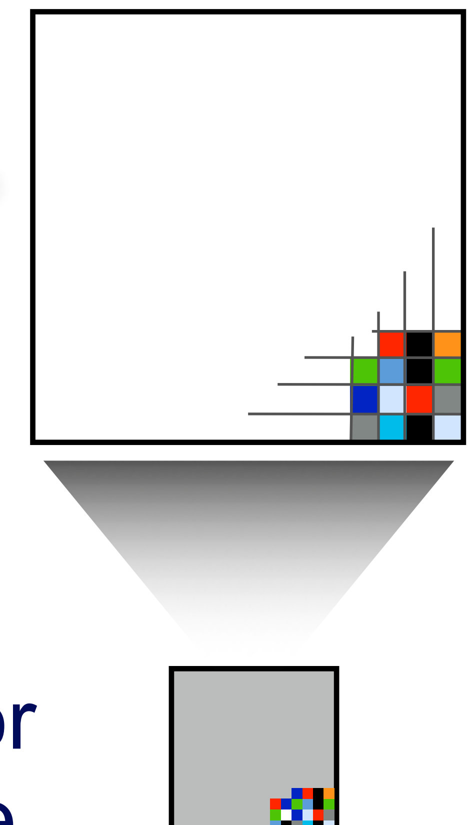
Universal (simulator) tile set



Temperature = 2

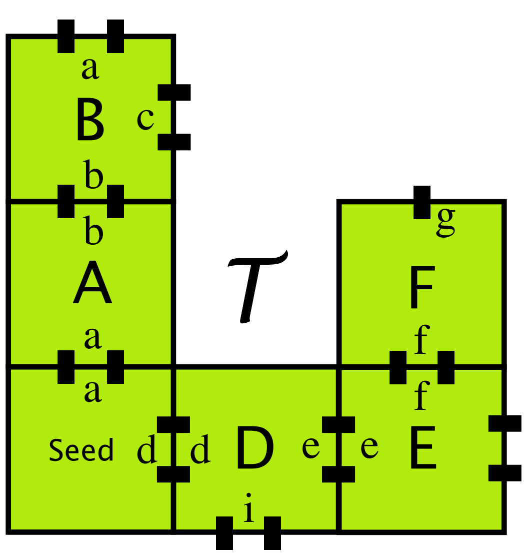


R

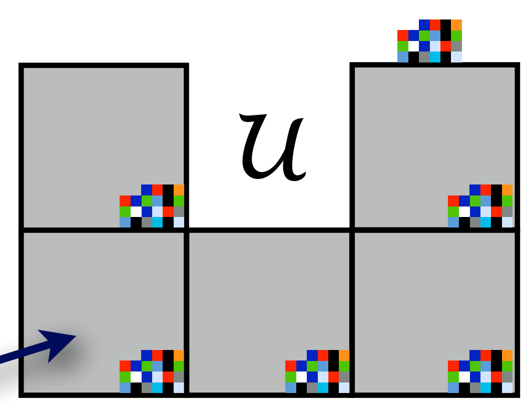


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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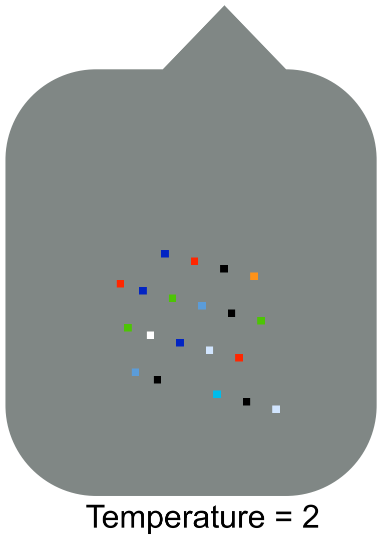
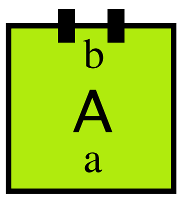
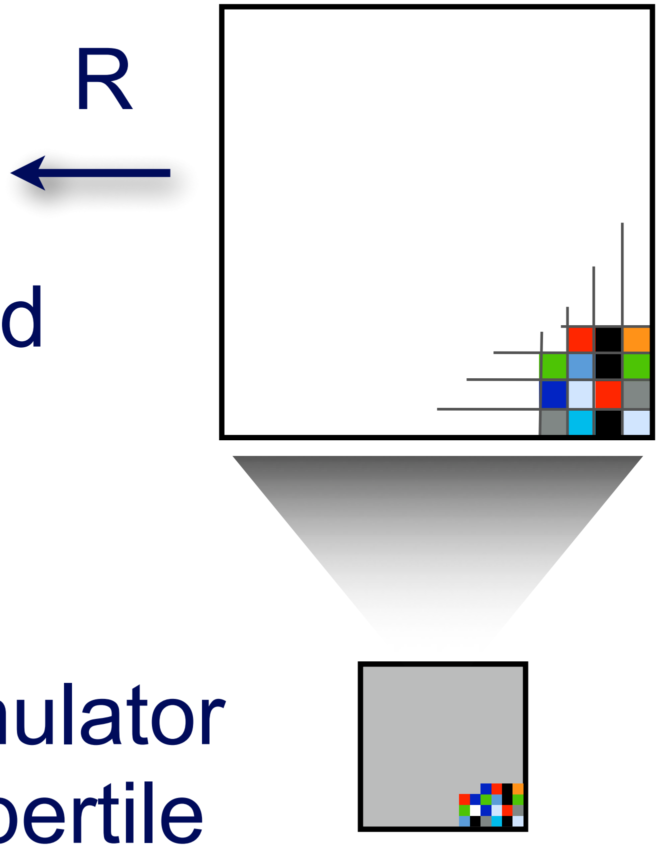
Simulation definition

Universal
(simulator)
tile set

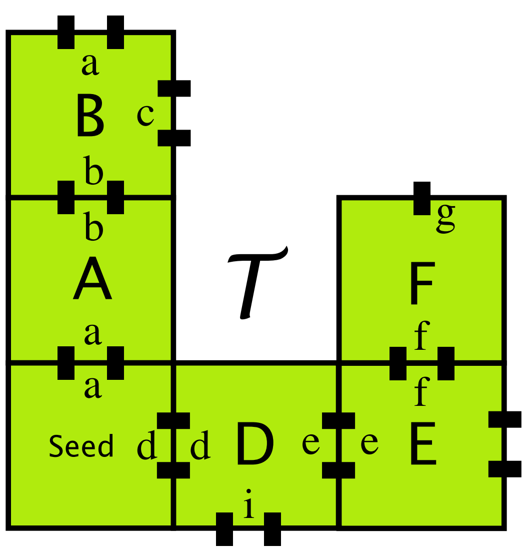




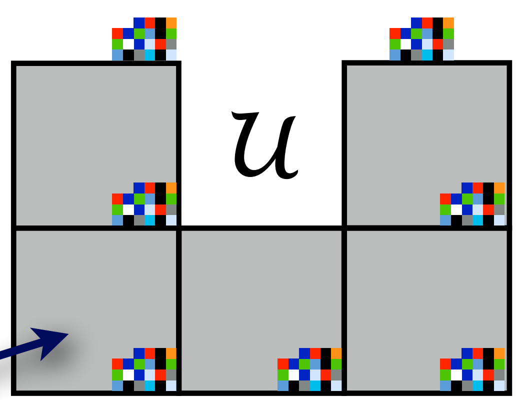
Simulated
tile



Simulator
supertile



Preassembled
seed structure
(encodes simulated
TAS)



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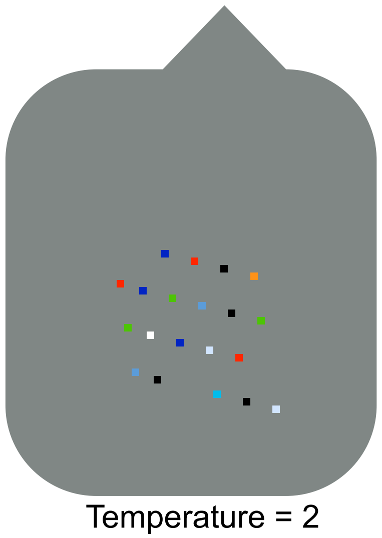
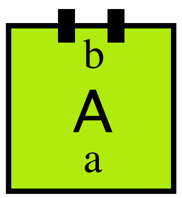
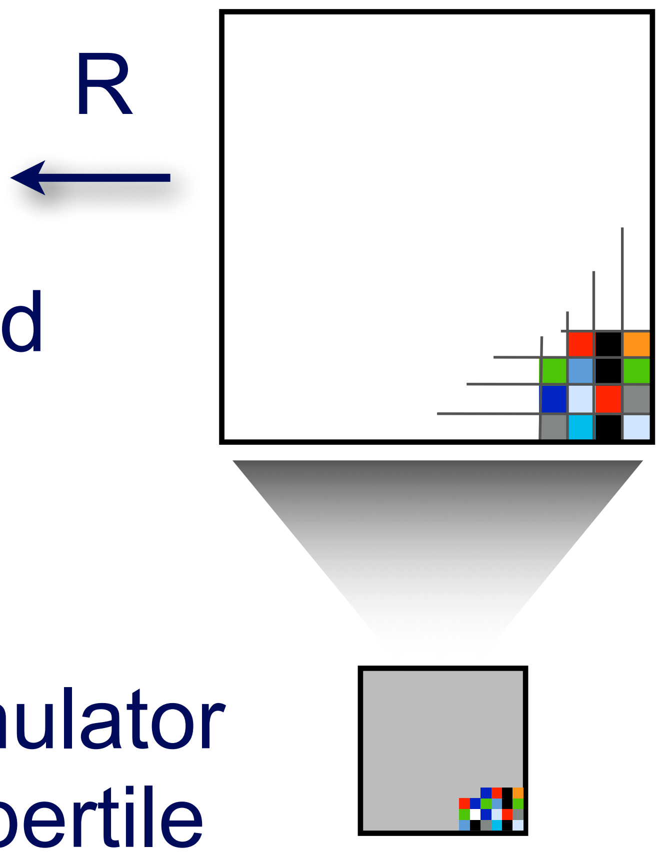
Simulation definition

Universal (simulator) tile set

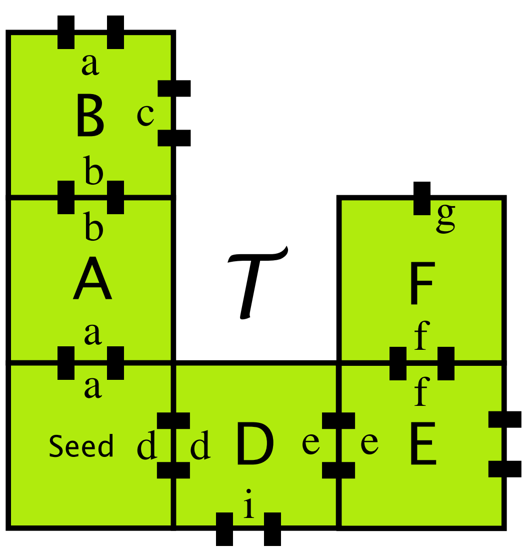




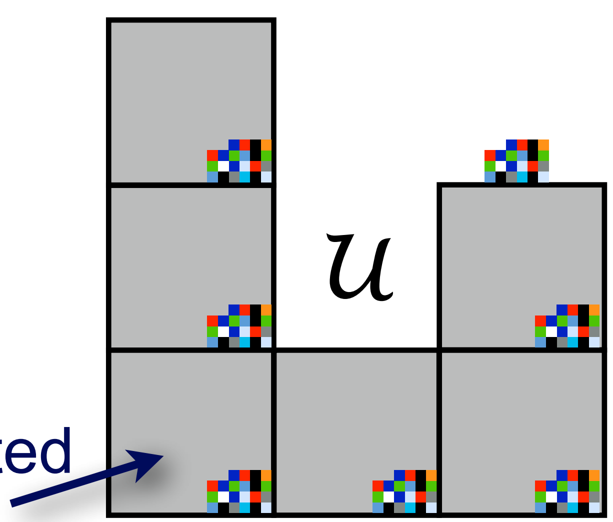


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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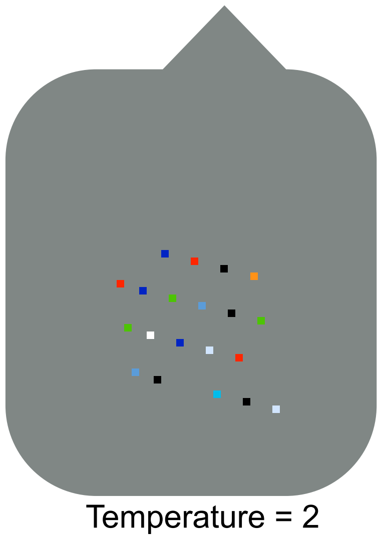
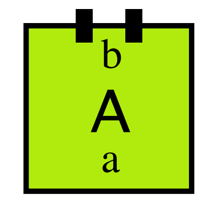
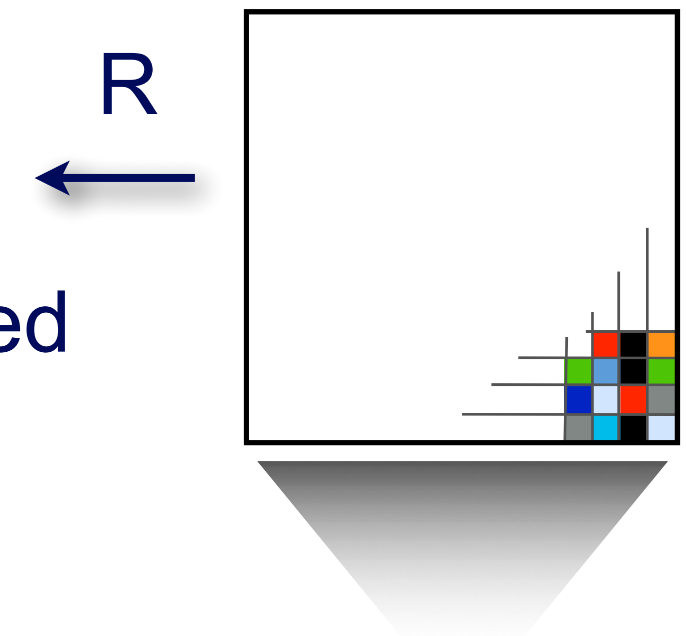
Simulation definition

Universal (simulator) tile set

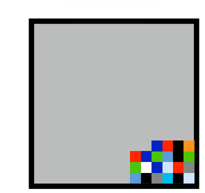


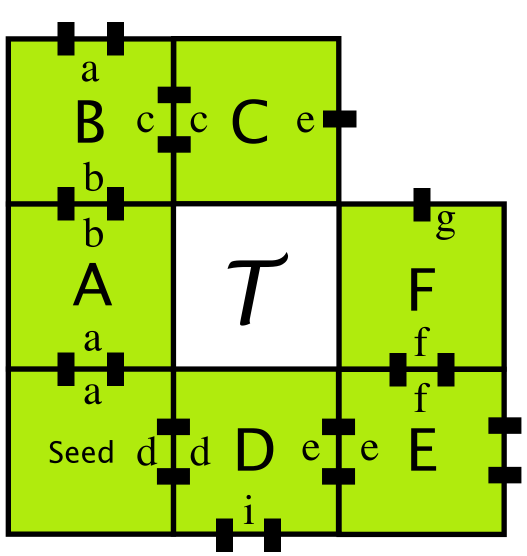




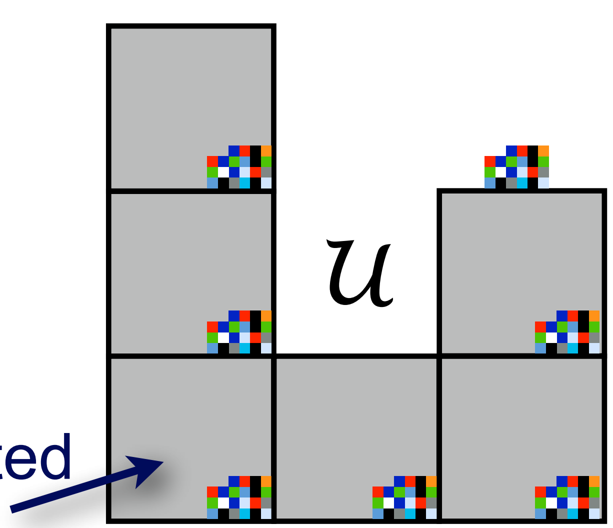
Simulated tile

Simulator supertile





Preassembled seed structure (encodes simulated TAS)



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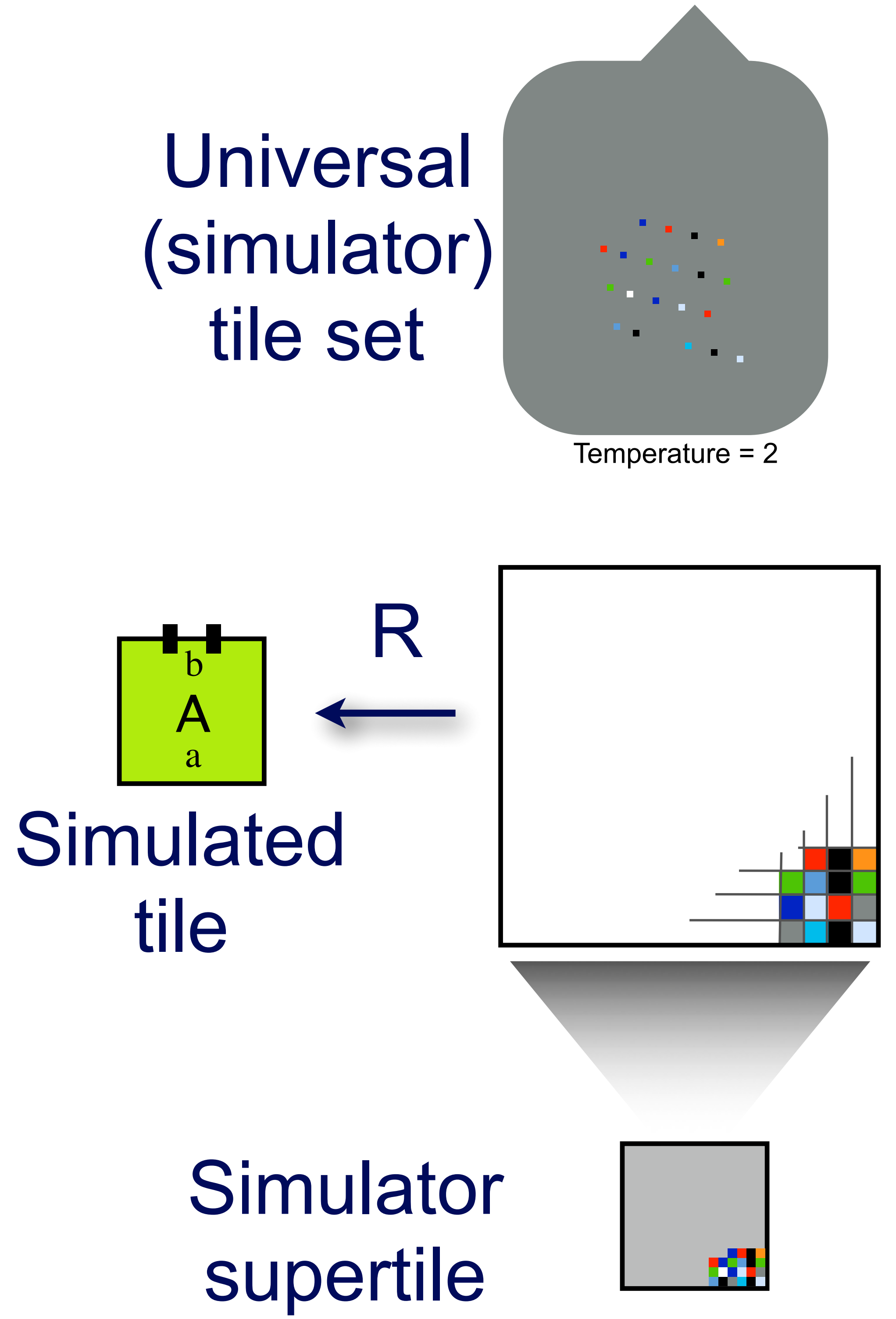
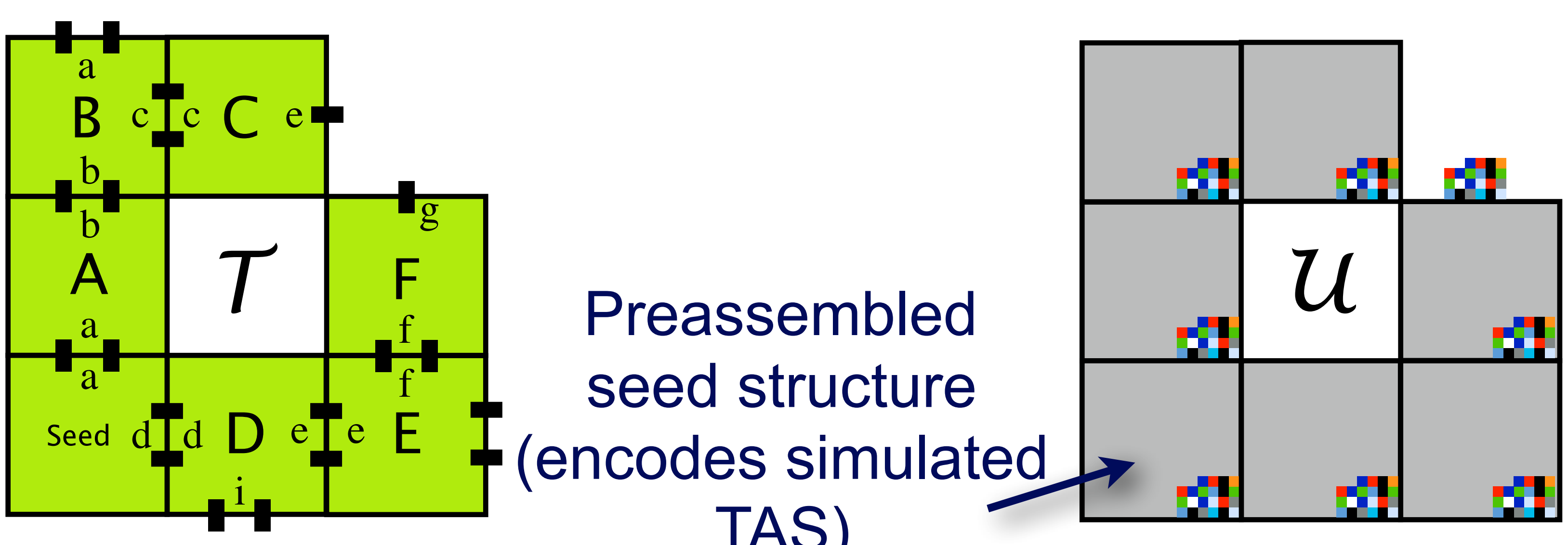
Simulation definition





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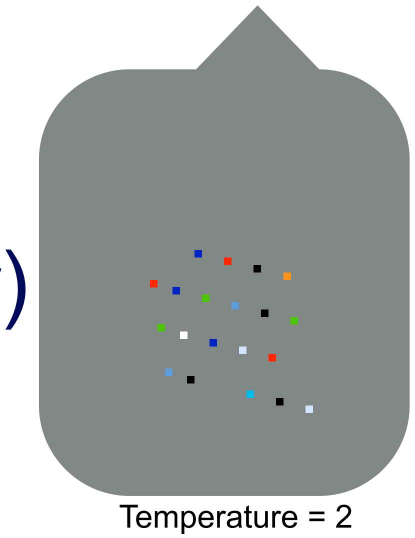
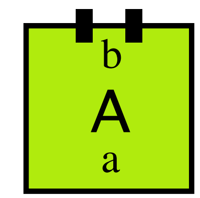
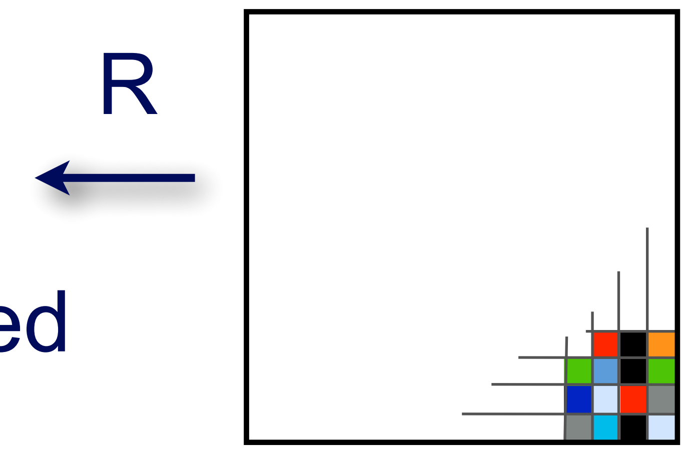
Simulation definition

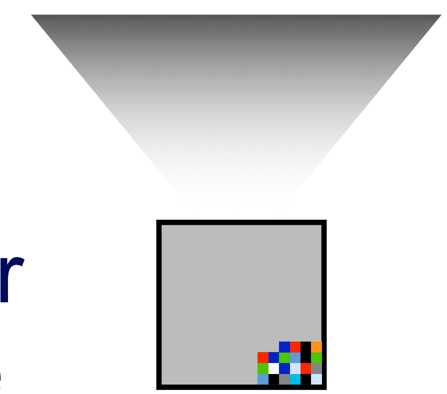
Universal (simulator) tile set



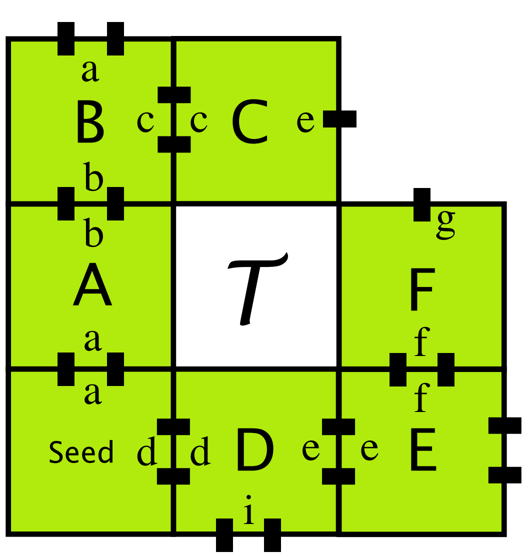




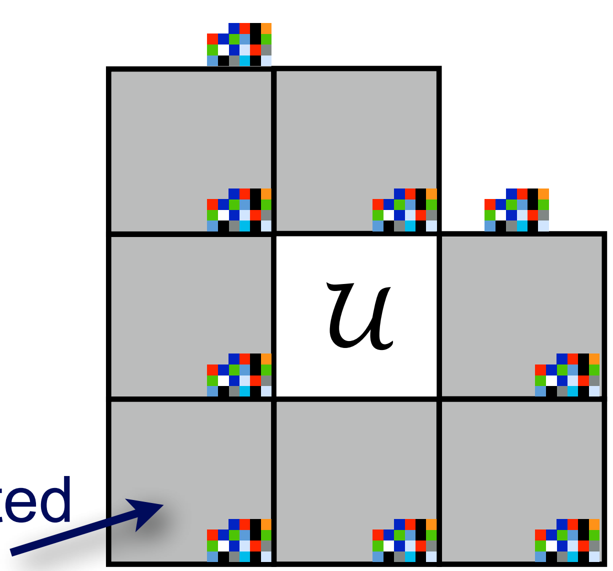
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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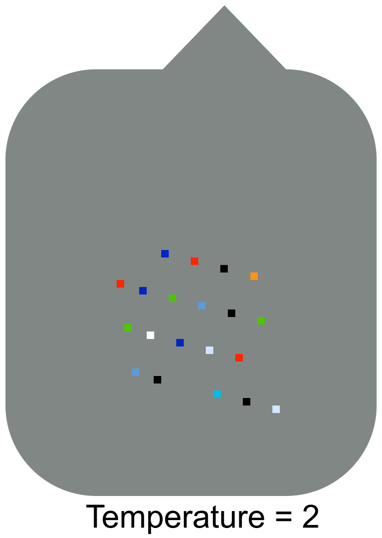
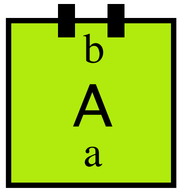
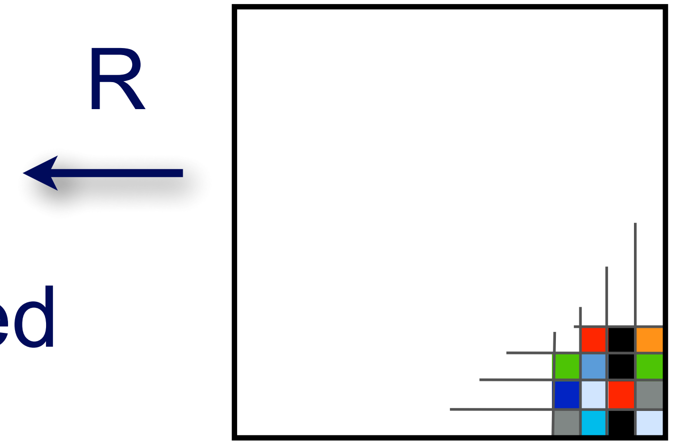
Simulation definition

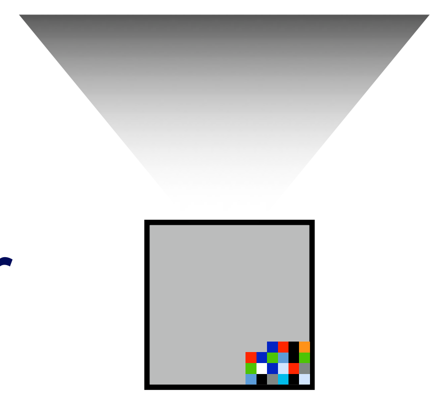
Universal (simulator) tile set



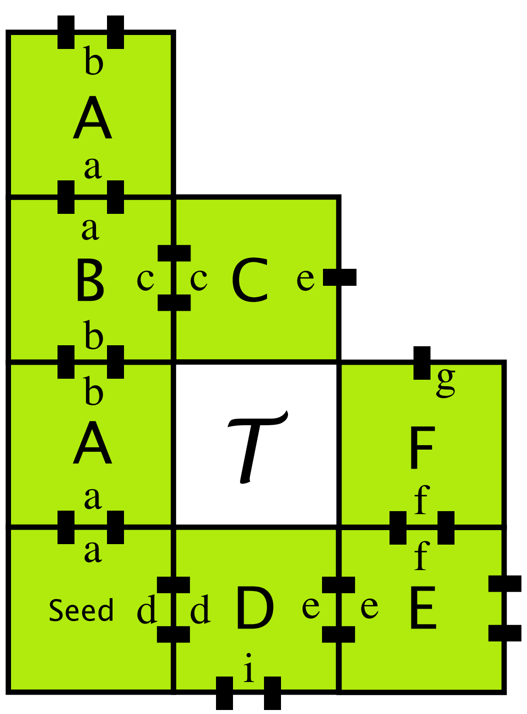




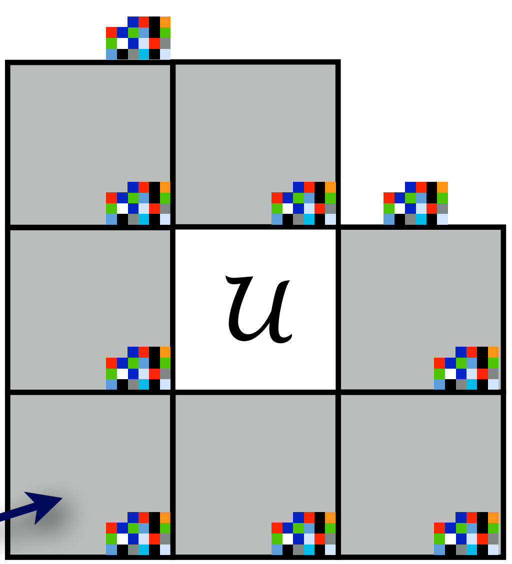
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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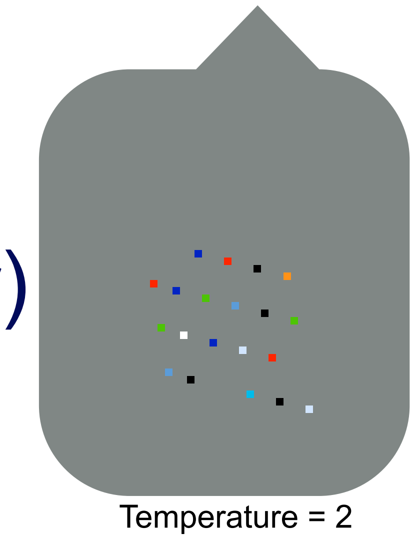
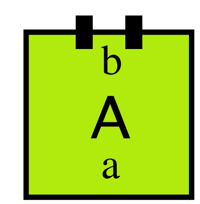
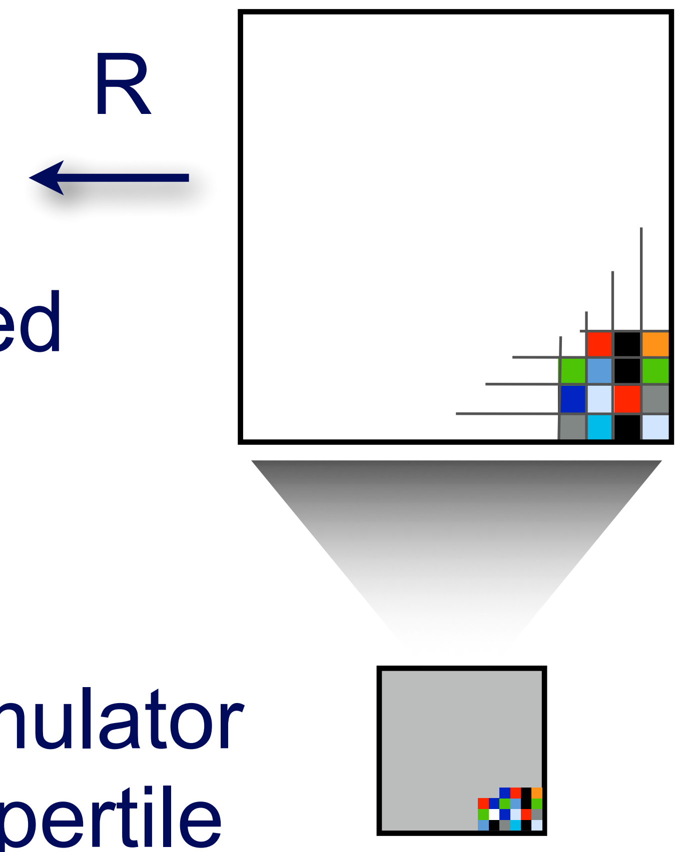
Simulation definition

Universal (simulator) tile set

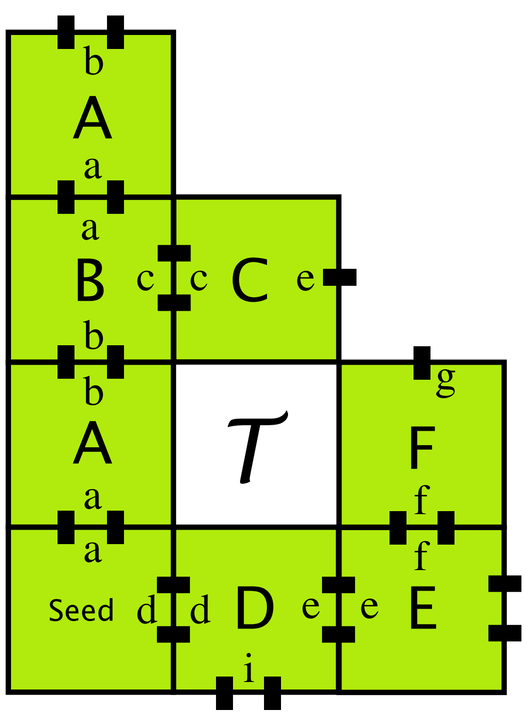




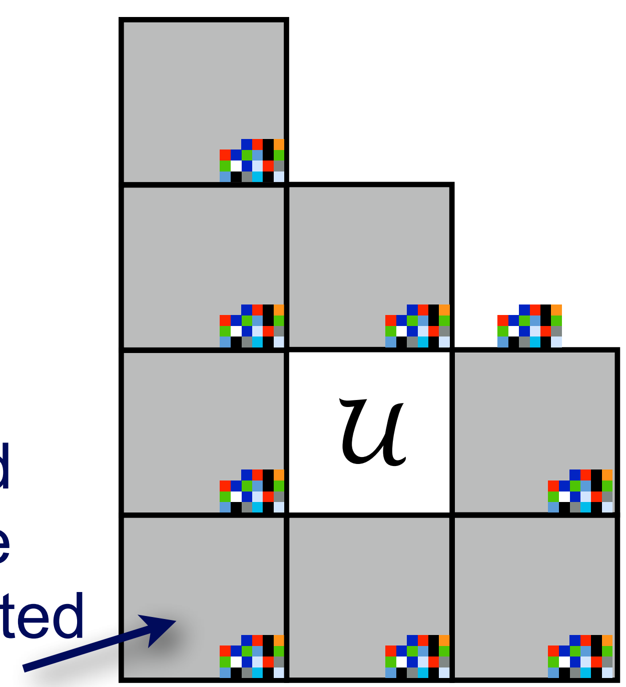


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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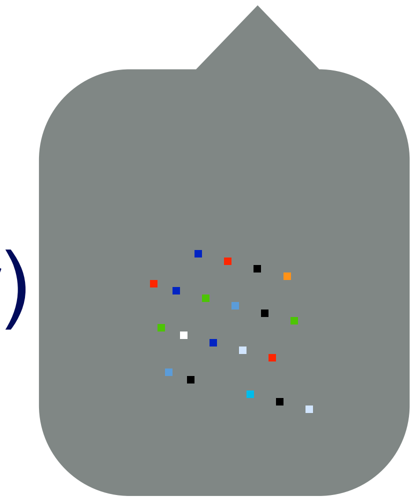
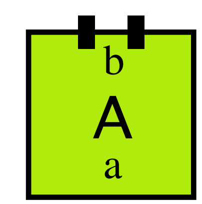
Simulation definition

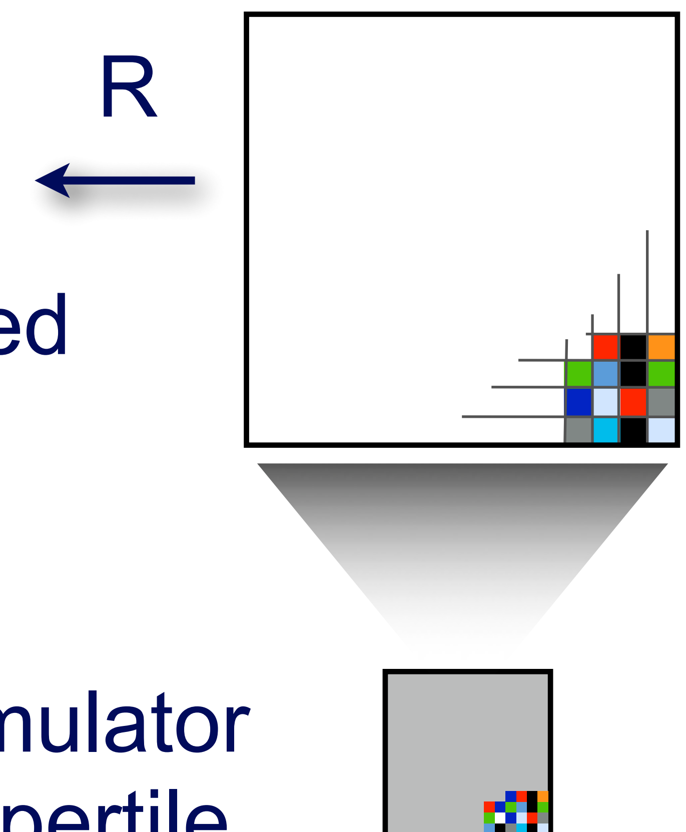
Universal (simulator) tile set



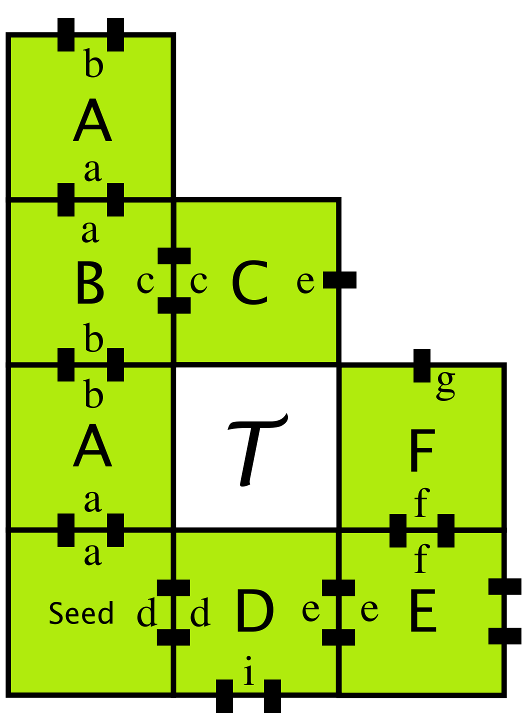
Temperature = 2



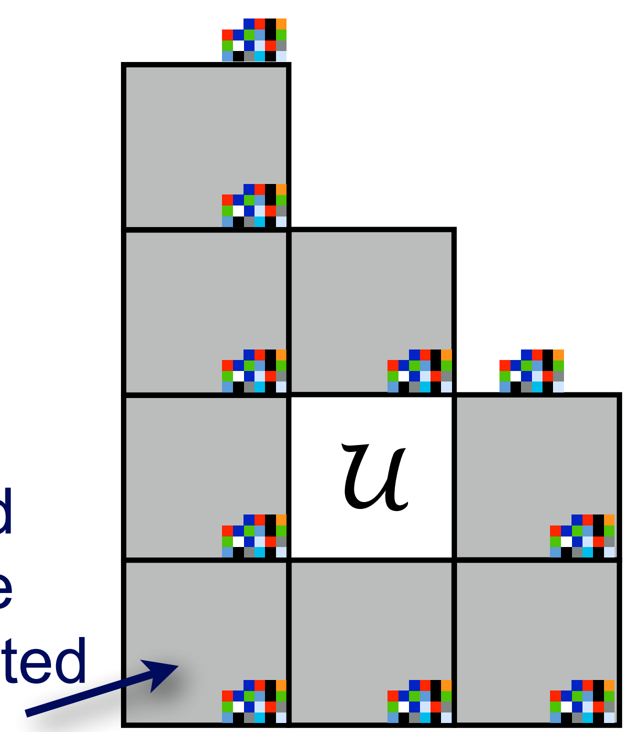
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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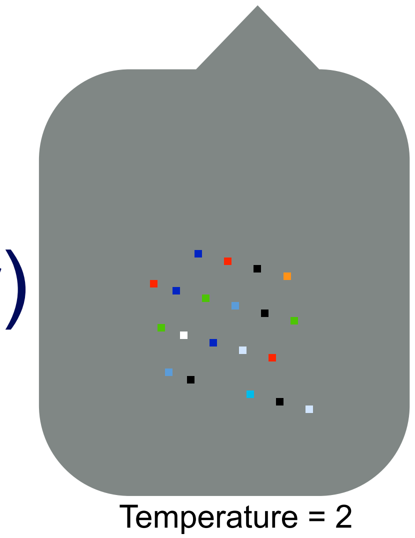
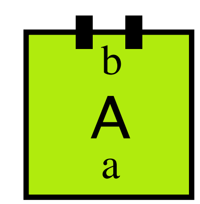
Simulation definition

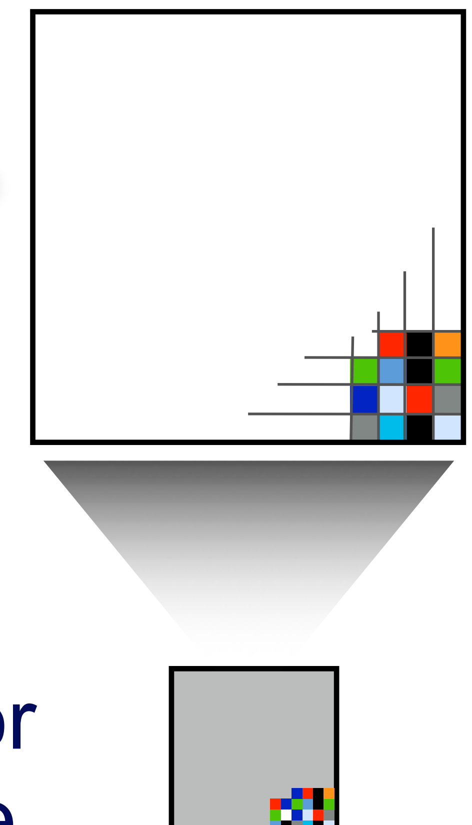
Universal (simulator) tile set



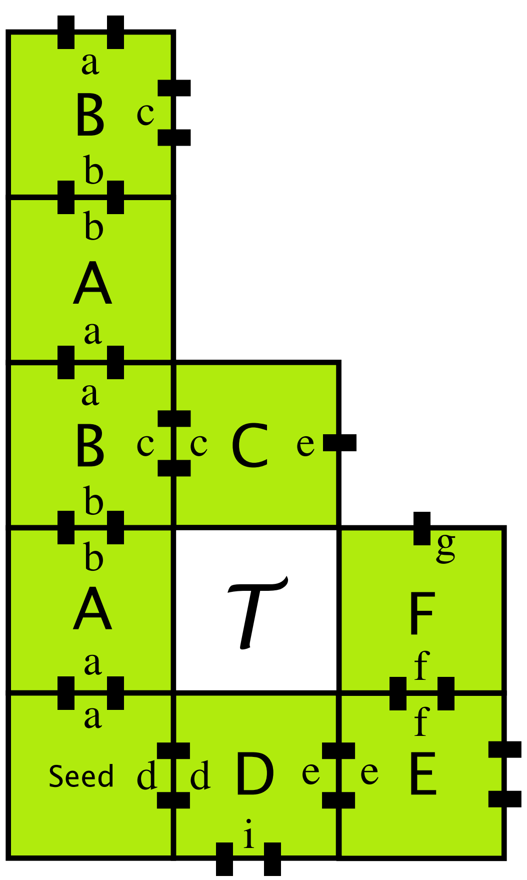


R

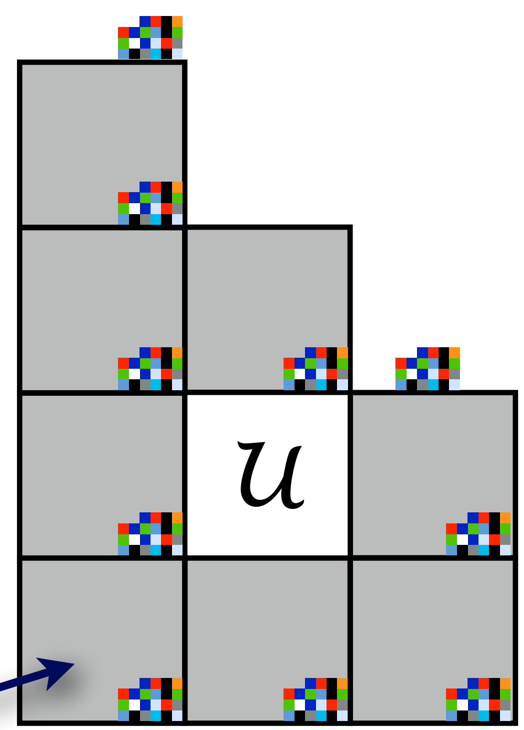
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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Simulation definition

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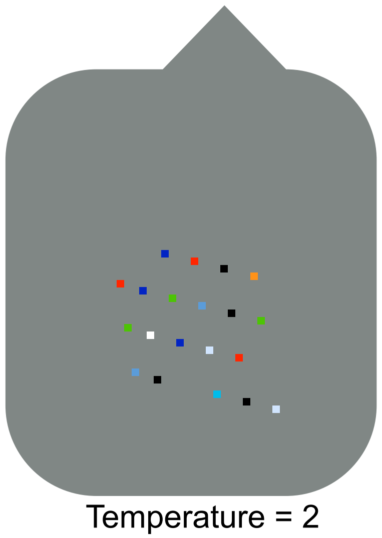
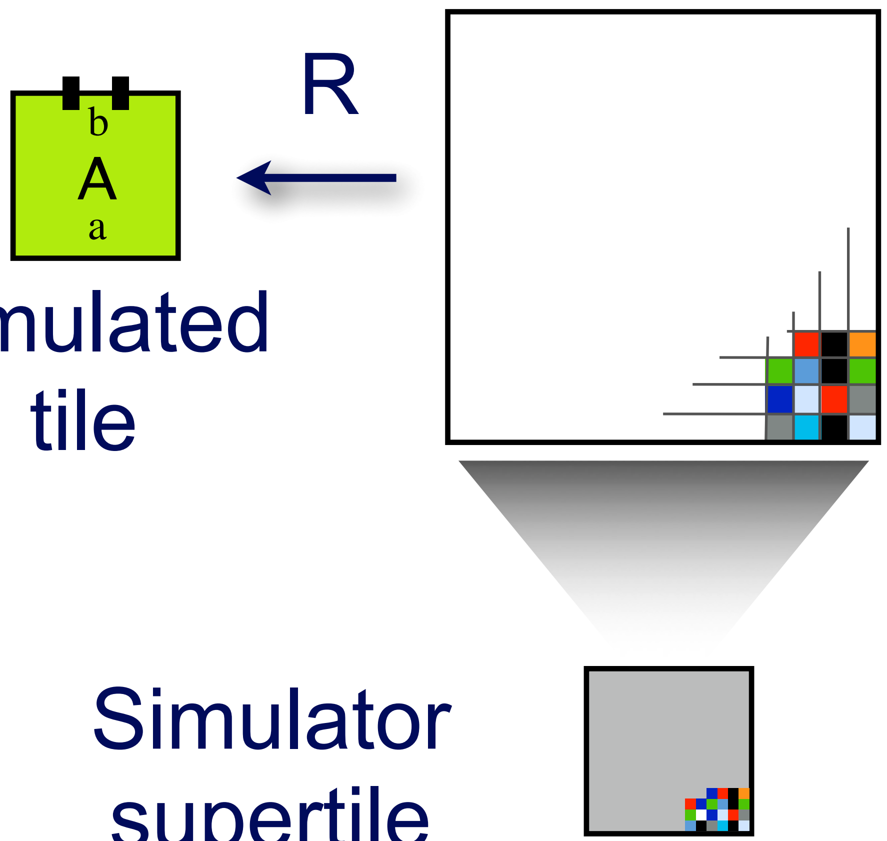
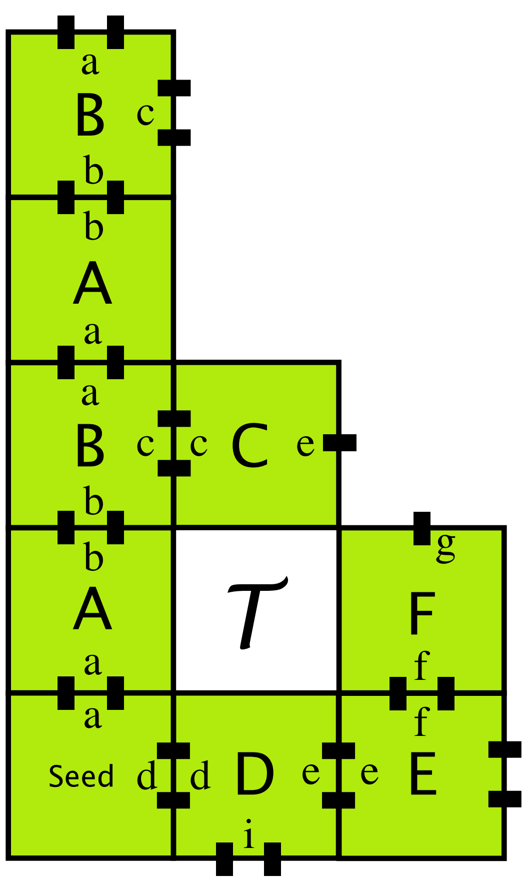


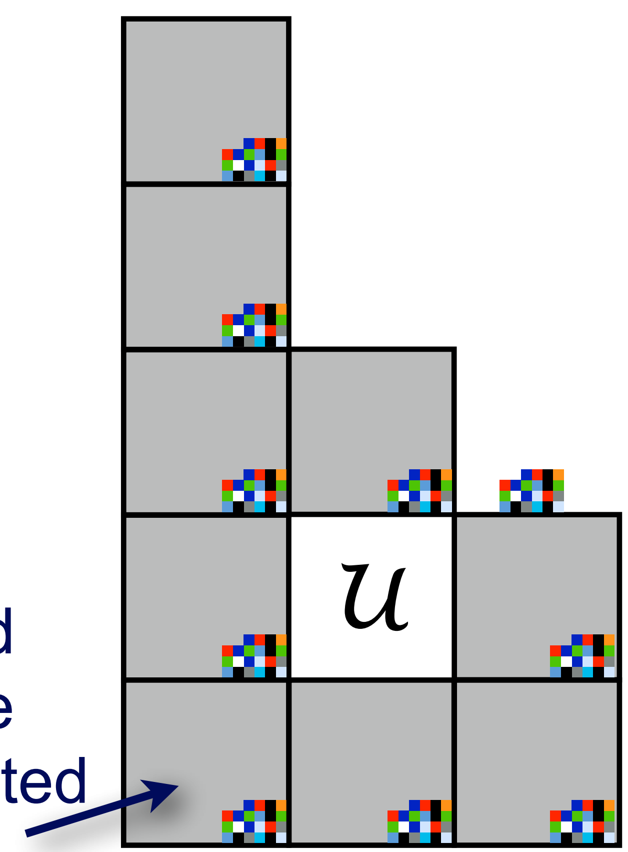


Simulated tile

Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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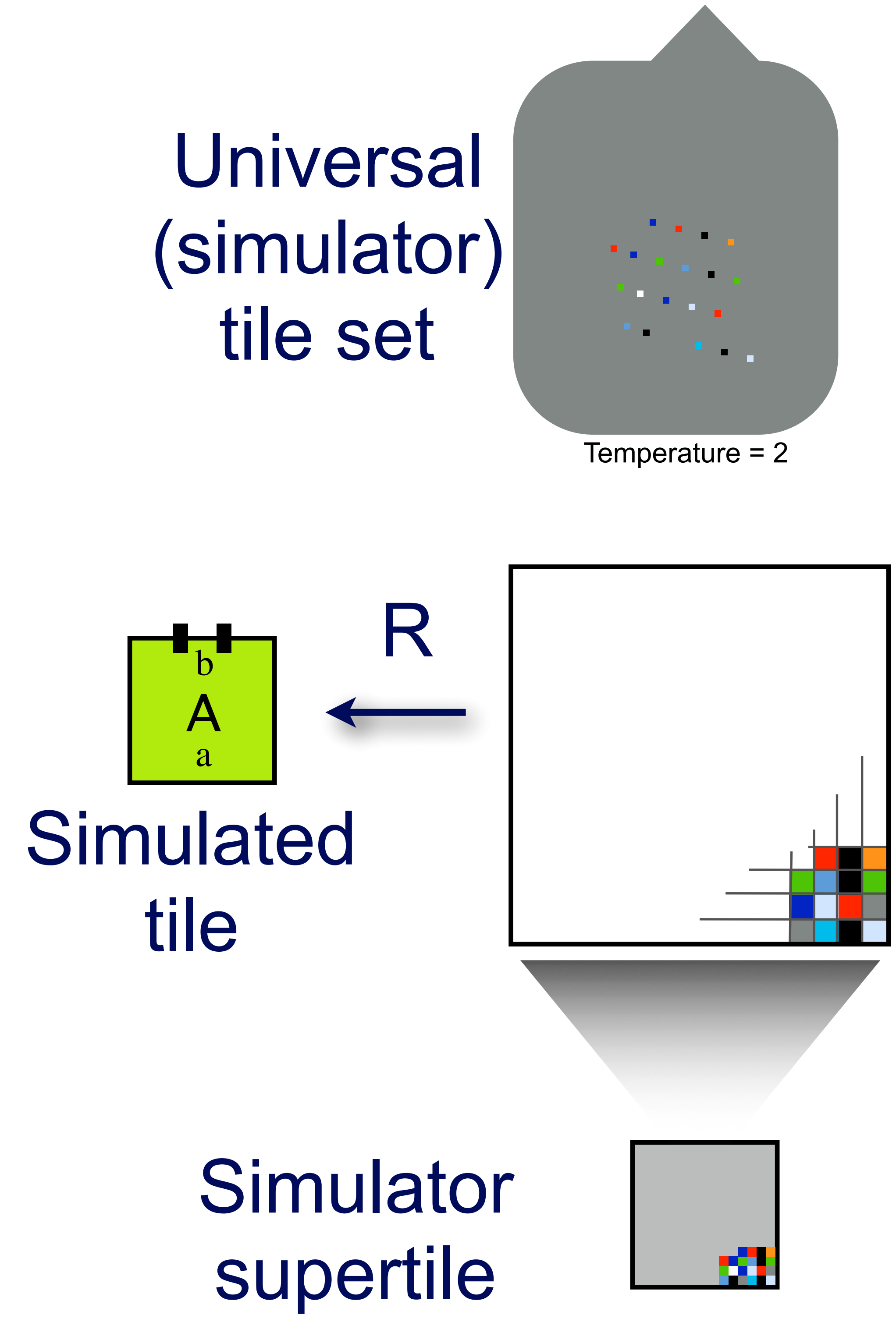

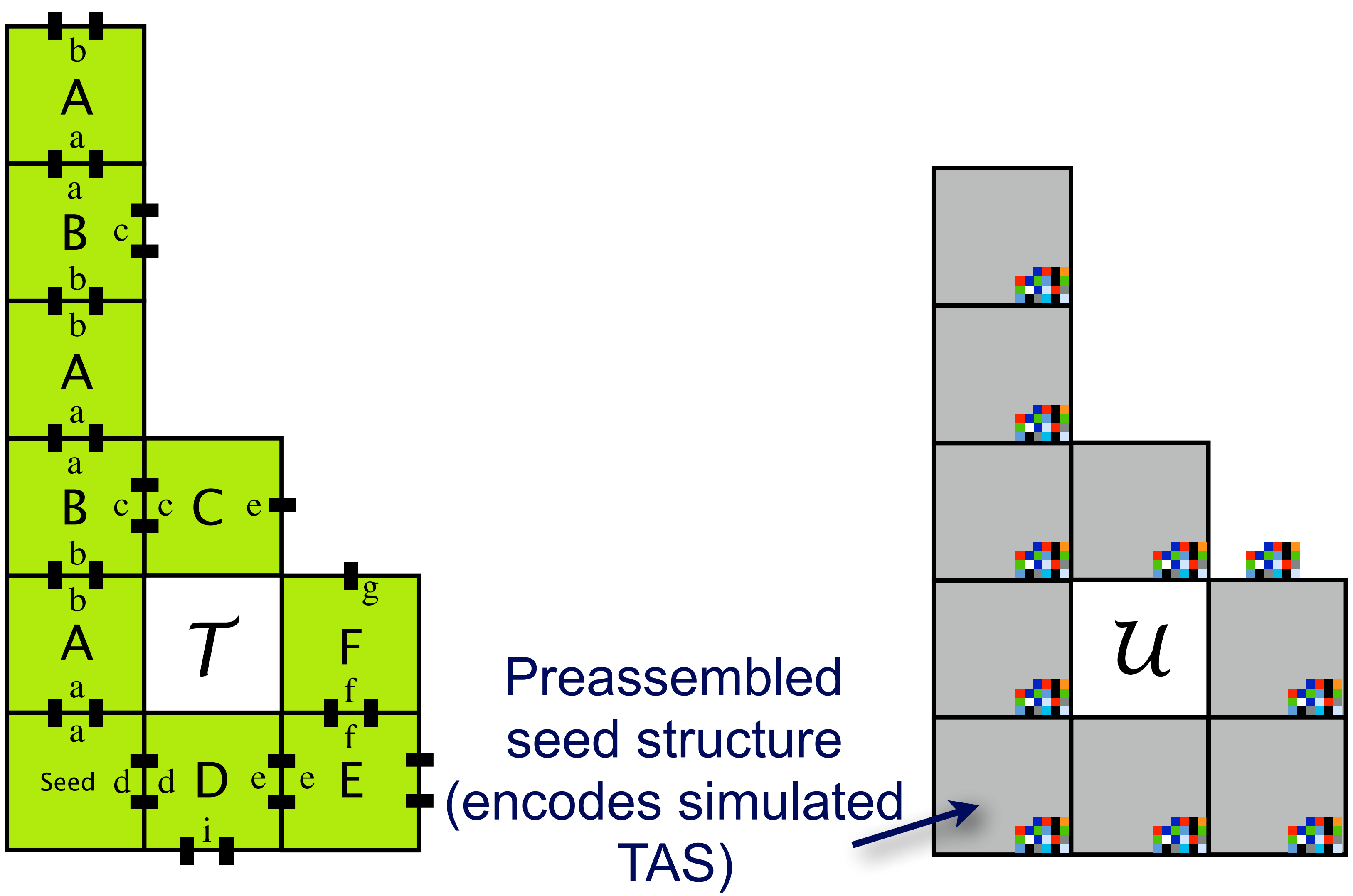

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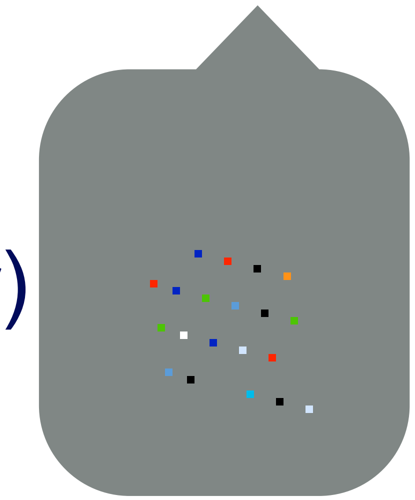

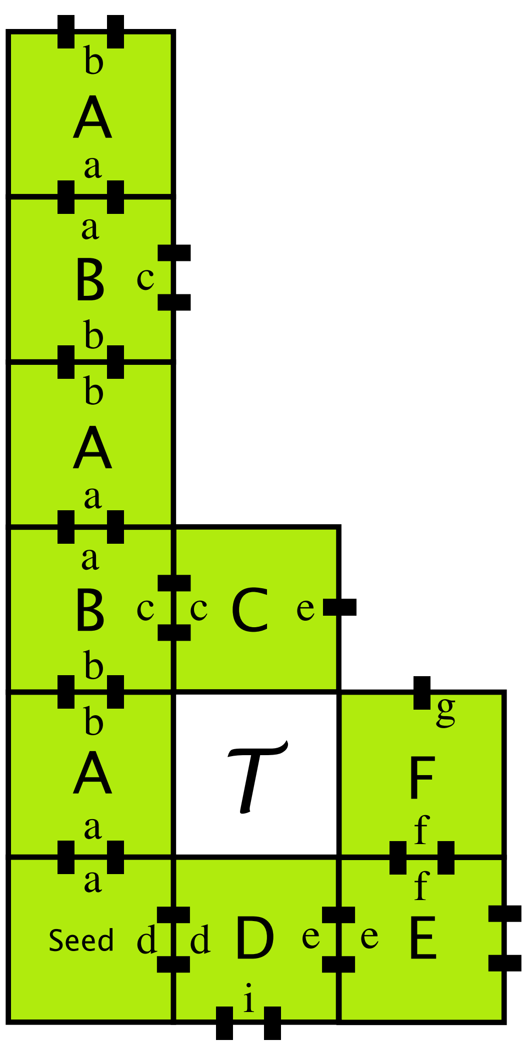

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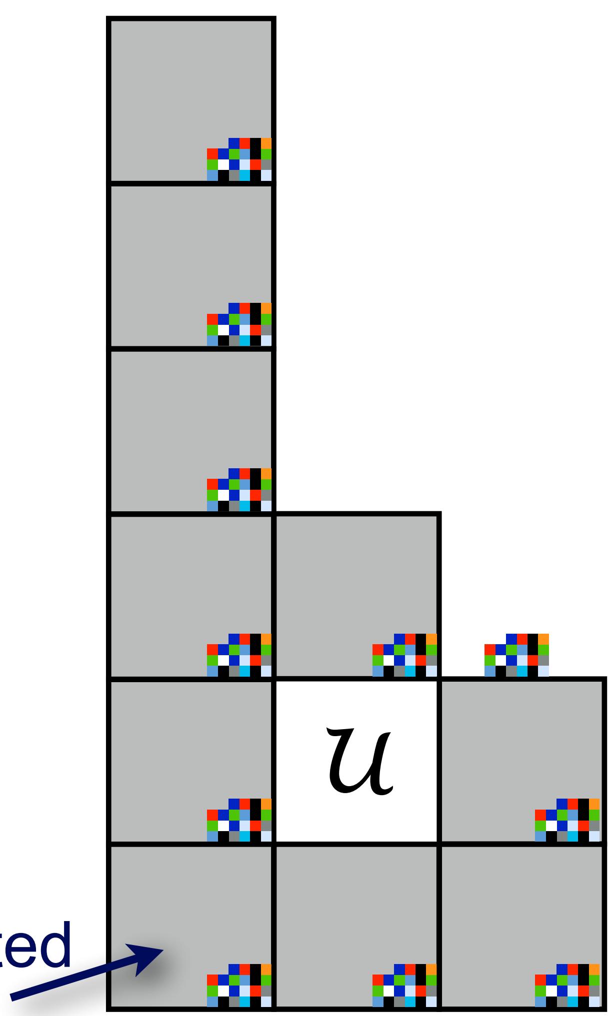

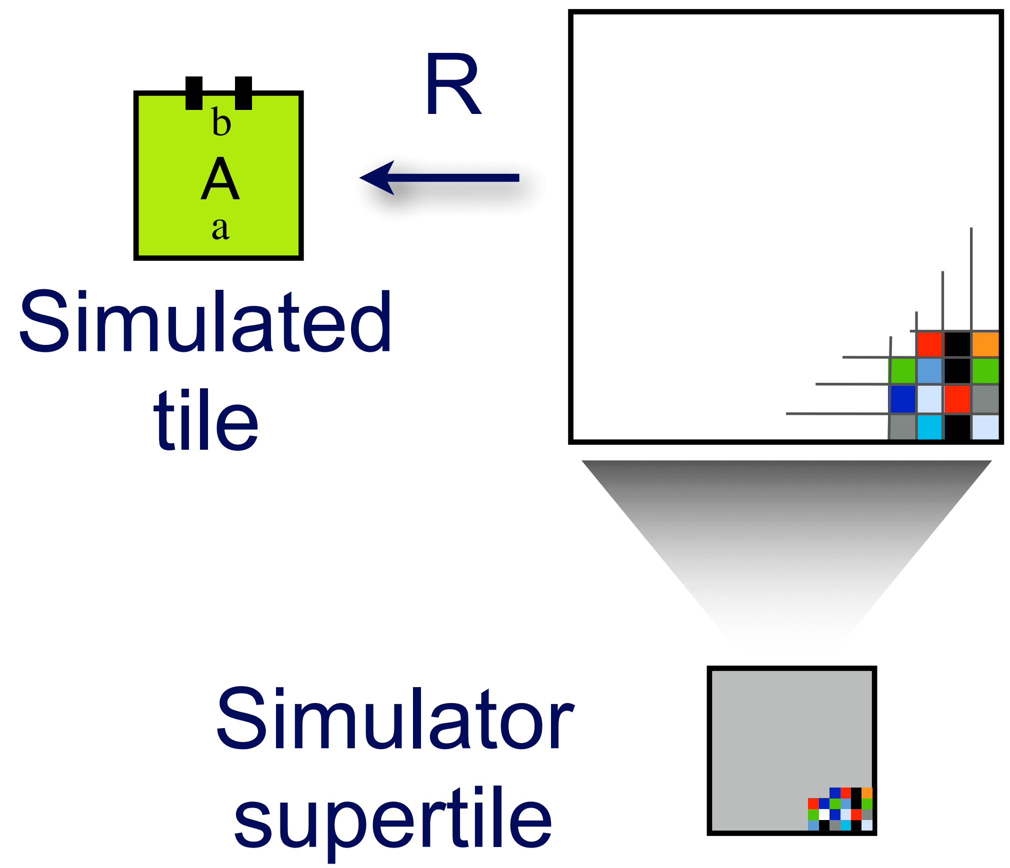

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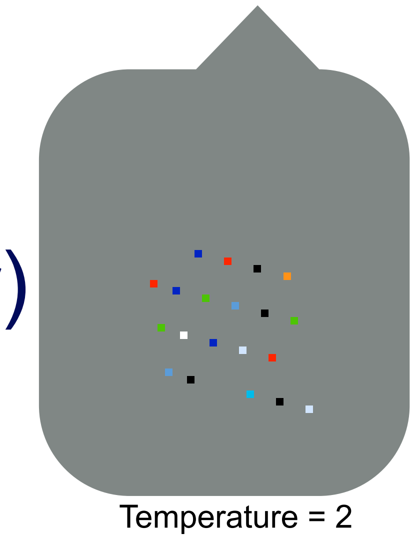
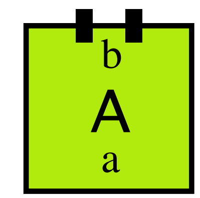
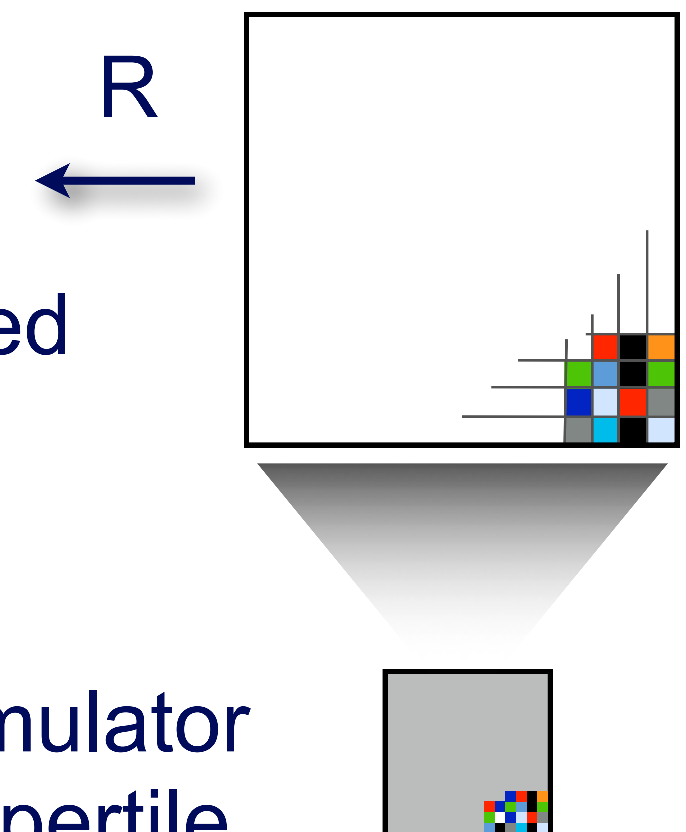


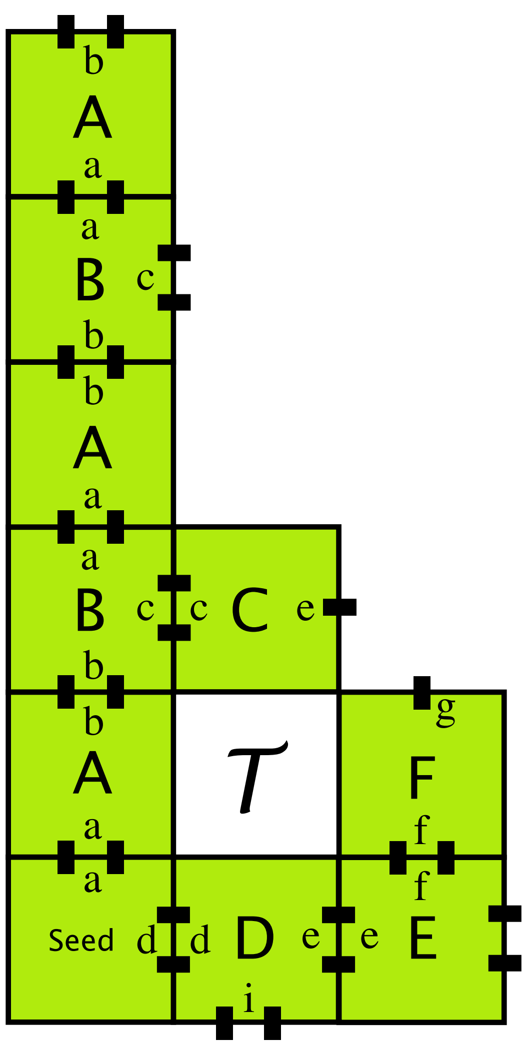


R

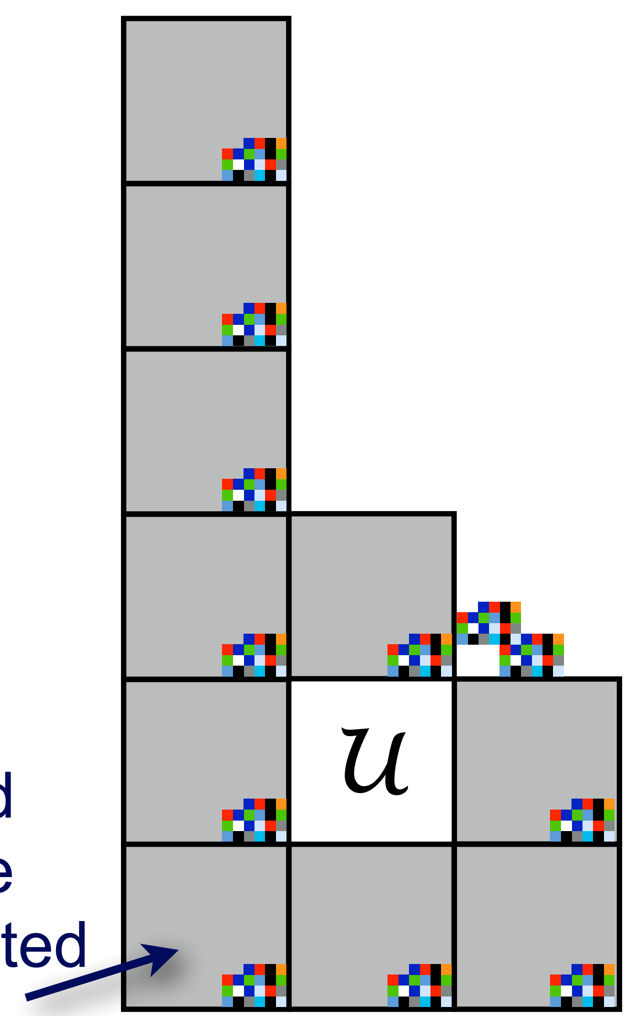
Simulated tile



Simulator supertile



Preassembled seed structure (encodes simulated TAS)



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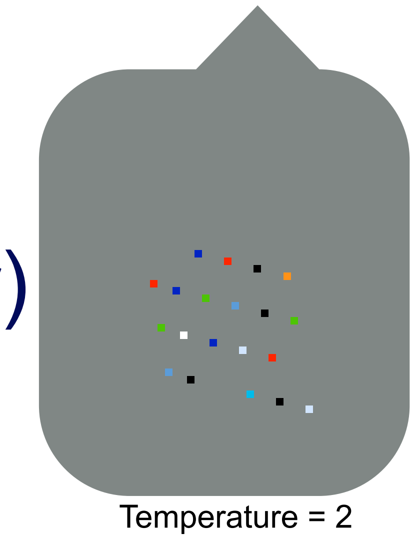

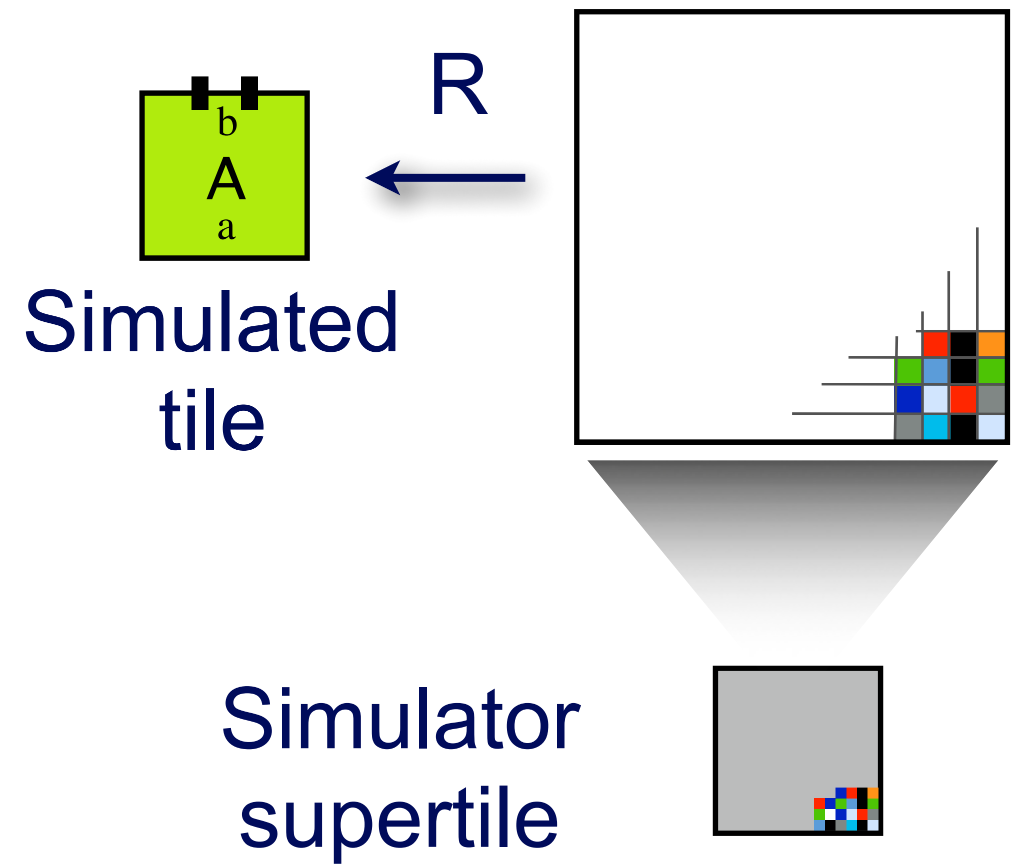

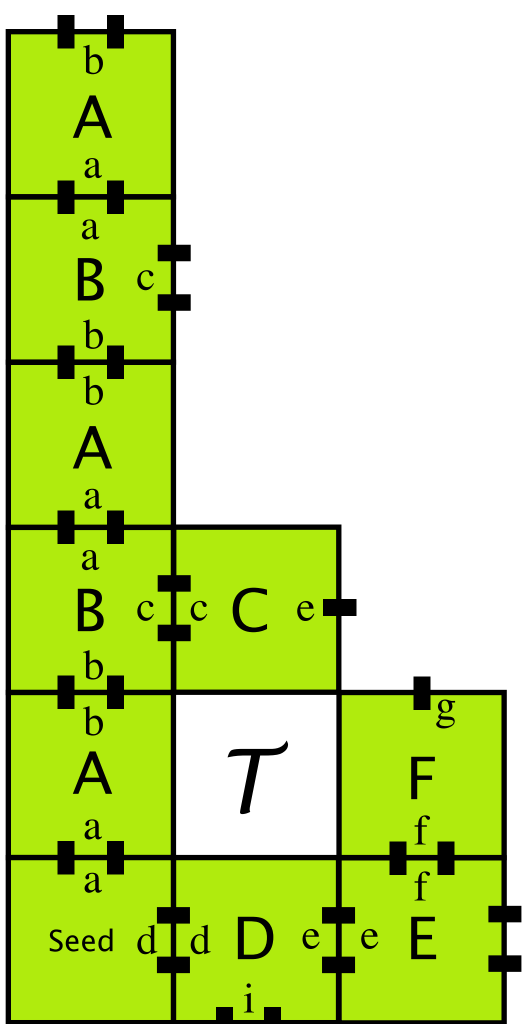

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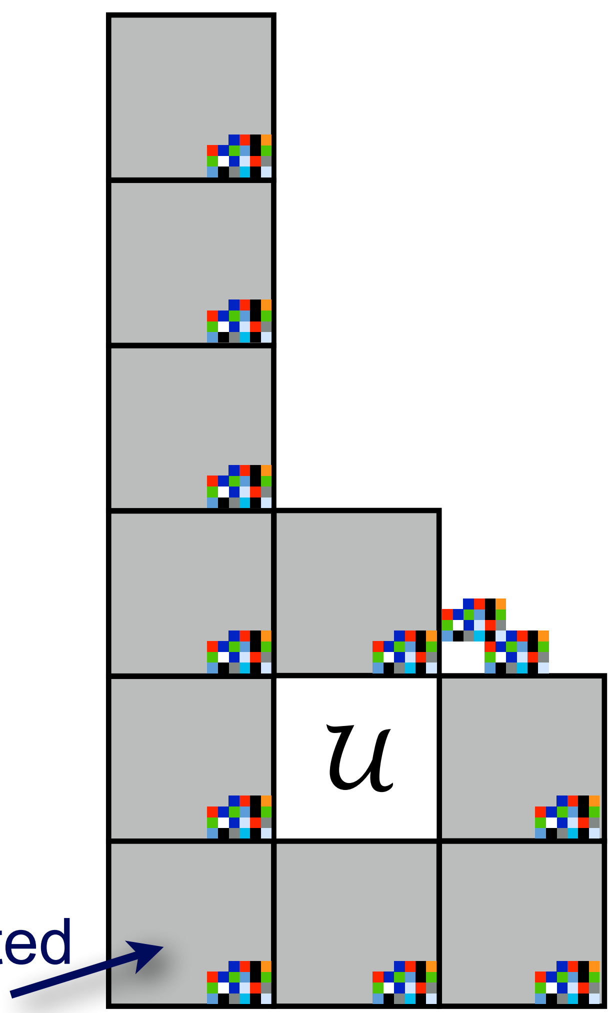

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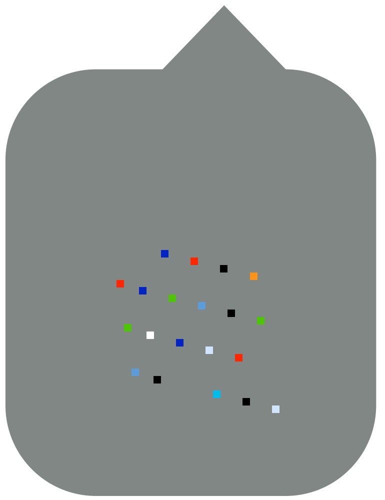
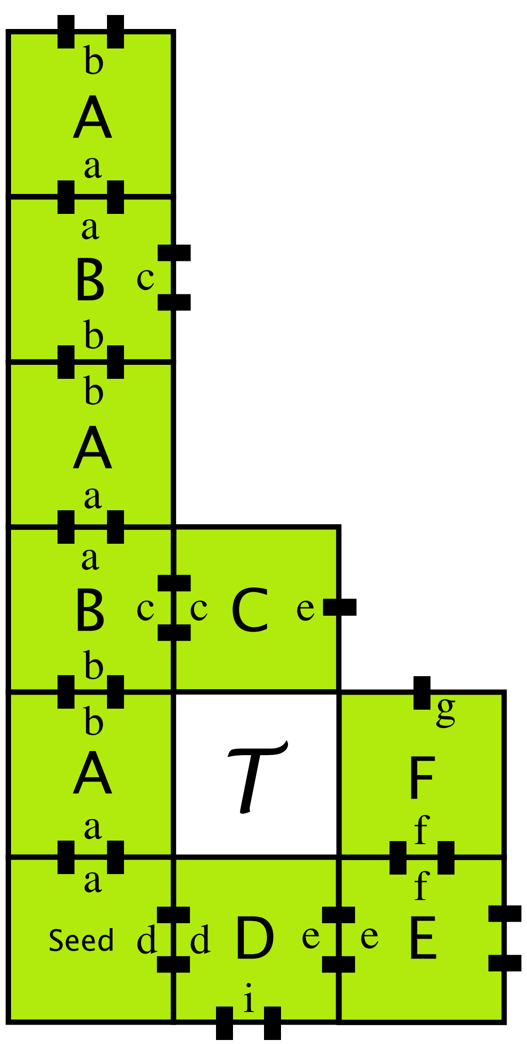
etc.

Simulation definition

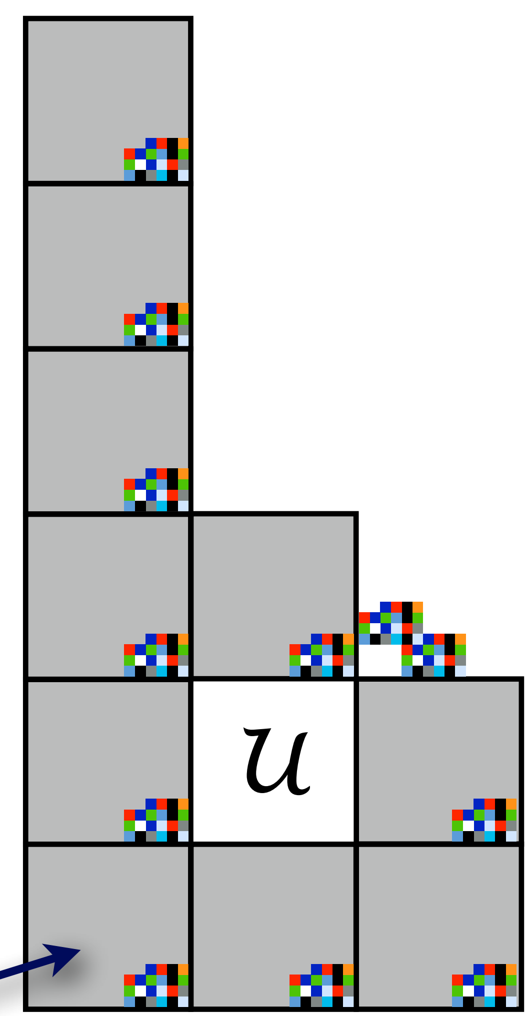
Ignoring *m* x *m* scaling, production & dynamics are equivalent in the simulated system and simulator

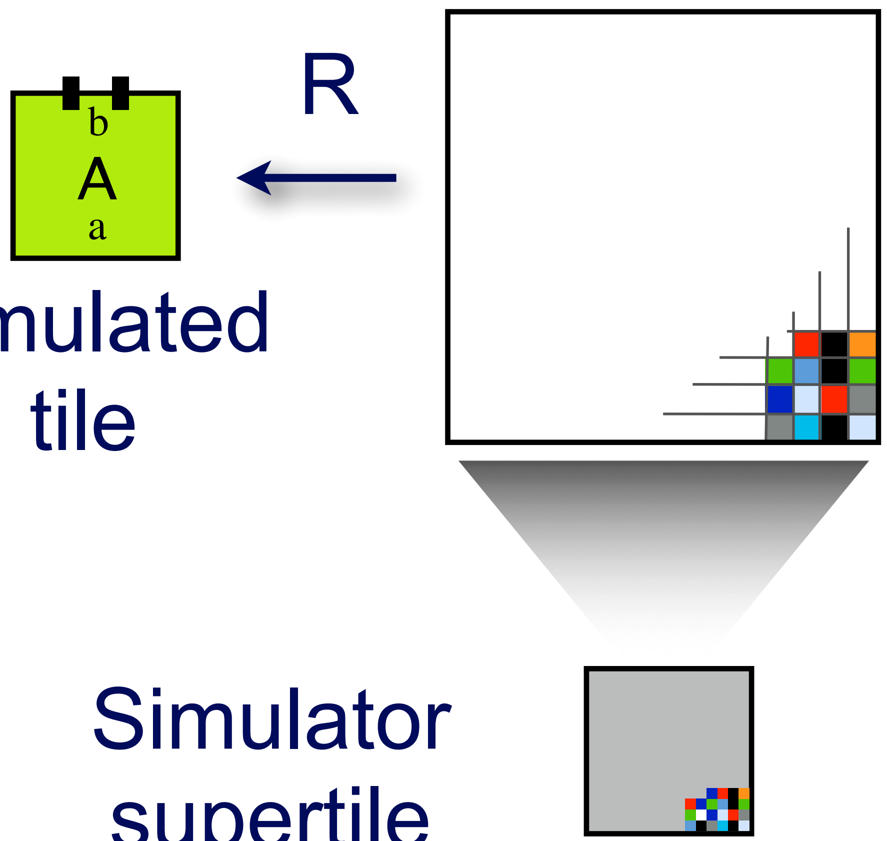
Universal (simulator) tile set





Preassembled seed structure (encodes simulated TAS)





Simulated tile

Simulator supertile

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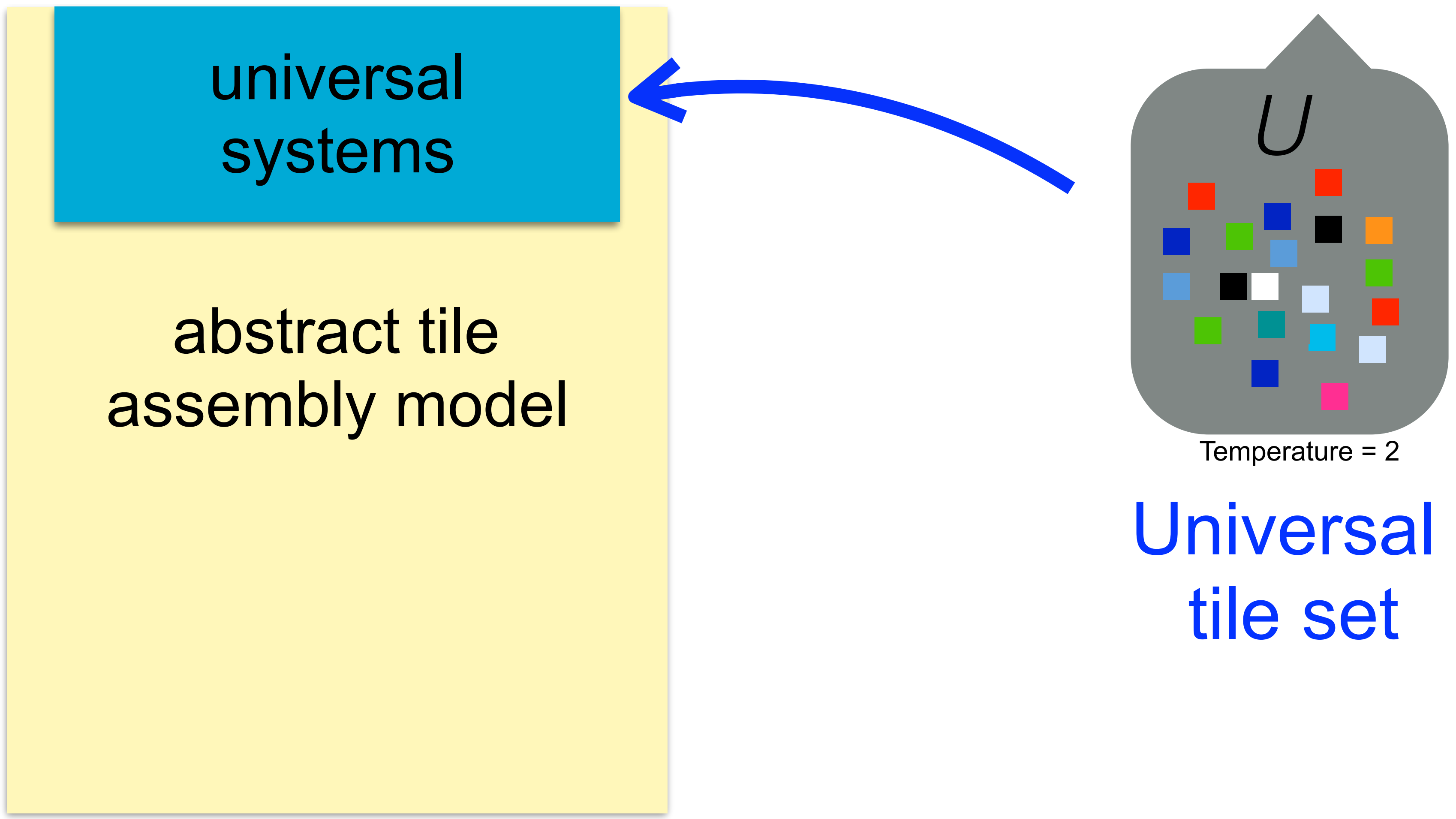
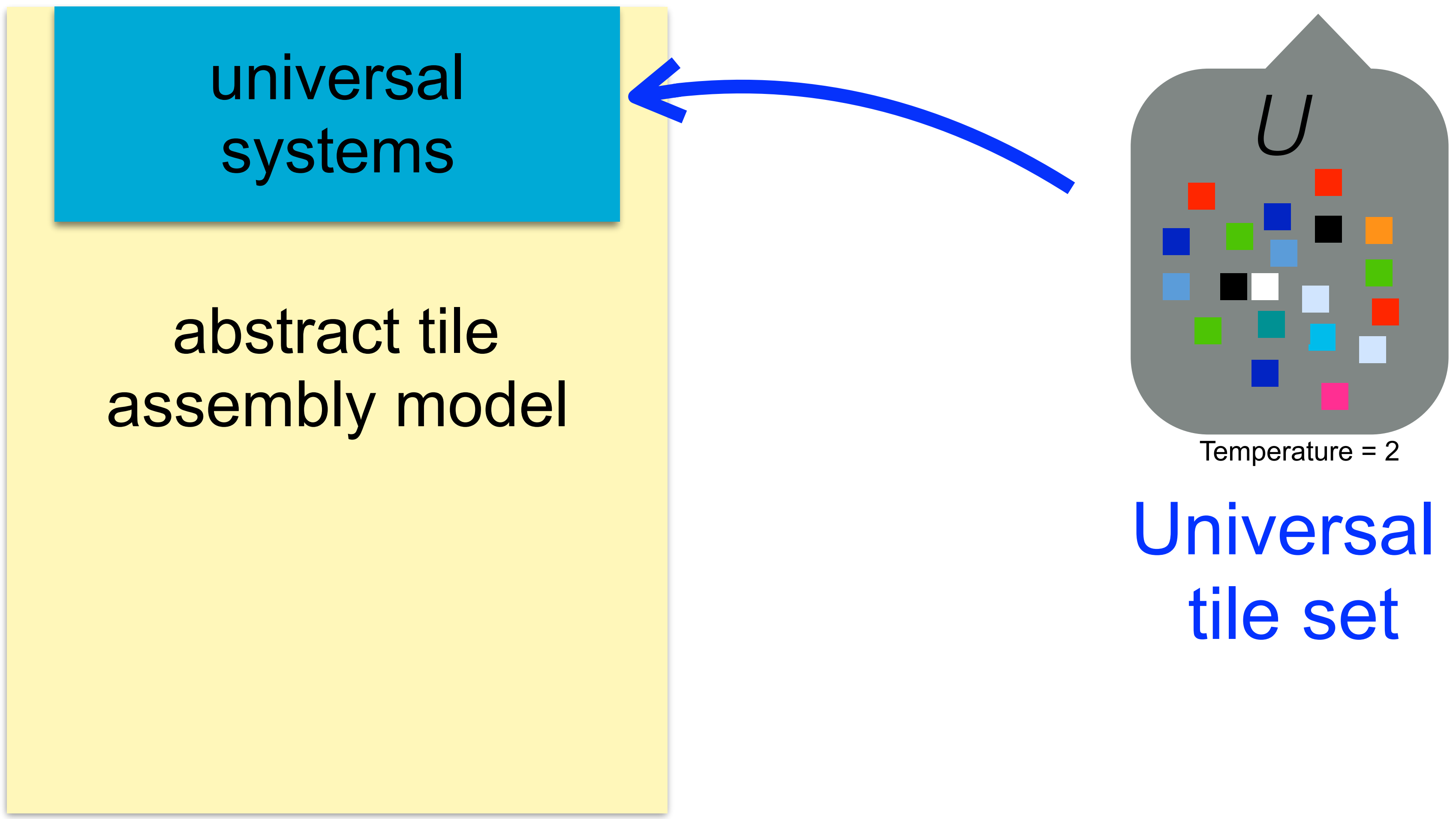
etc.

Is the abstract tile assembly model intrinsically universal?



Is the abstract tile assembly model intrinsically universal? **Yes!**

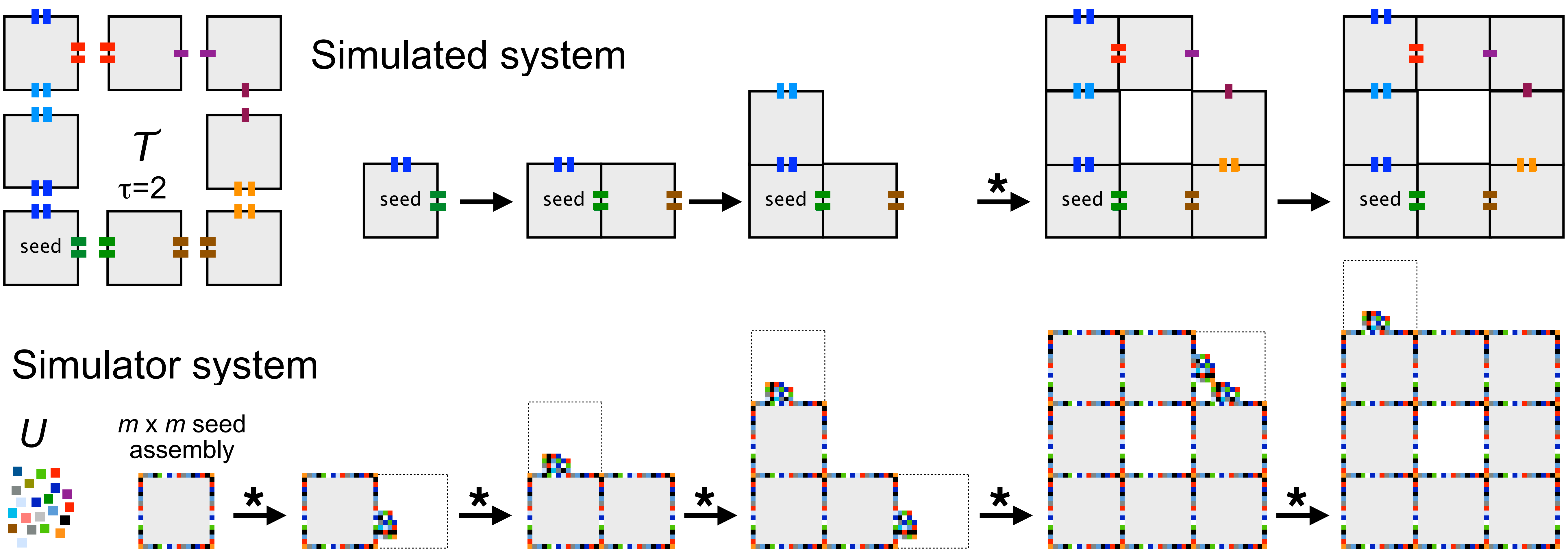


Theorem: There is a single intrinsically universal tile set *U* that simulates *any* tile assembly system

Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

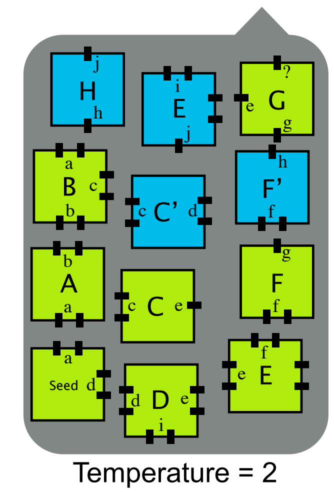
Simulation

- For (any) simulated tile assembly system $\mathcal{T}$
  - $\mathcal{T}$ = (tileset T, seed assembly σ, temperature τ)
- Tile assembly system $\mathcal{U}$ simulates $\mathcal{T}$ if:
  - **Tiles** from $\mathcal{T}$ are represented by ***m* x *m* supertiles** in $\mathcal{U}$
  - Assemblies produced by $\mathcal{U}$ **represent exactly** assemblies produced by $\mathcal{T}$ (via a representation function R : Blocks of tiles from *U* ⟶ tiles from T)
  - **Dynamics are equivalent** in $\mathcal{U}$ and $\mathcal{T}$, ignoring *m* x *m* scaling



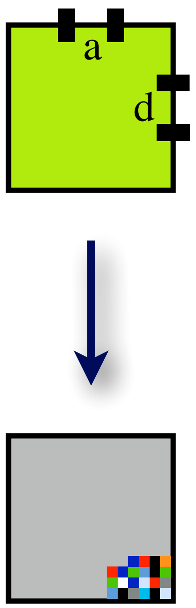
Superside

Encoding of the entire simulated **tile assembly system** written down using tiles from the simulator *U*

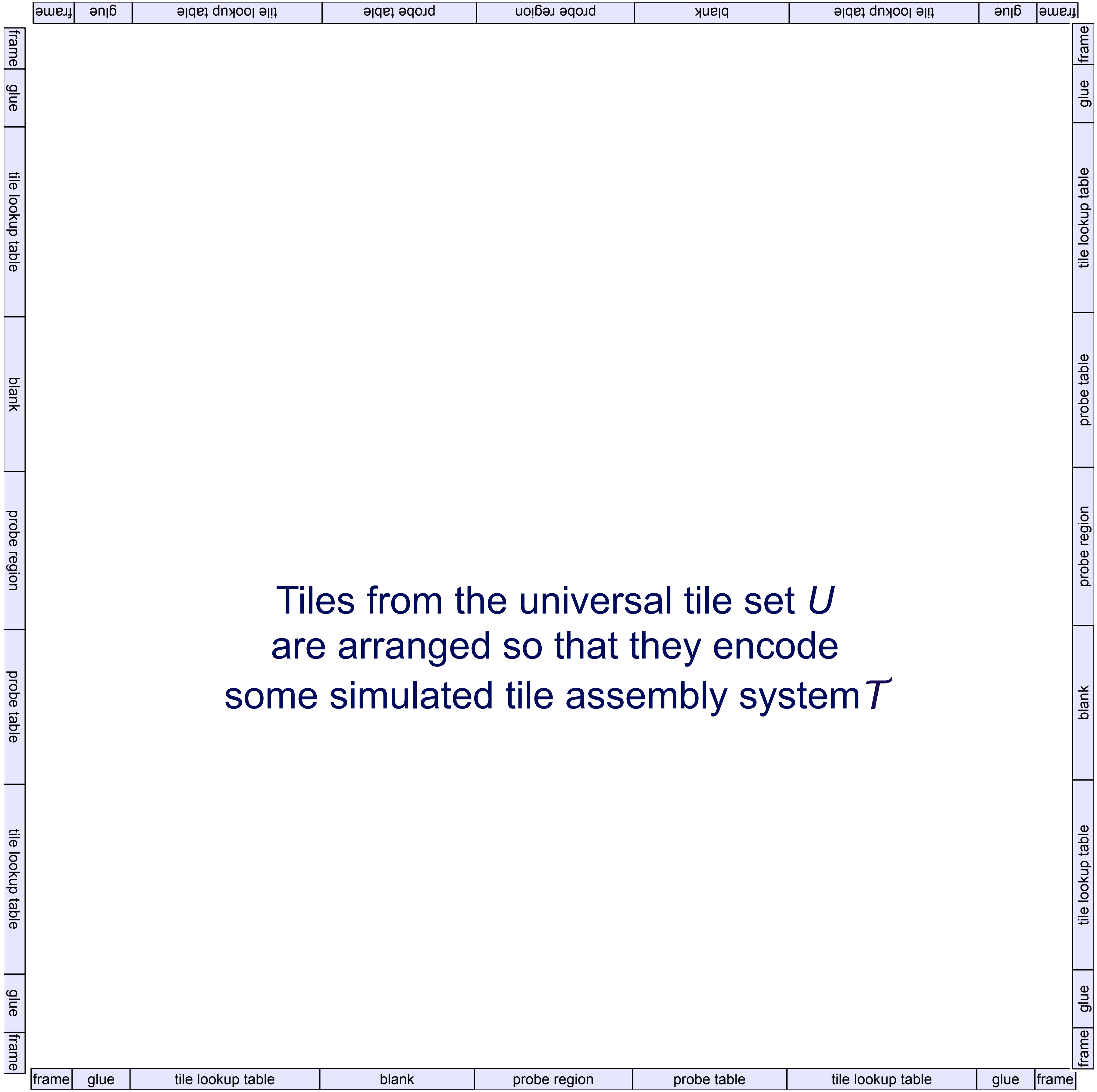


frame	glue	tile lookup table	blank	probe region	probe table	tile lookup table	glue	frame
4	$O(\log \lvert T \rvert)$	$O(\lvert T \rvert^4 \log \lvert T \rvert)$	$O(\lvert T \rvert^2)$	$O(\lvert T \rvert^2)$	$O(\lvert T \rvert^2)$	$O(\lvert T \rvert^4 \log \lvert T \rvert)$	$O(\log \lvert T \rvert)$	4

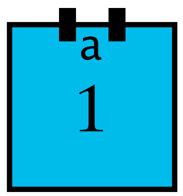
$|T|$ is number of tiles in the simulated tileset T.



Preassembled
seed supertile

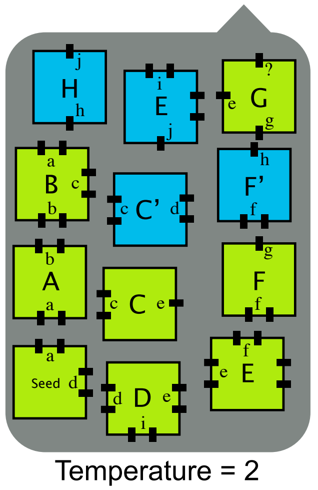


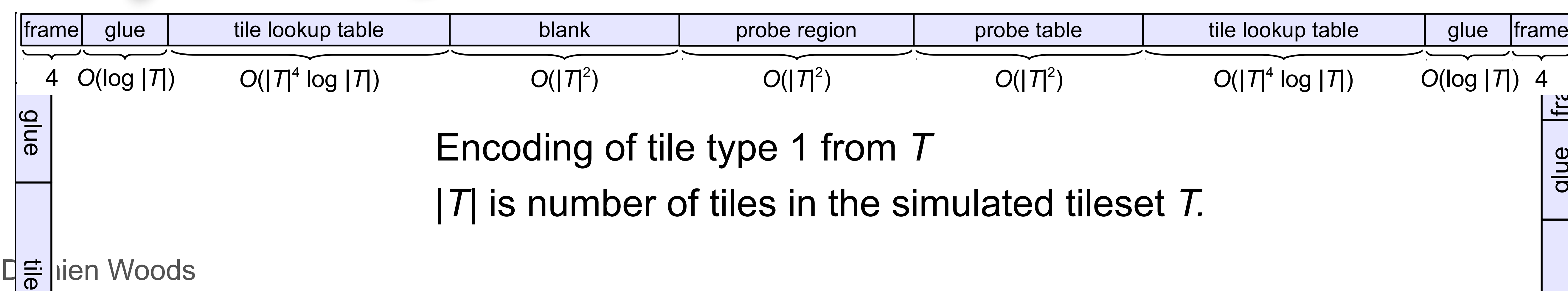
Superside



Encoded **glue** of **this** superside (e.g. “a”)

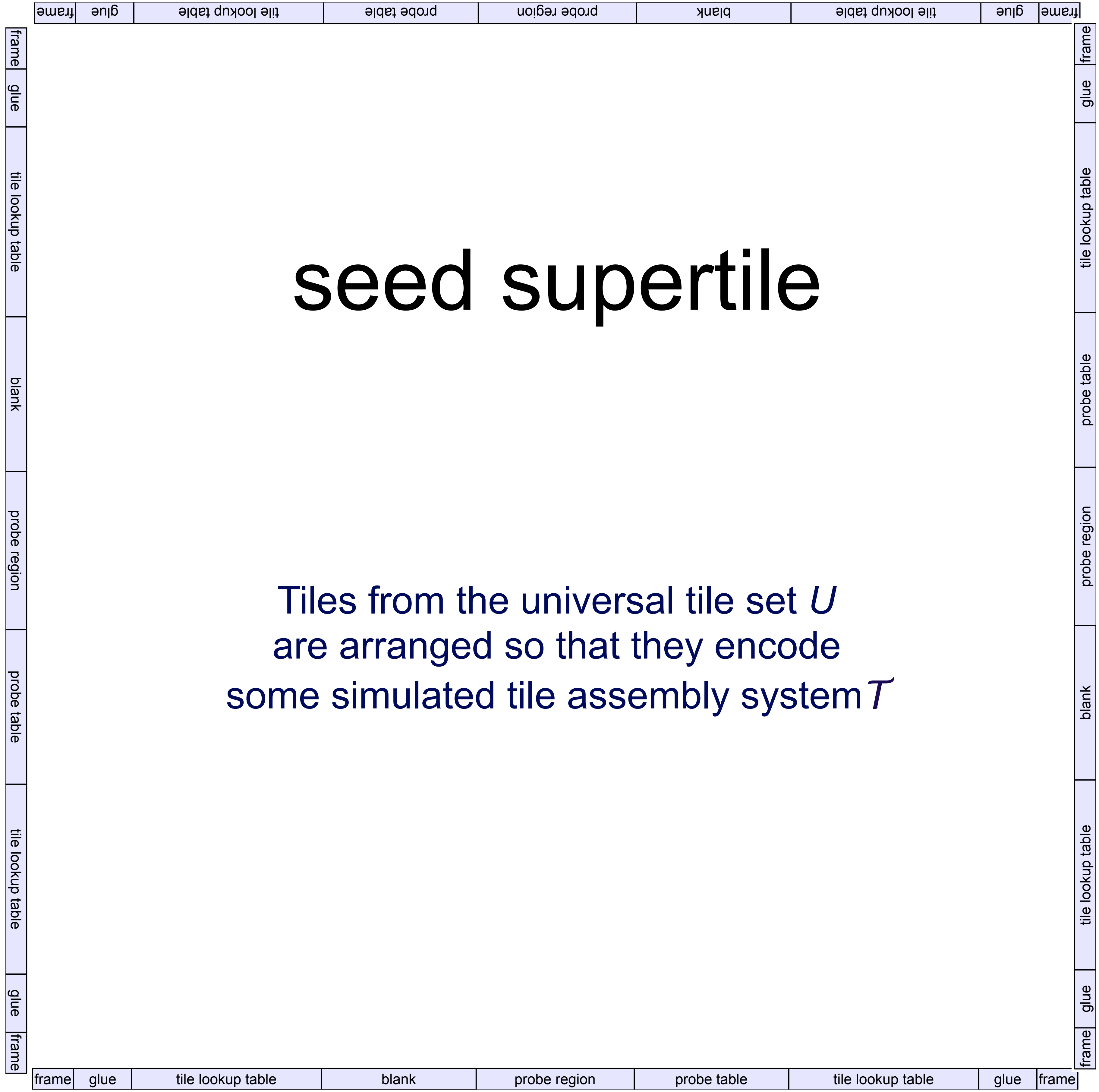
Encoding of the entire simulated **tile assembly system** written down using tiles from the simulator *U*





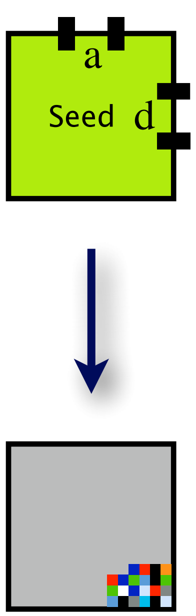
Encoding of tile type 1 from *T*

$|T|$ is number of tiles in the simulated tileset *T*.



seed supertile

Tiles from the universal tile set U
are arranged so that they encode
some simulated tile assembly system $\mathcal{T}$

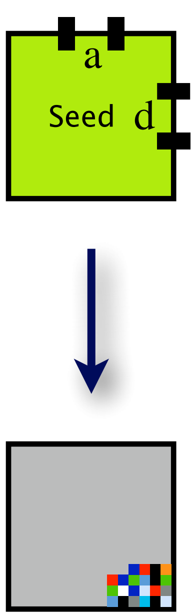


Preassembled
seed supertile

seed supertile

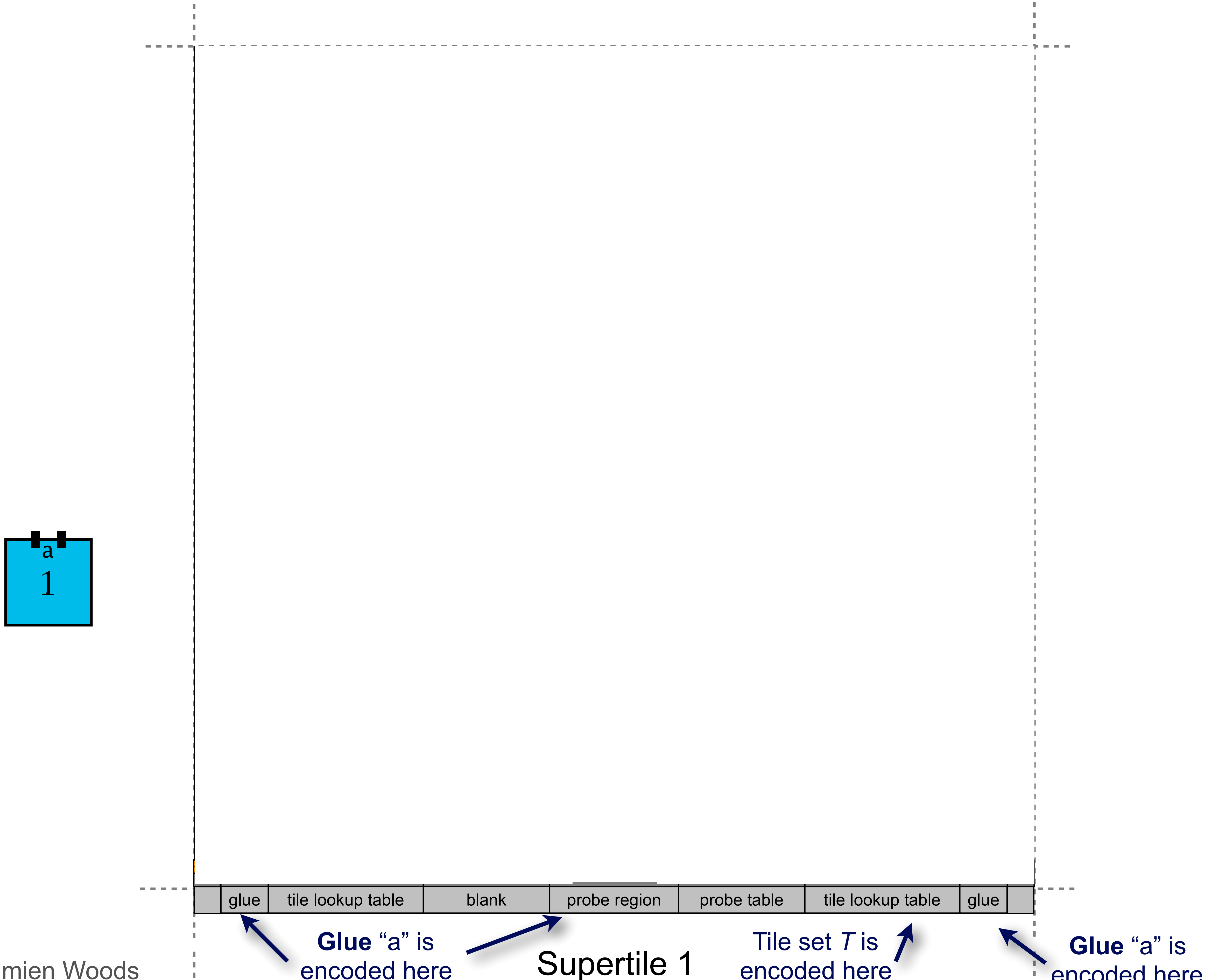
Growth begins from here!

Tiles from the universal tile set *U*
are arranged so that they encode
some simulated tile assembly system $\mathcal{T}$

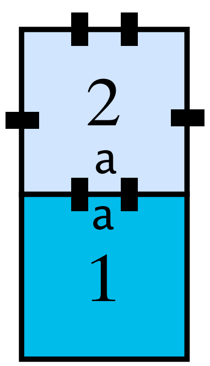


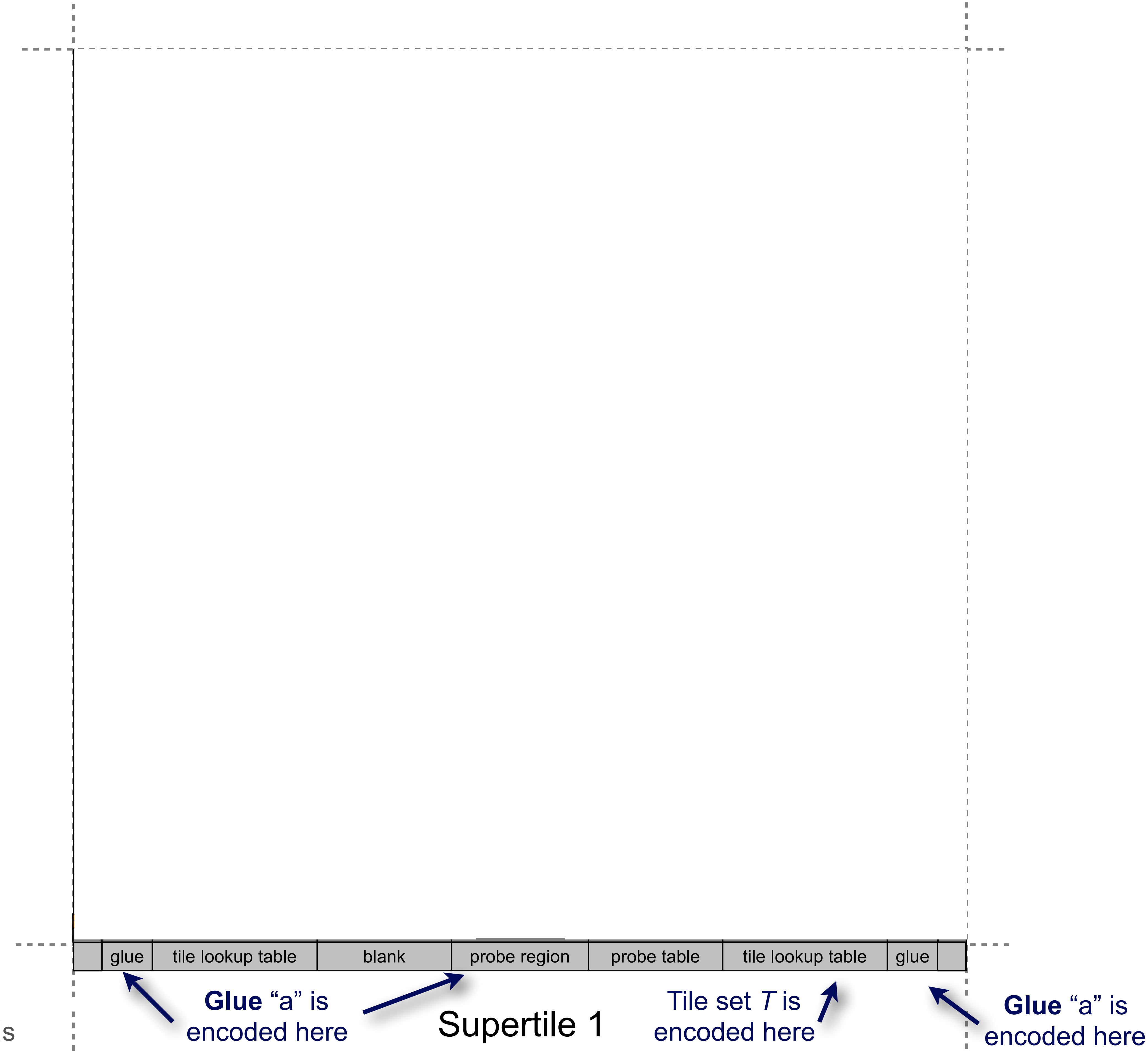
Preassembled
seed supertile

One-sided binding with a single strength-τ south superside

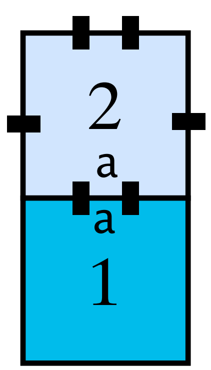


One-sided binding with a single strength-τ south superside





One-sided binding with a single strength-τ south superside



Goal: place a description of *T* **and "tile 2"** around the 4 super-edges

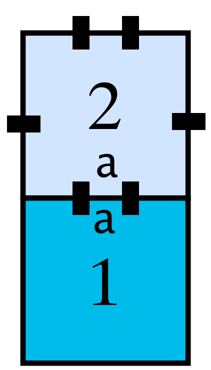
Glue "a" is encoded here

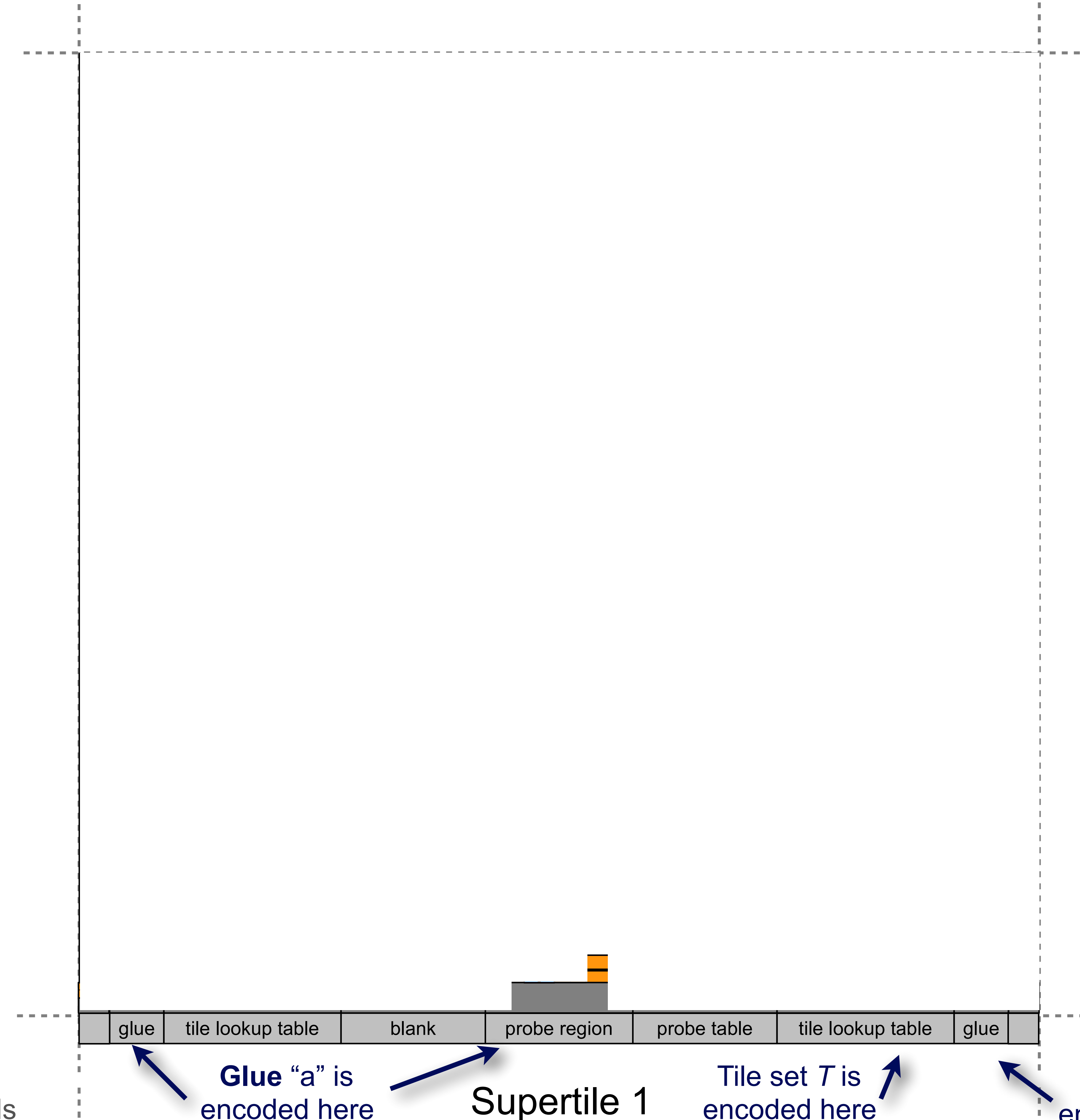
Supertile 1

Tile set *T* is encoded here

Glue "a" is encoded here

One-sided binding with a single strength-τ south superside

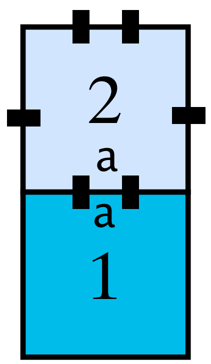


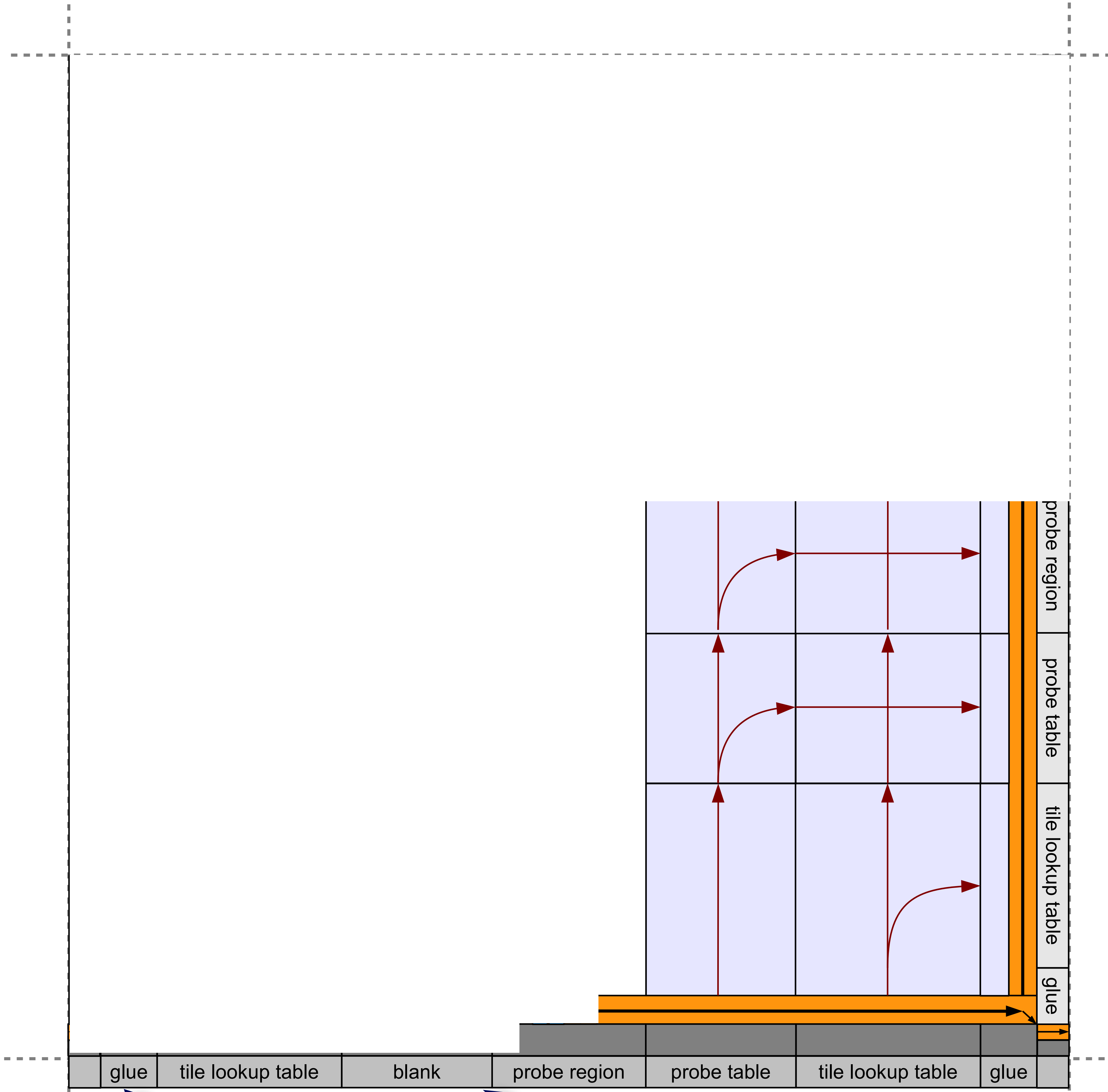


Goal: place a description of *T* **and "tile 2"** around the 4 super-edges

One-sided binding with a single strength-τ south superside

Goal: place a description of *T* **and "tile 2"** around the 4 super-edges





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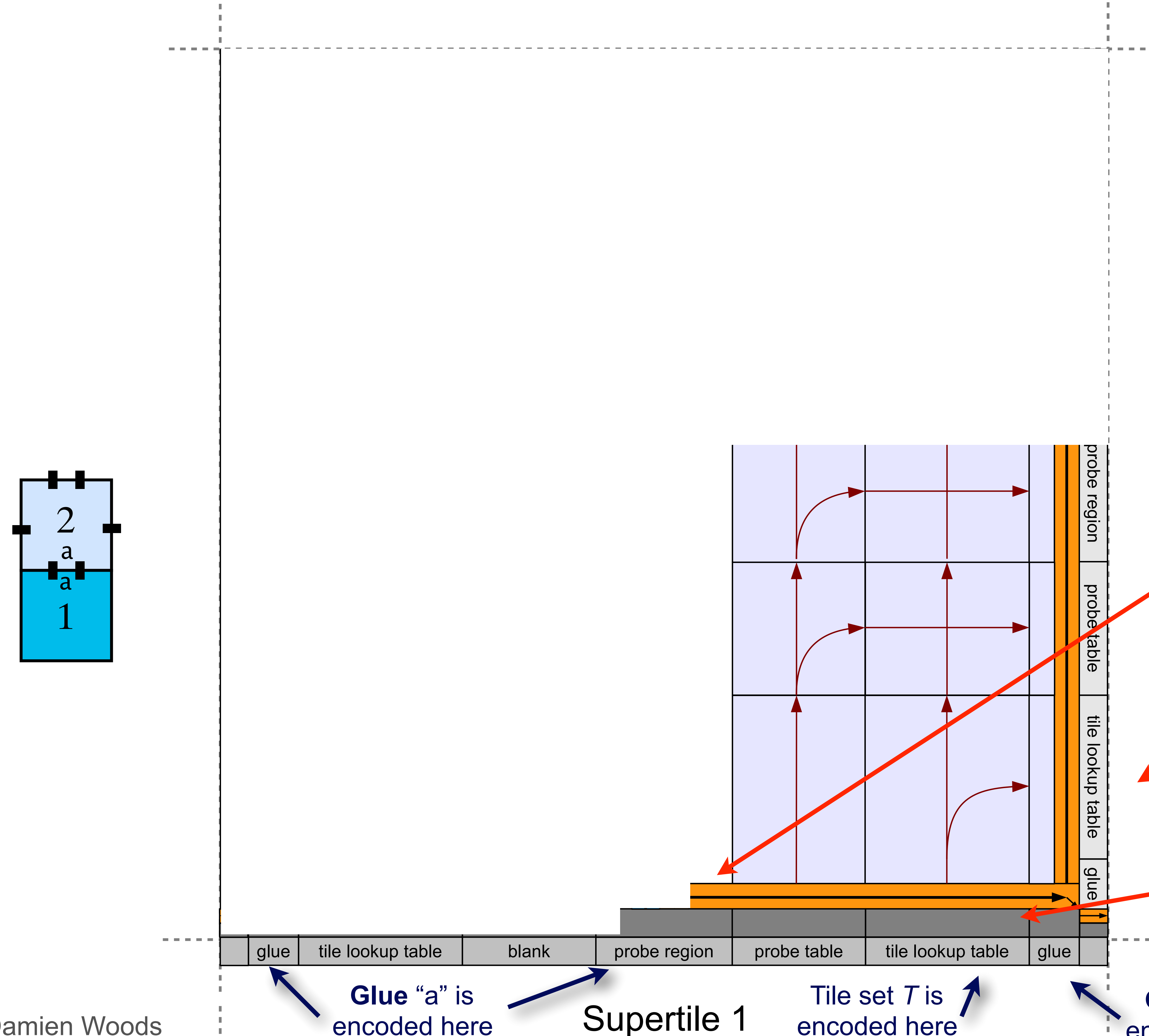
One-sided binding with a single strength-τ south superside



Goal: place a description of *T* **and "tile 2"** around the 4 super-edges

crawler encodes glue of south superside

"Genome" is copied

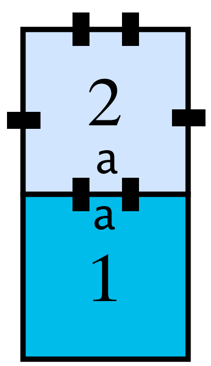
"Genome" is read

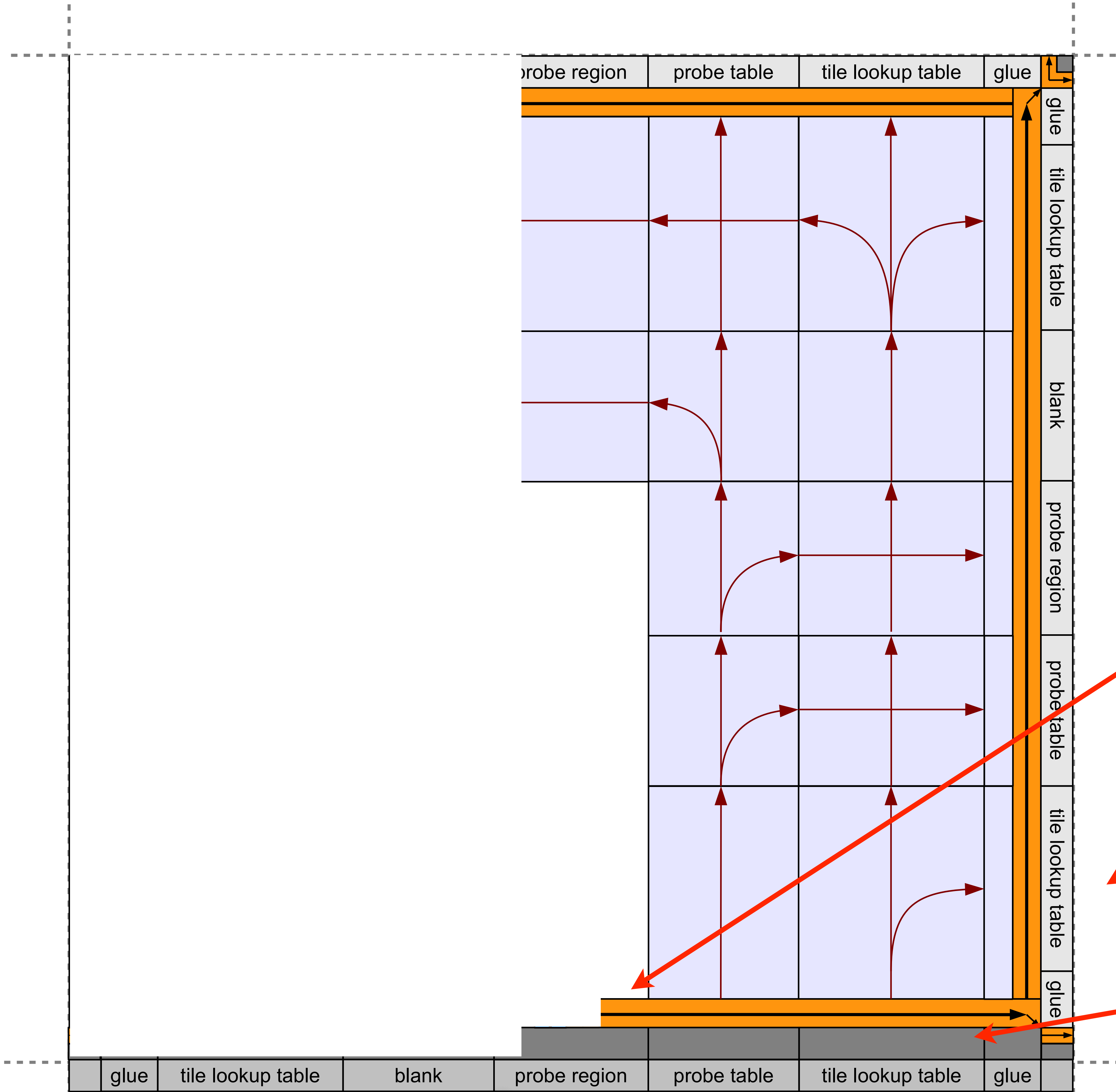
Glue "a" is encoded here

Tile set *T* is encoded here

Glue "a" is encoded here

One-sided binding with a single strength-τ south superside





Goal: place a description of *T* **and "tile 2"** around the 4 super-edges

crawler encodes glue of south superside

"Genome" is copied

"Genome" is read

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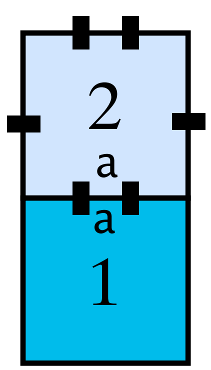
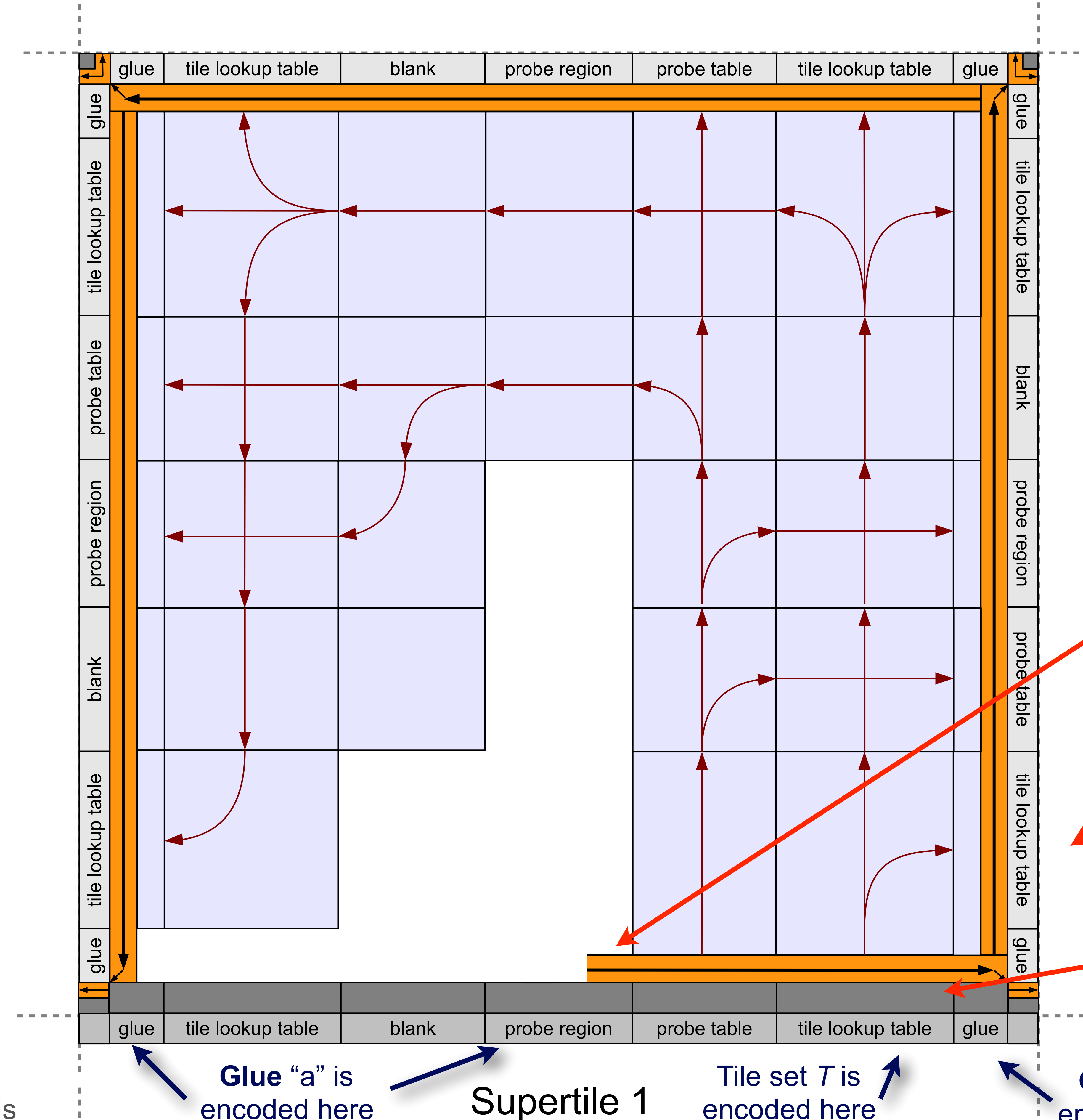




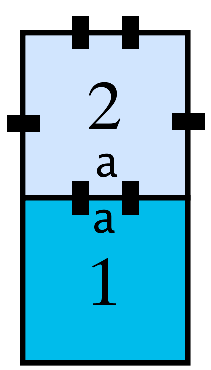
Goal: place a description of *T* **and "tile 2"** around the 4 super-edges

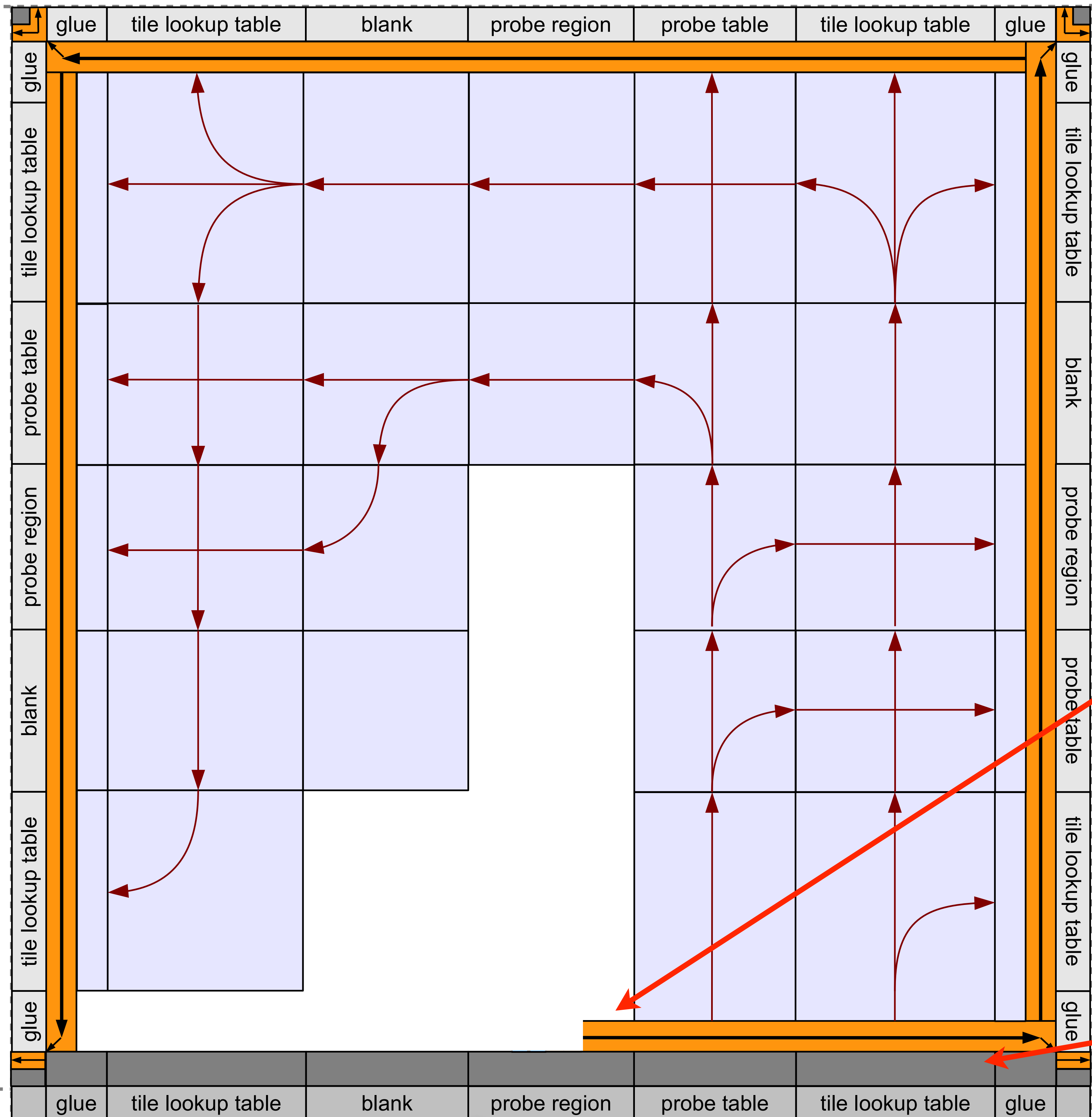
crawler encodes glue of south superside

"Genome" is copied

"Genome" is read

One-sided binding with a single strength-τ south superside





Goal: place a description of T **and "tile 2"** around the 4 super-edges

Nondeterminism
Rotations

crawler encodes glue of south superside

"Genome" is copied

"Genome" is read

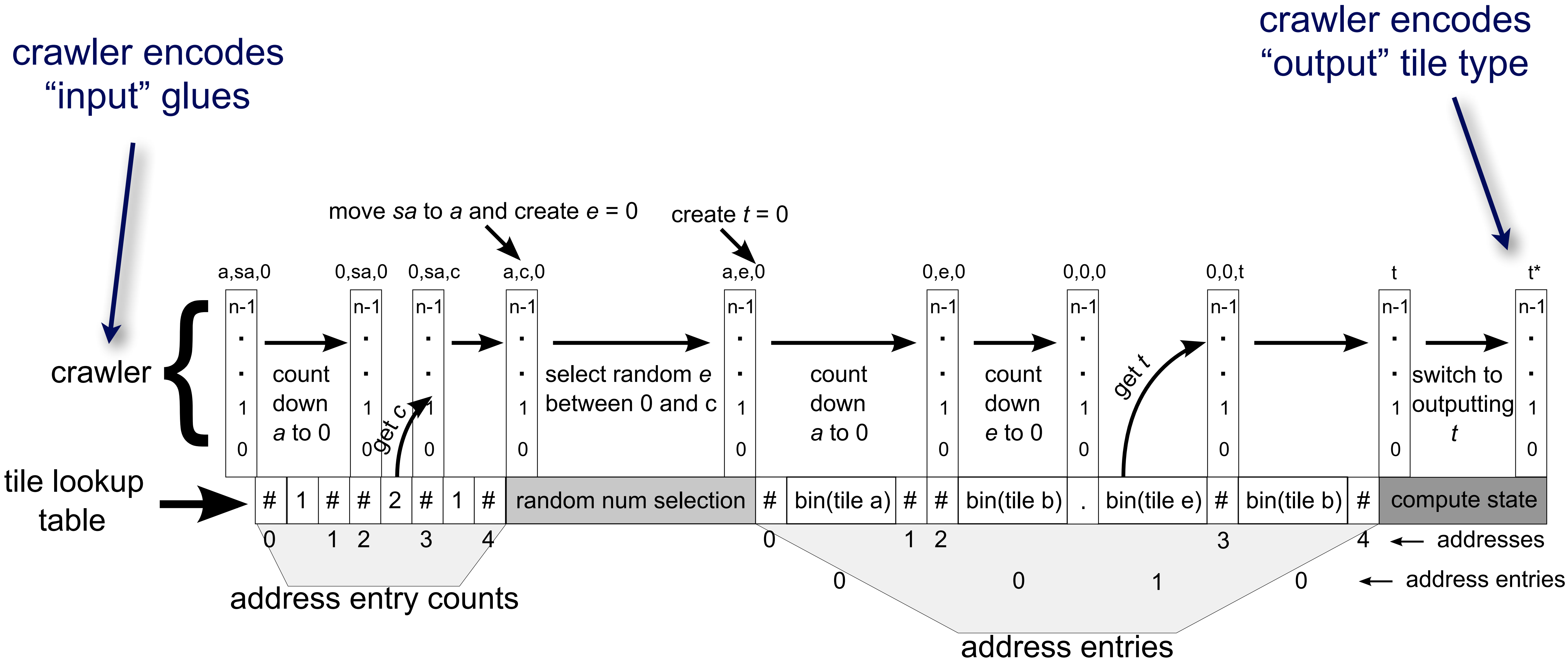
Glue "a" is encoded here

Supertile 1

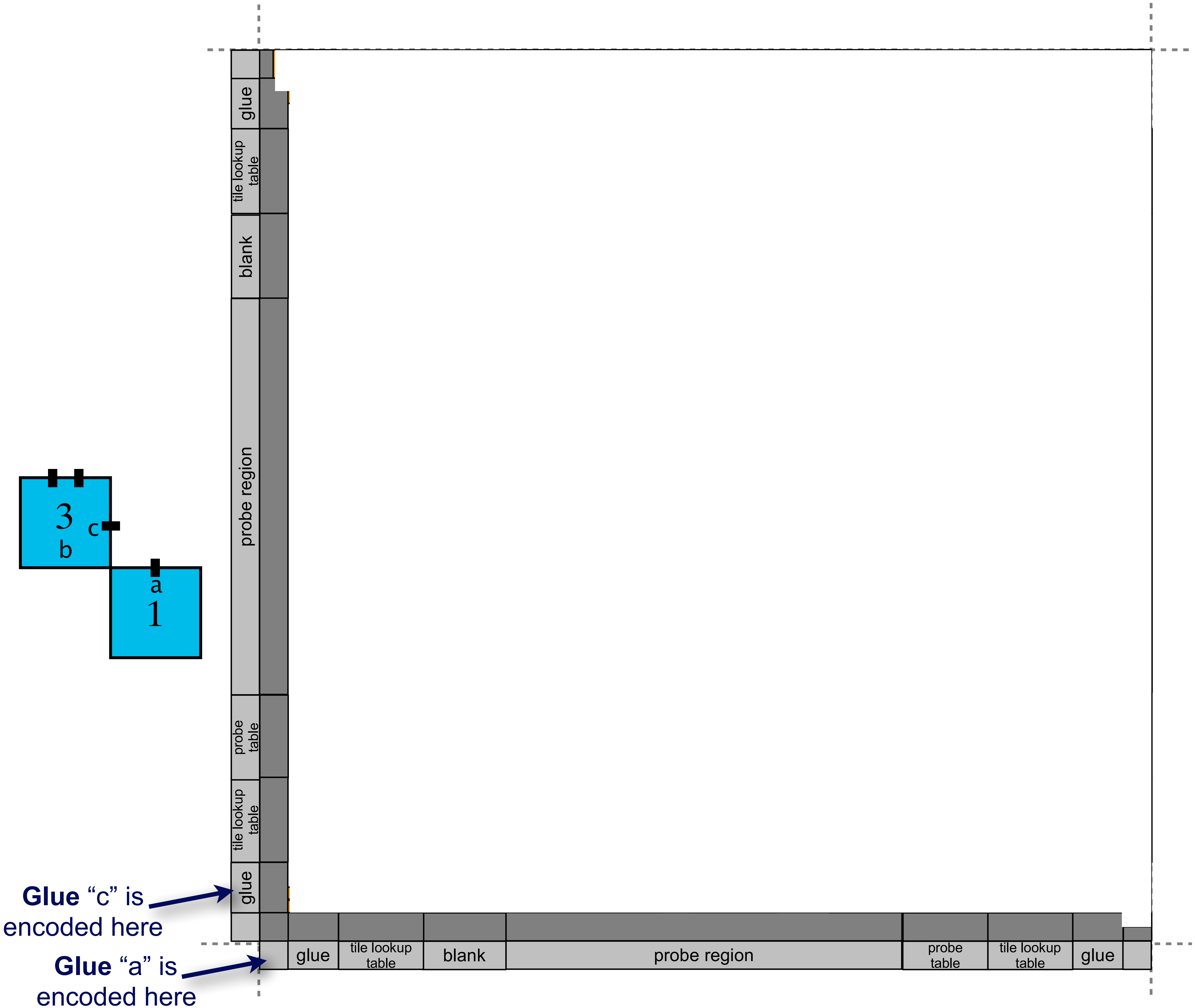
Tile set T is encoded here

Glue "a" is encoded here

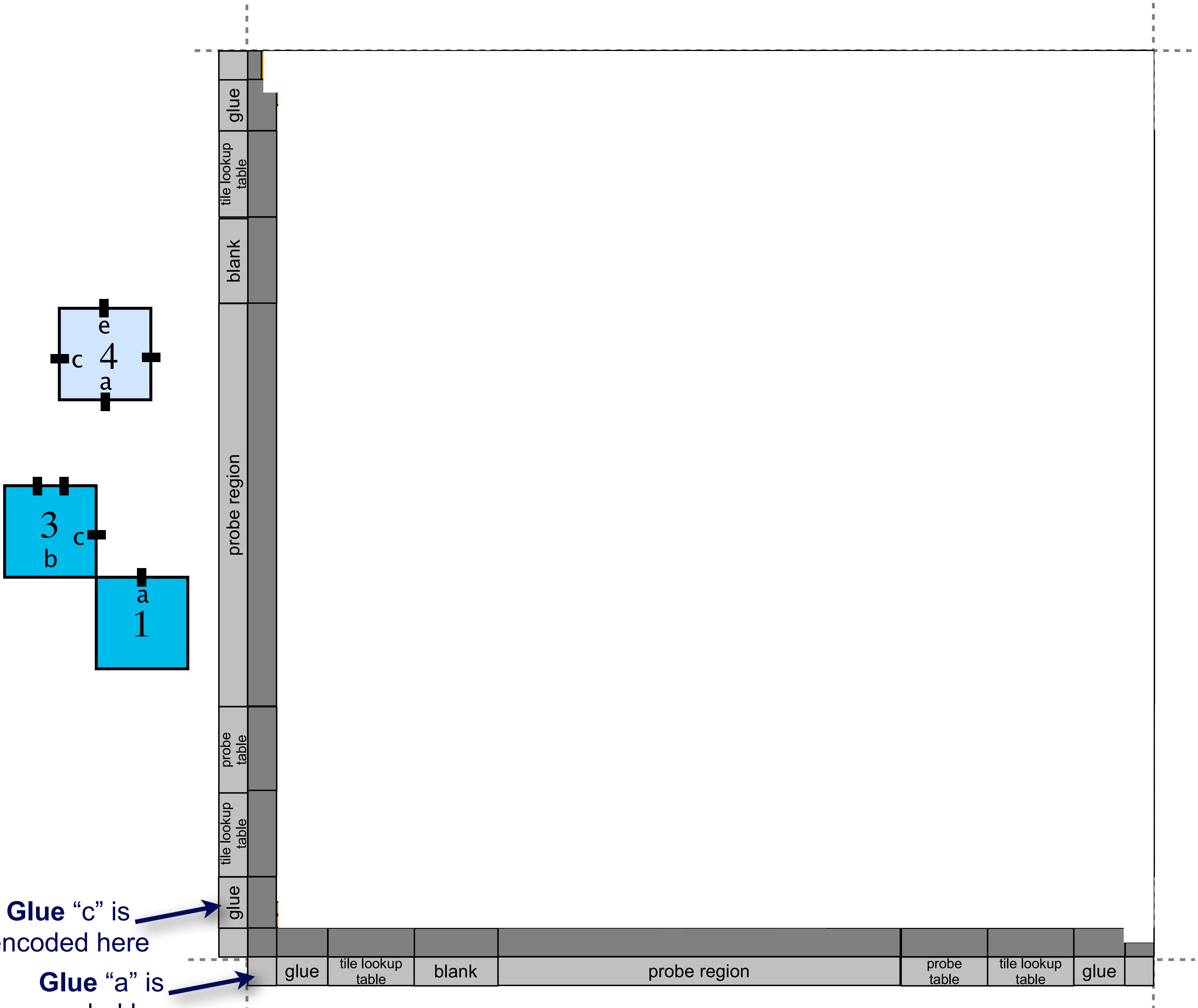
Crawler doing a tile lookup



Two-sided binding with adjacent cooperating supersides



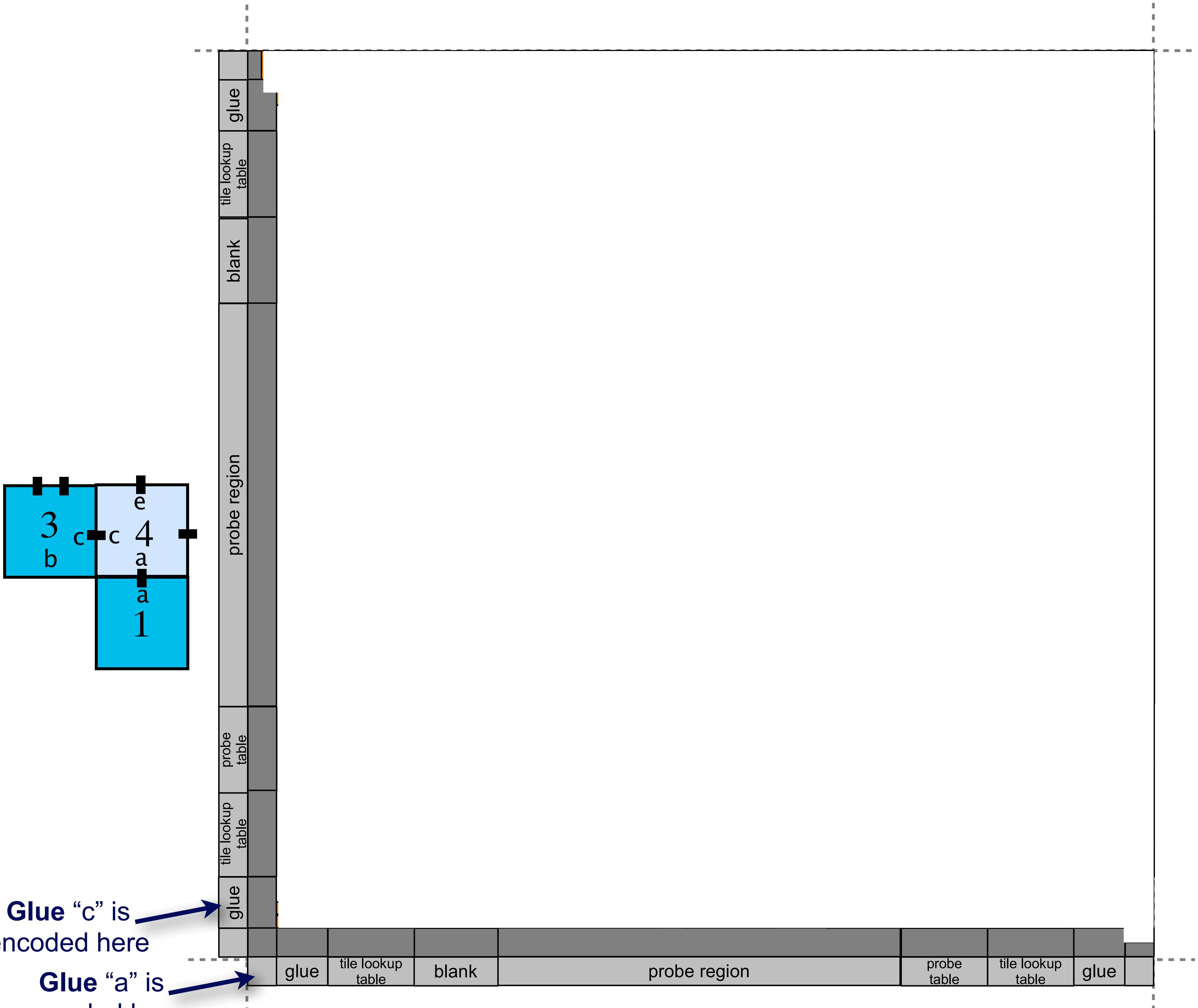
Two-sided binding with adjacent cooperating supersides



Glue “c” is encoded here

Glue “a” is encoded here

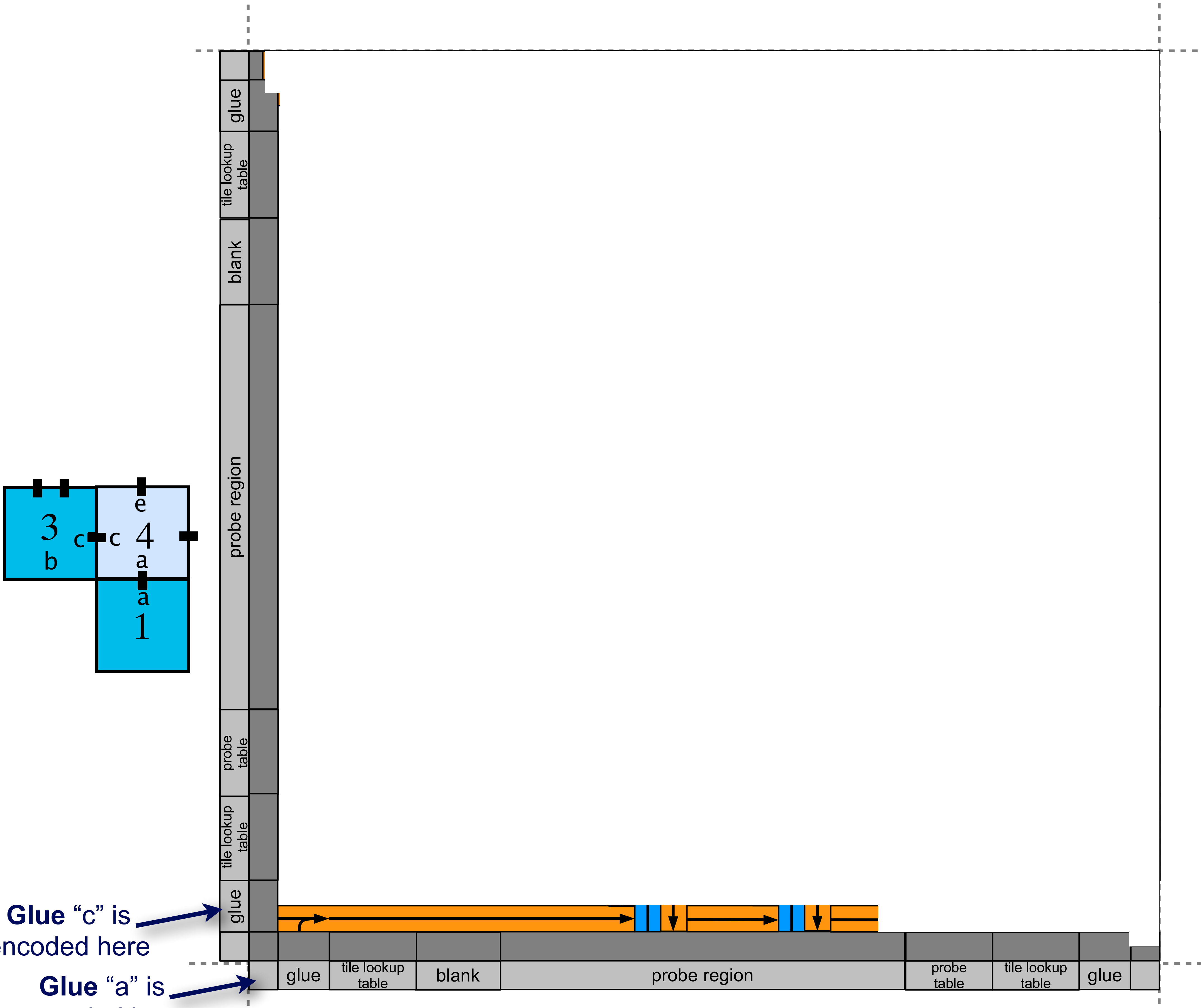
Two-sided binding with adjacent cooperating supersides



Glue “c” is encoded here

Glue “a” is encoded here

Two-sided binding with adjacent cooperating supersides



Glue “c” is encoded here

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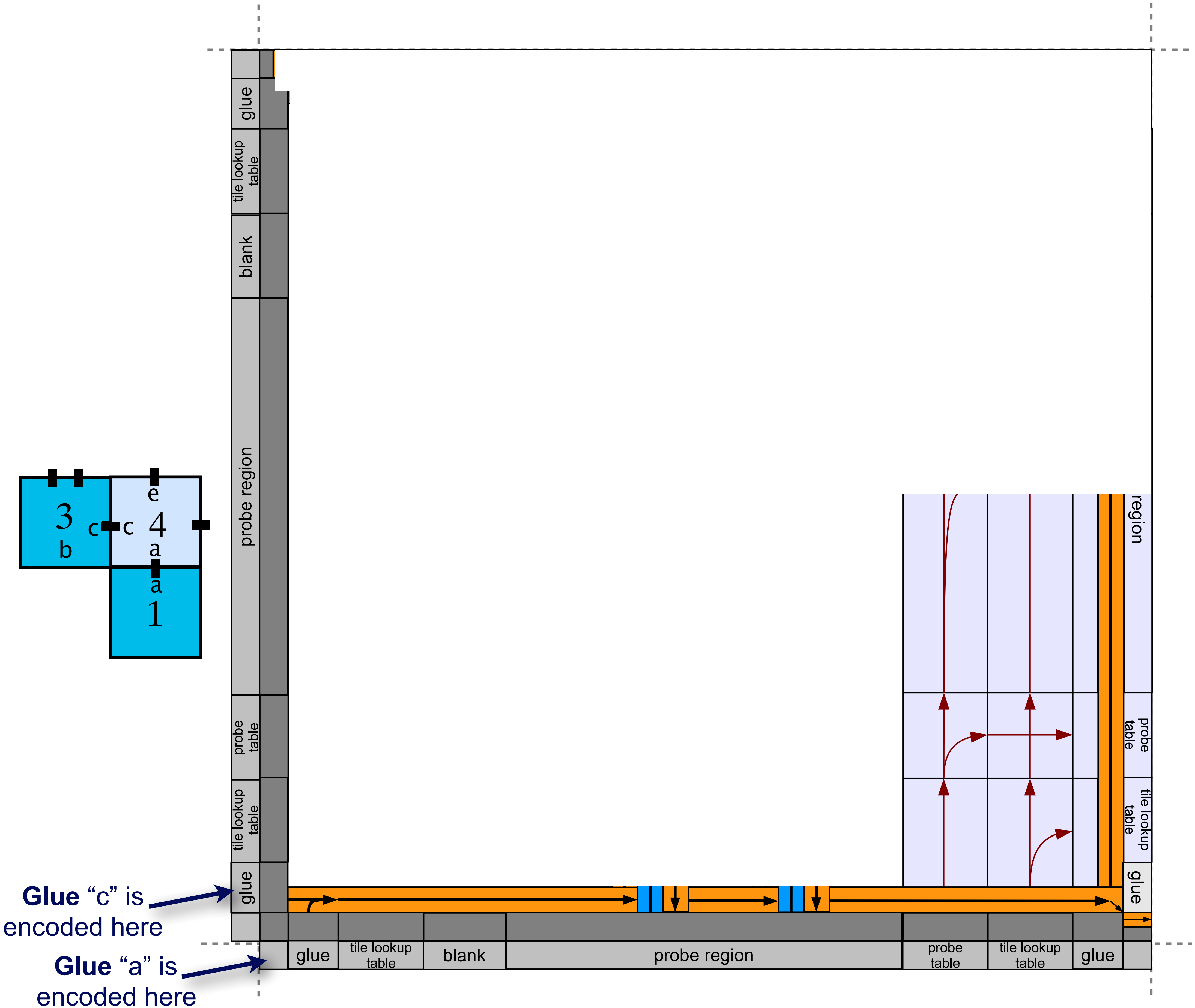
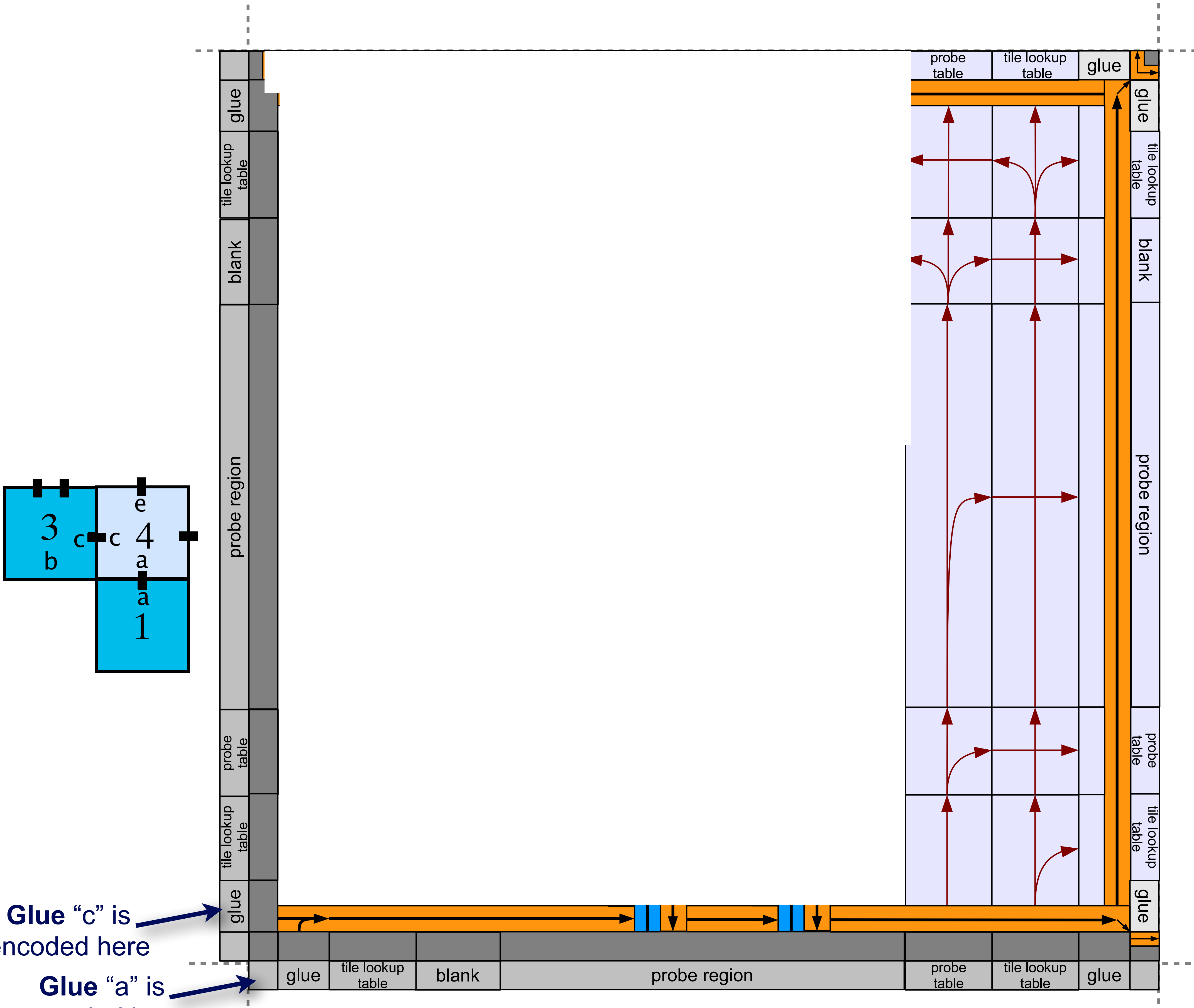


Two-sided binding with adjacent cooperating supersides



Glue "c" is encoded here

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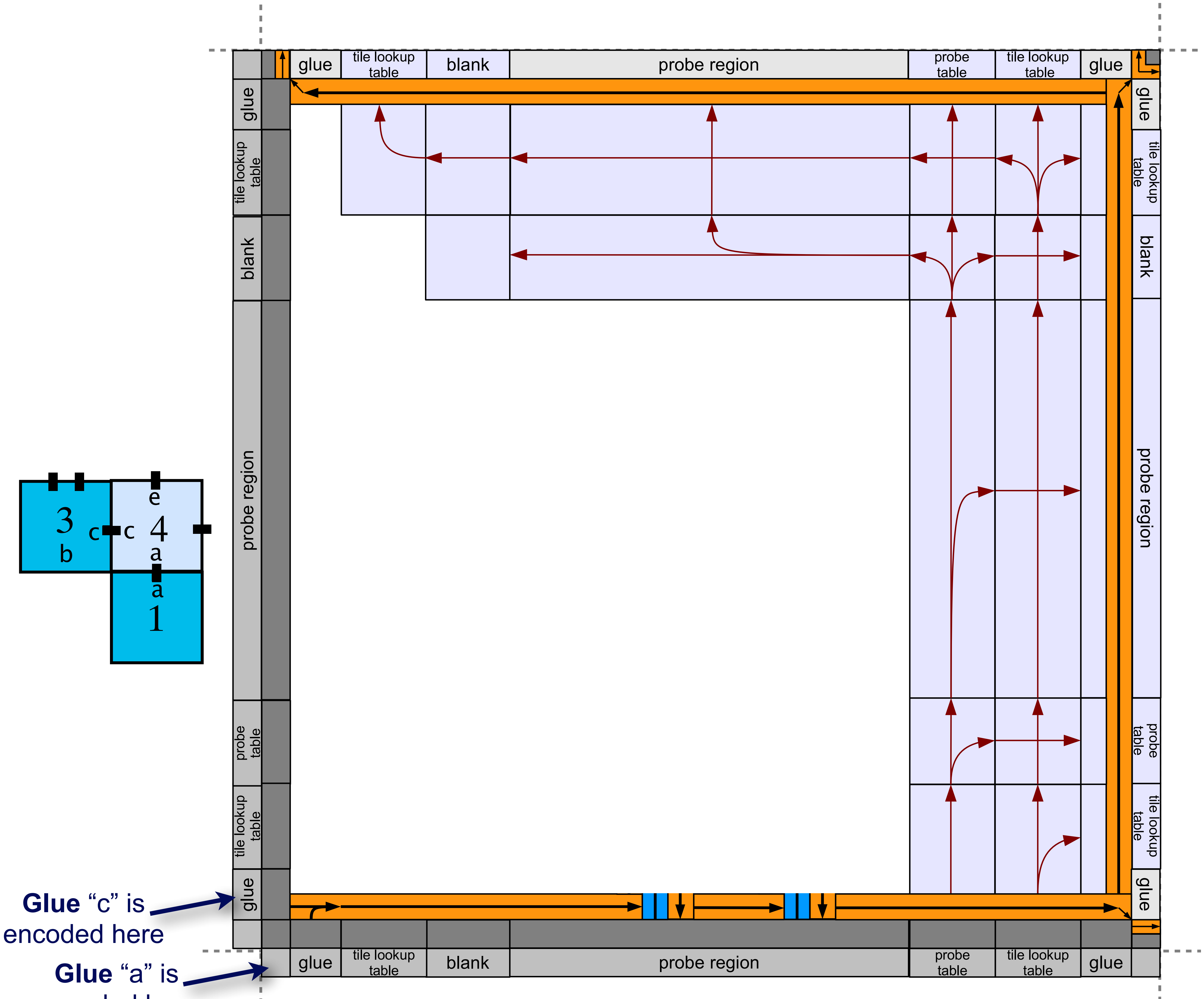
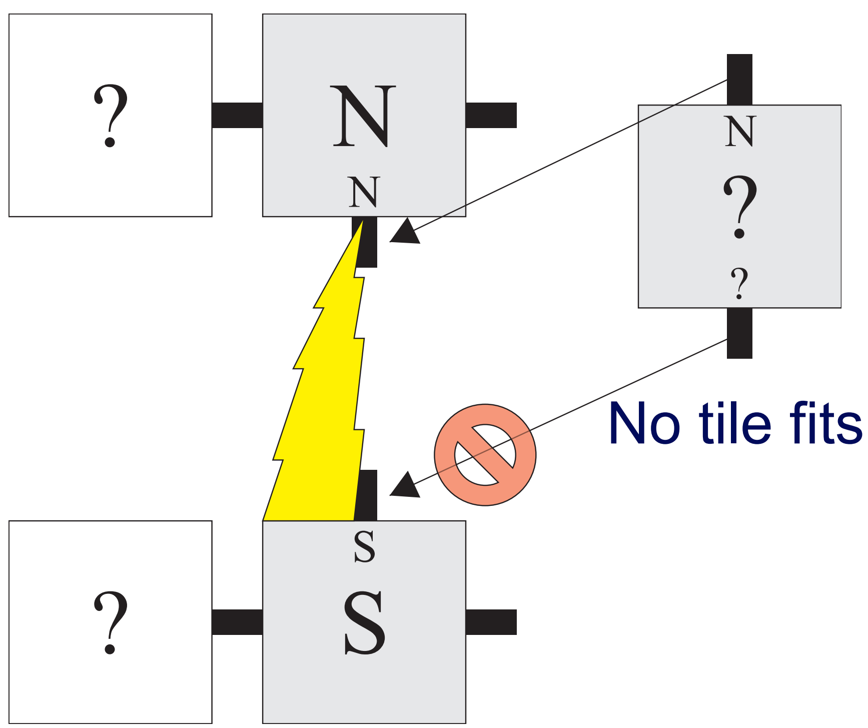
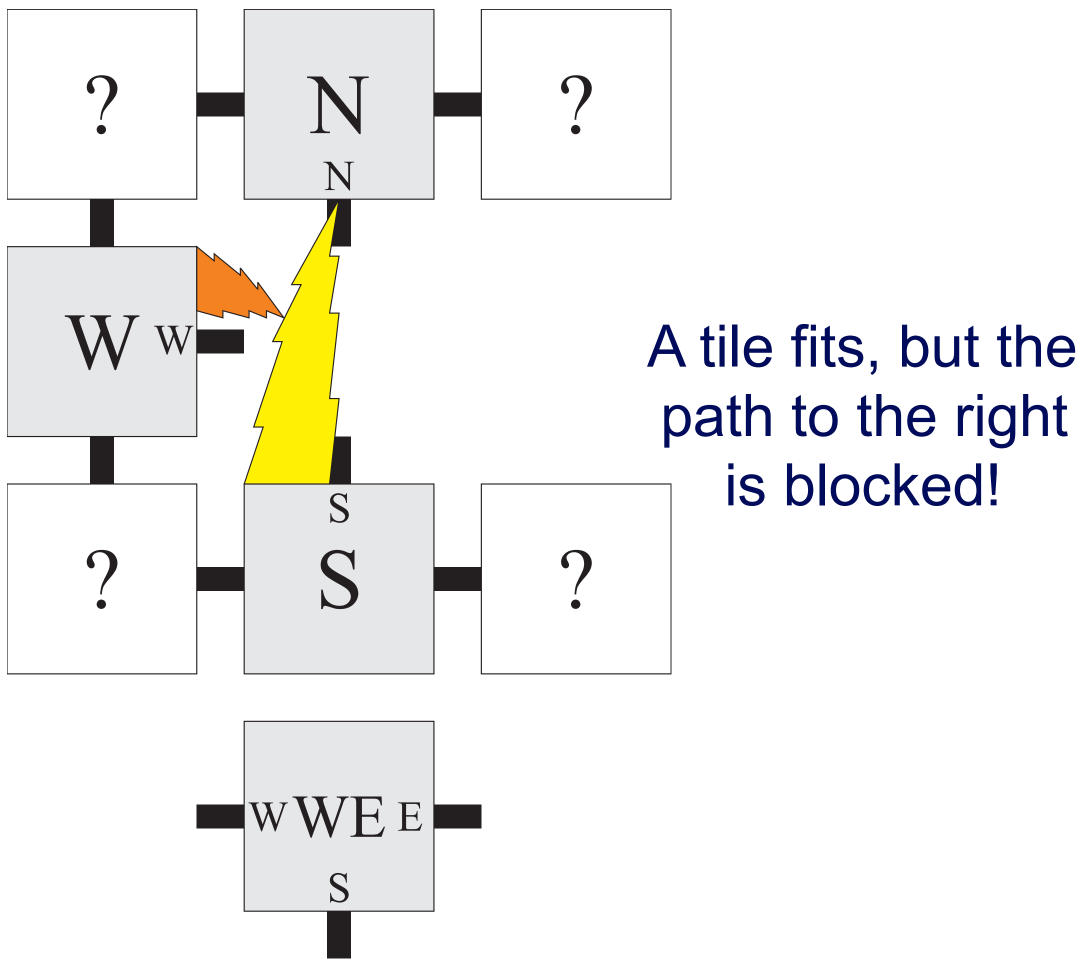
Two-sided binding with adjacent cooperating supersides



A key problem

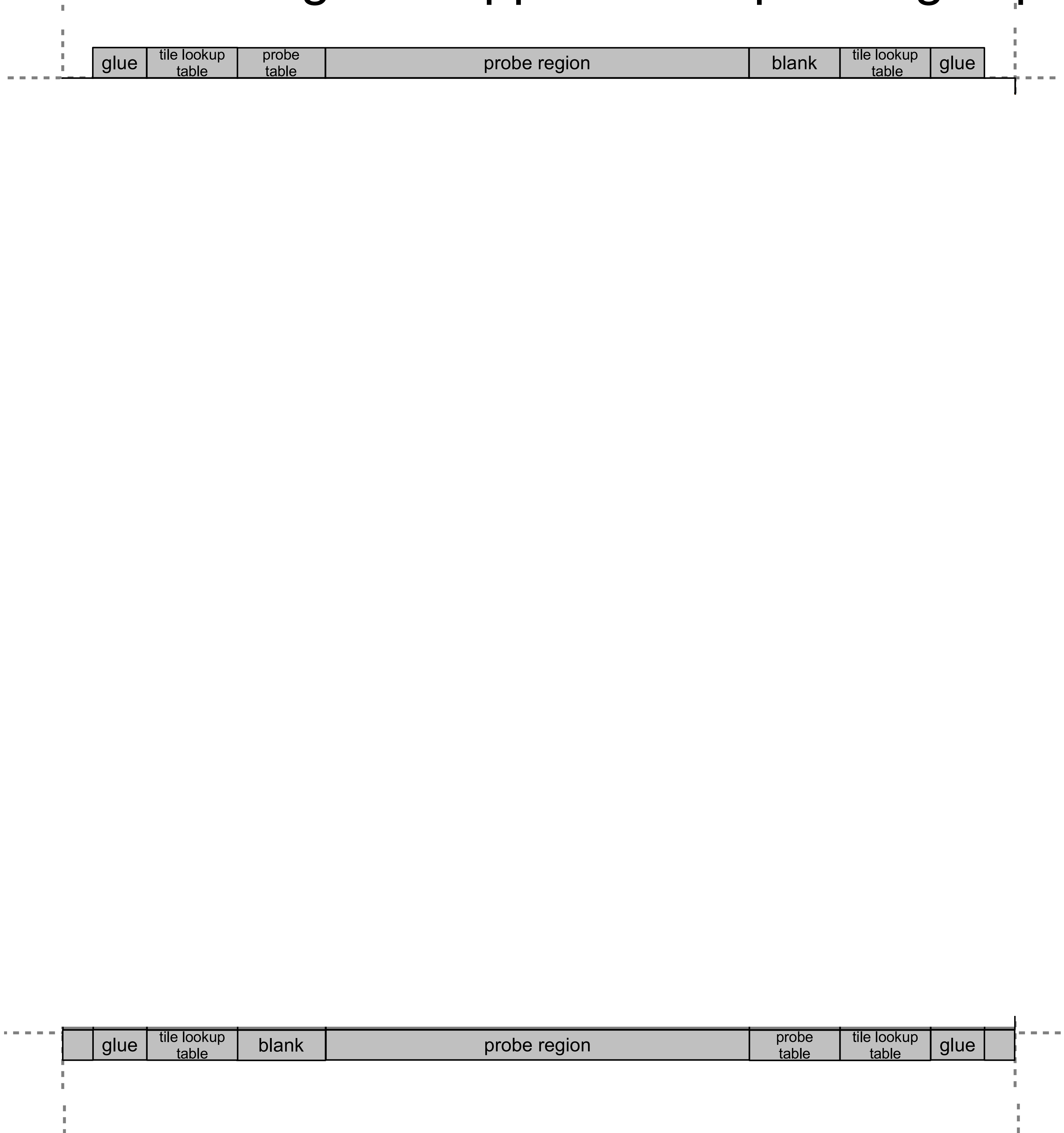


Better luck next time!

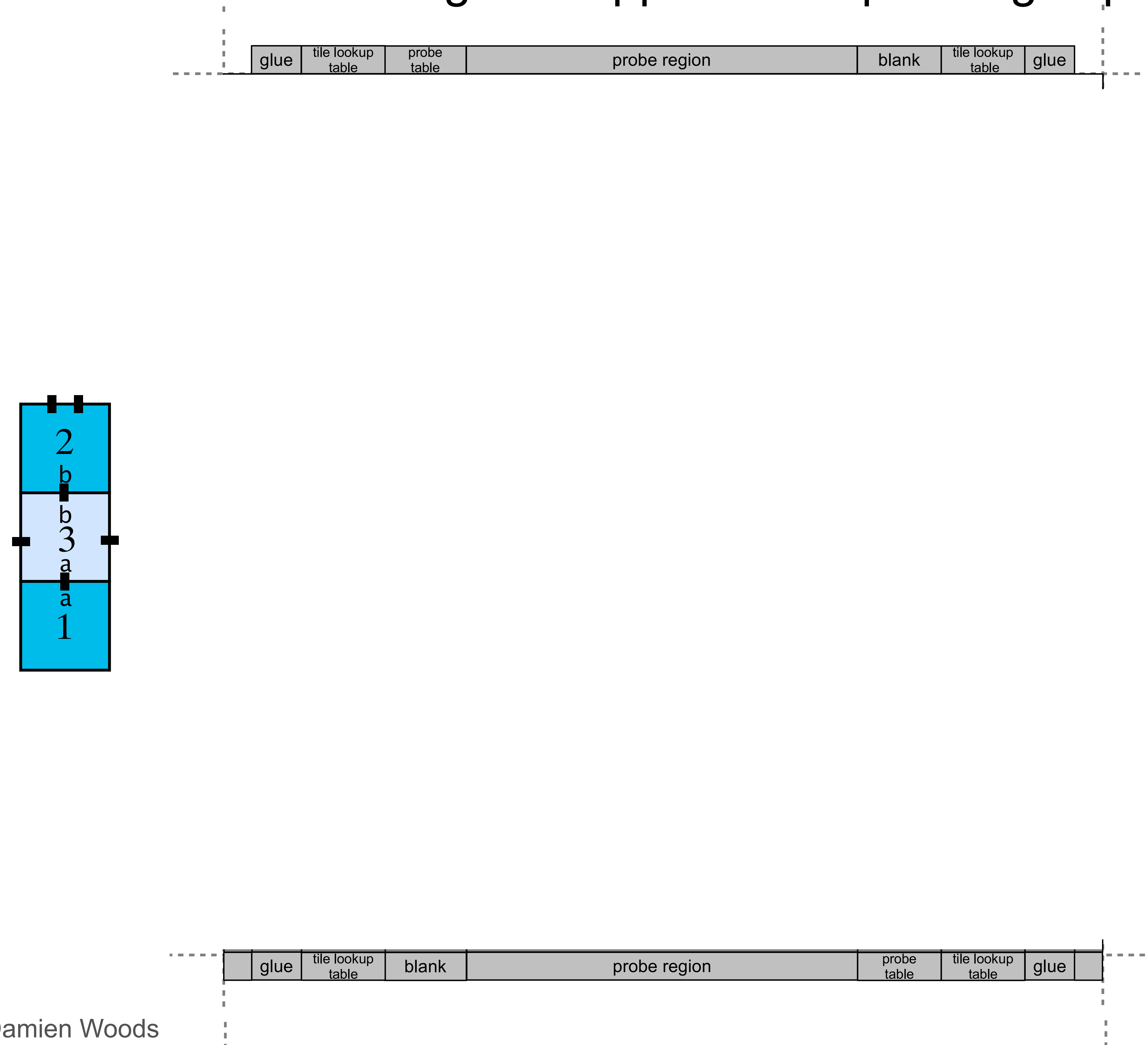


Uh oh!

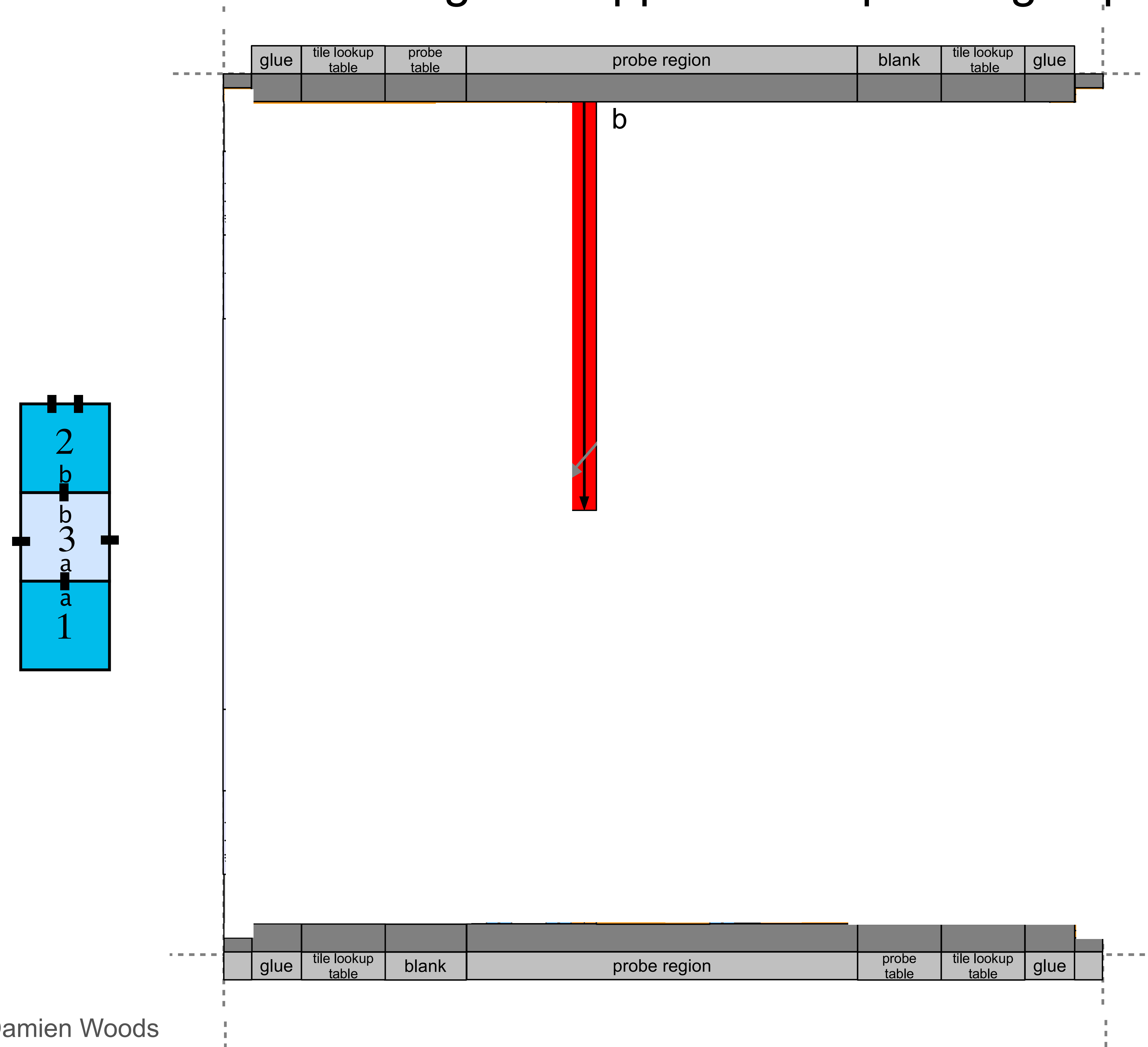
Two-sided binding with opposite cooperating supersides



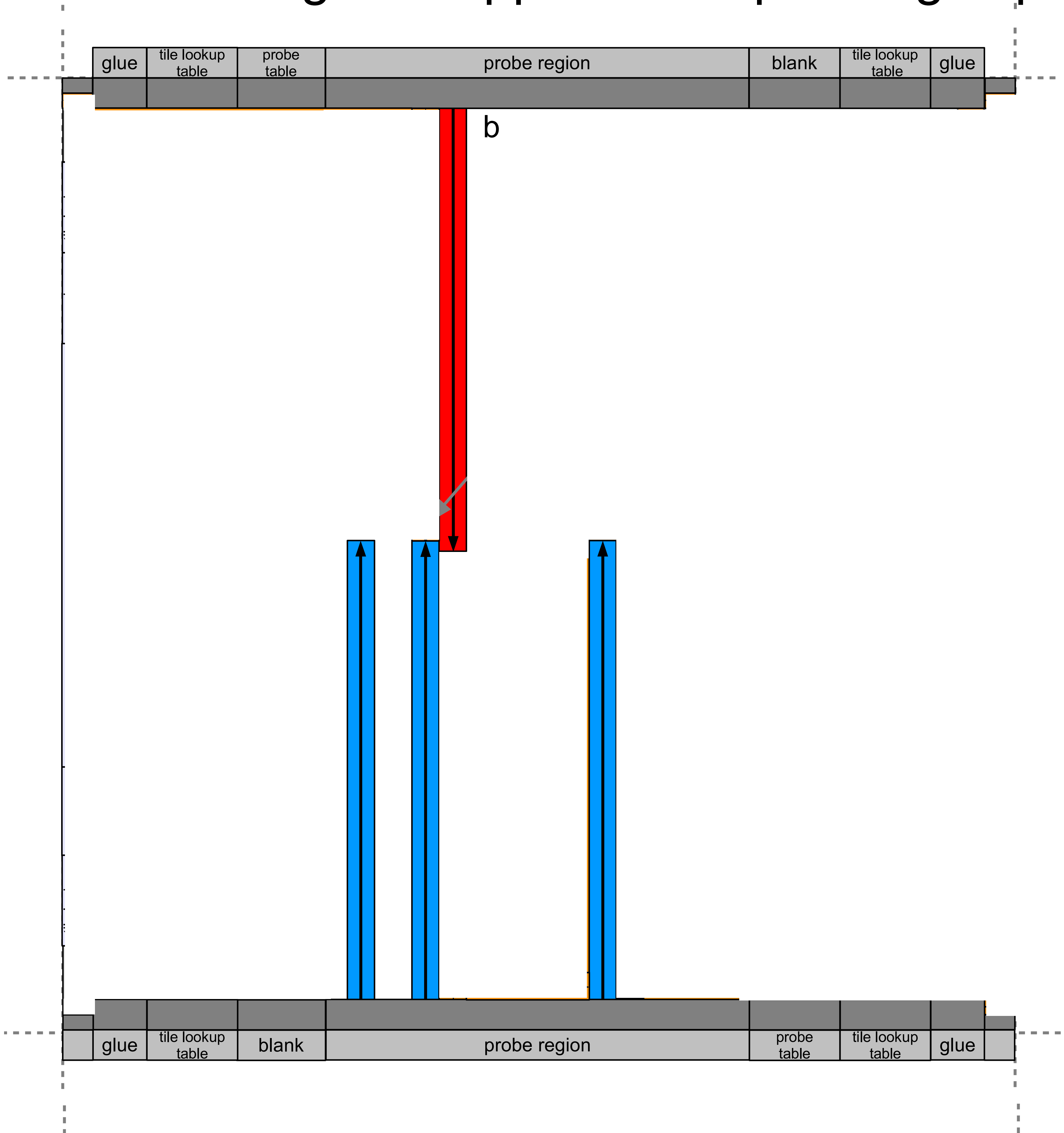
Two-sided binding with opposite cooperating supersides

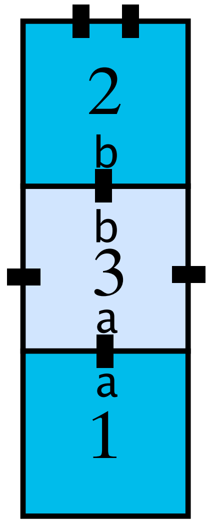


Two-sided binding with opposite cooperating supersides



Two-sided binding with opposite cooperating supersides





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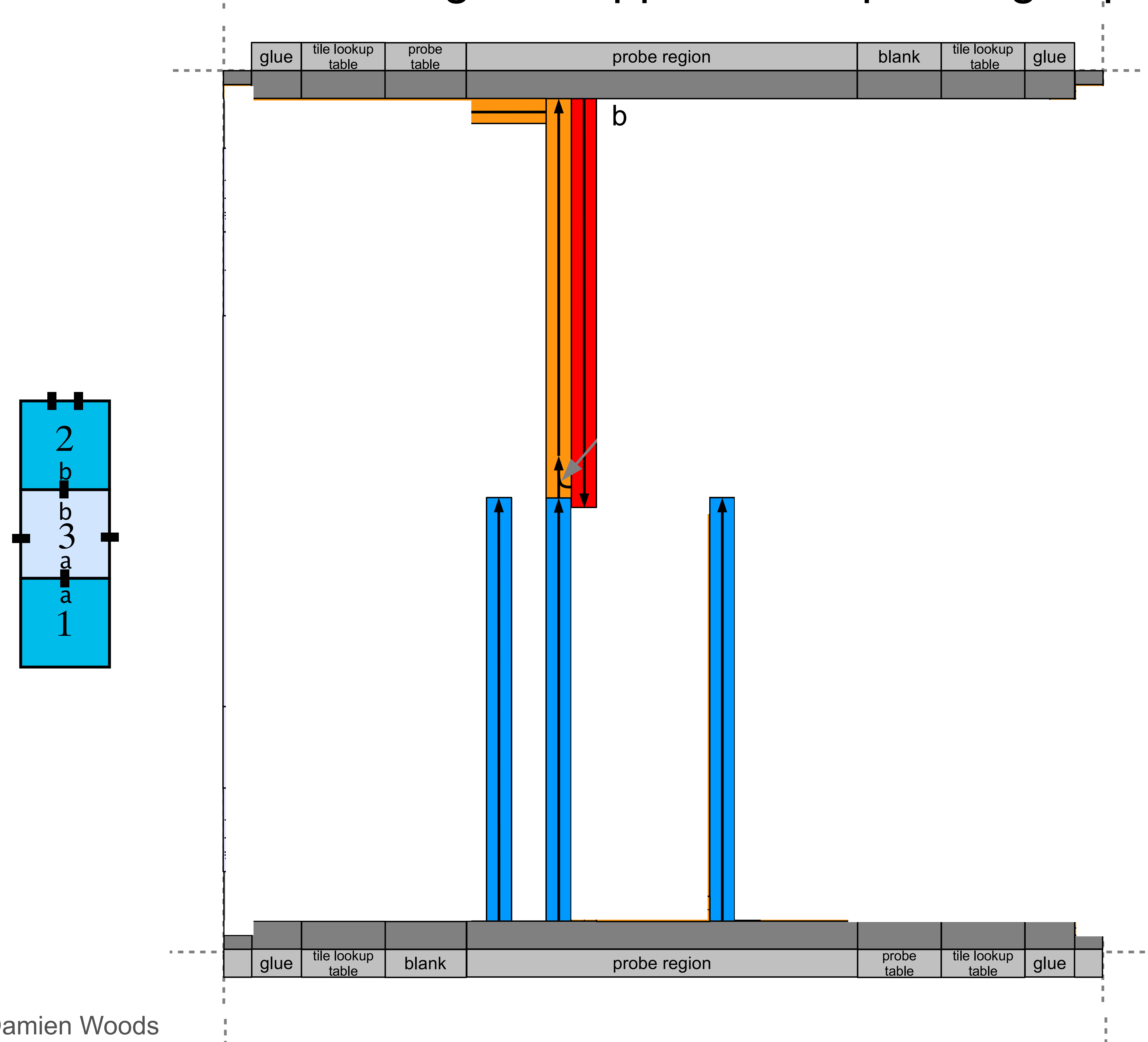
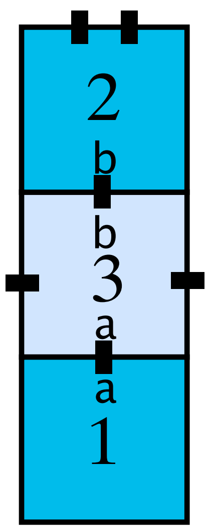
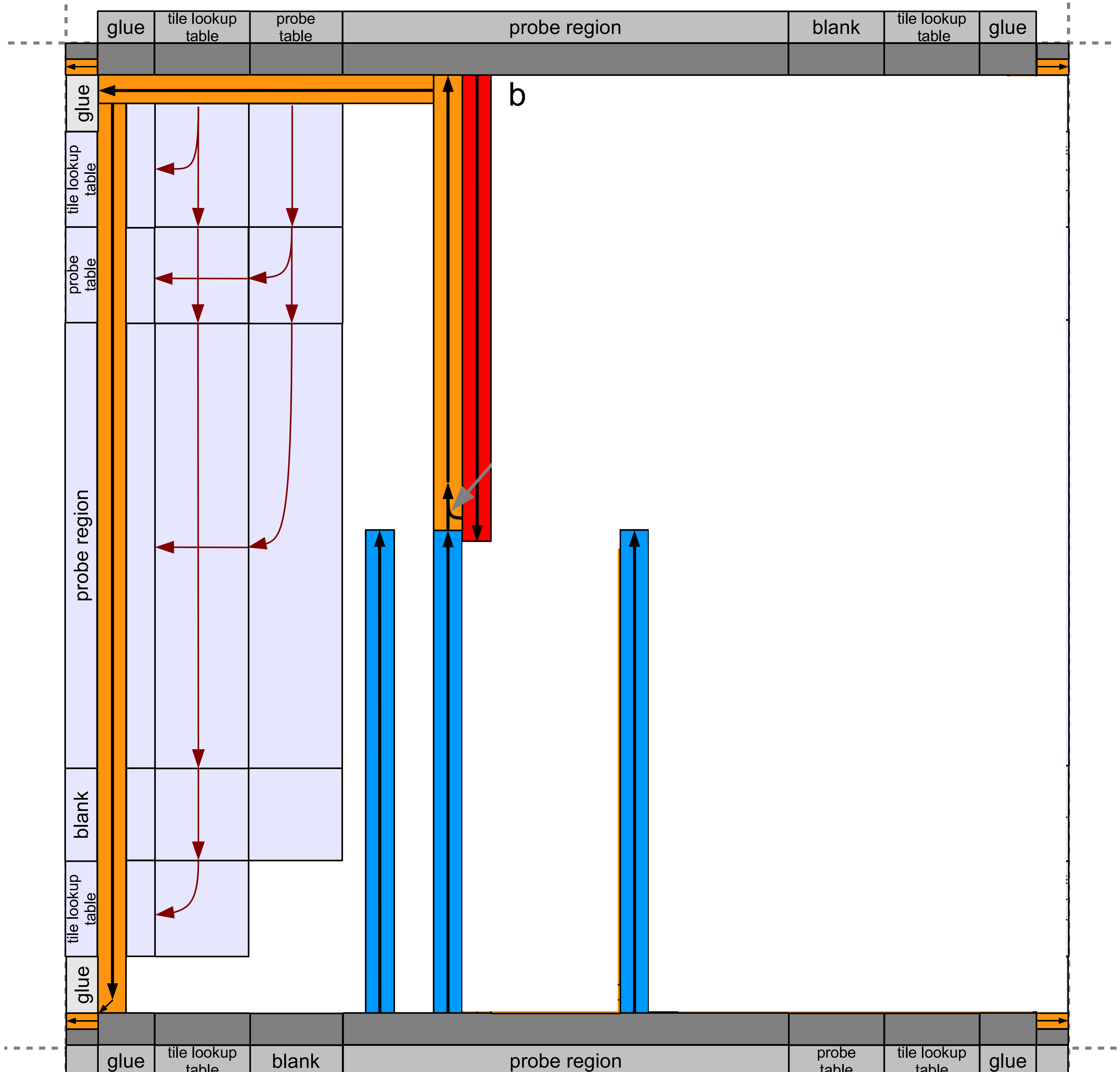
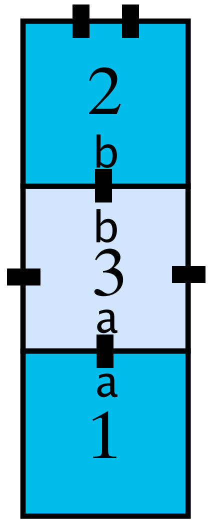


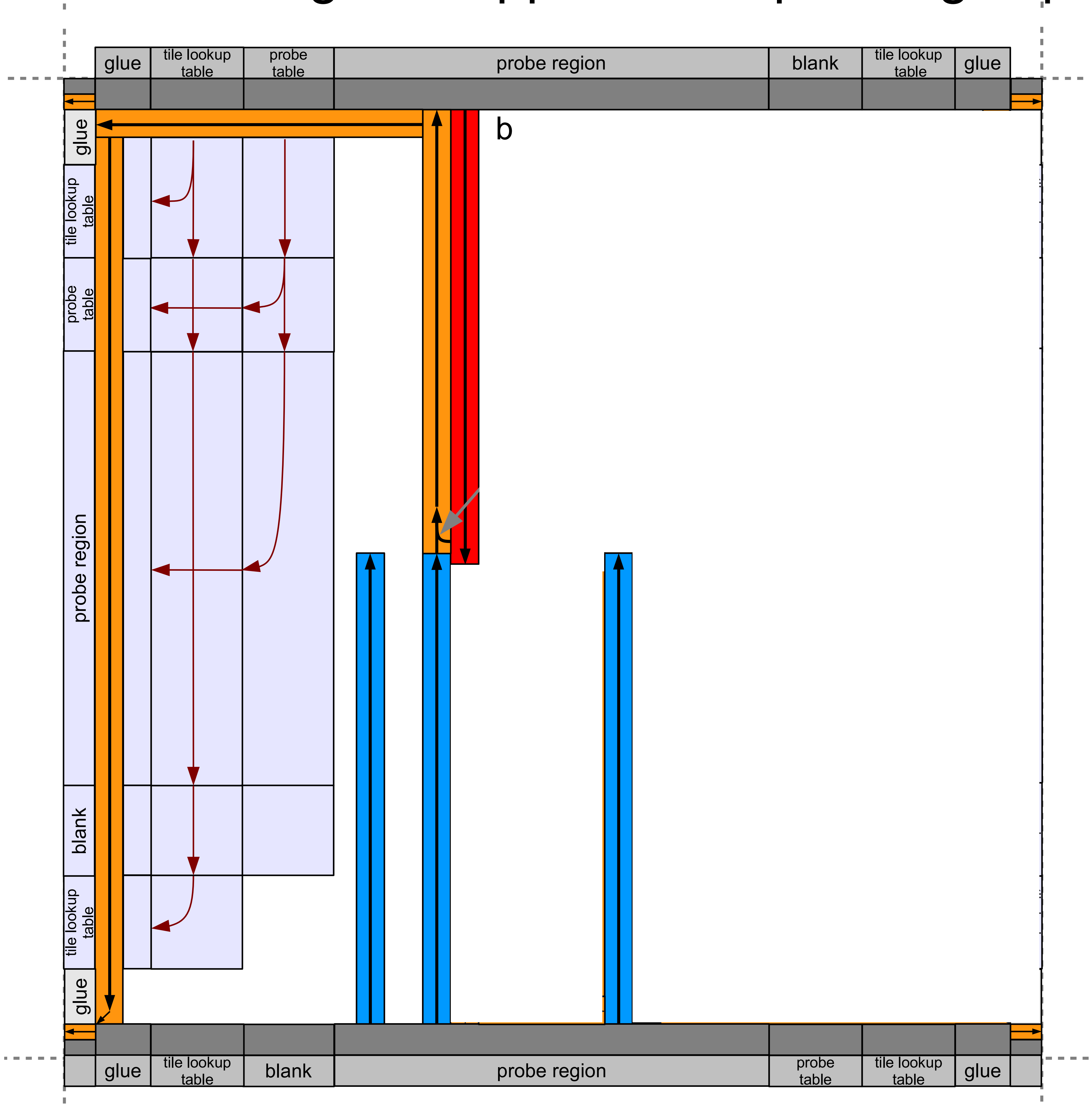
Two-sided binding with opposite cooperating supersides



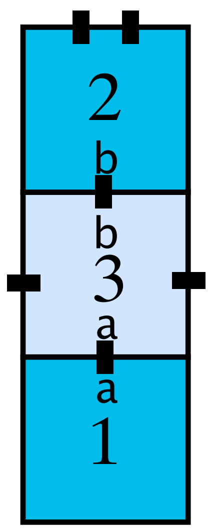


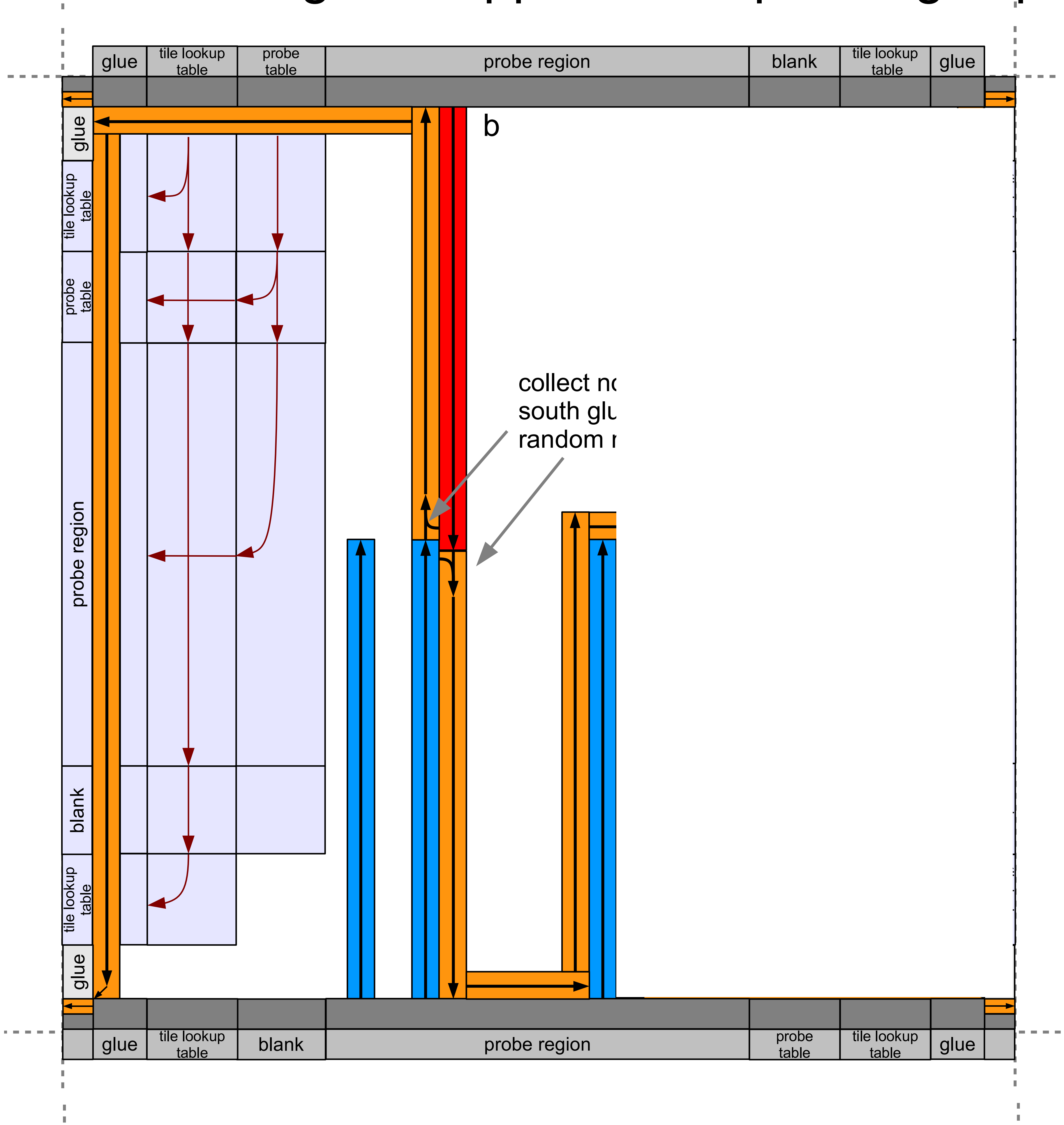
Two-sided binding with opposite cooperating supersides



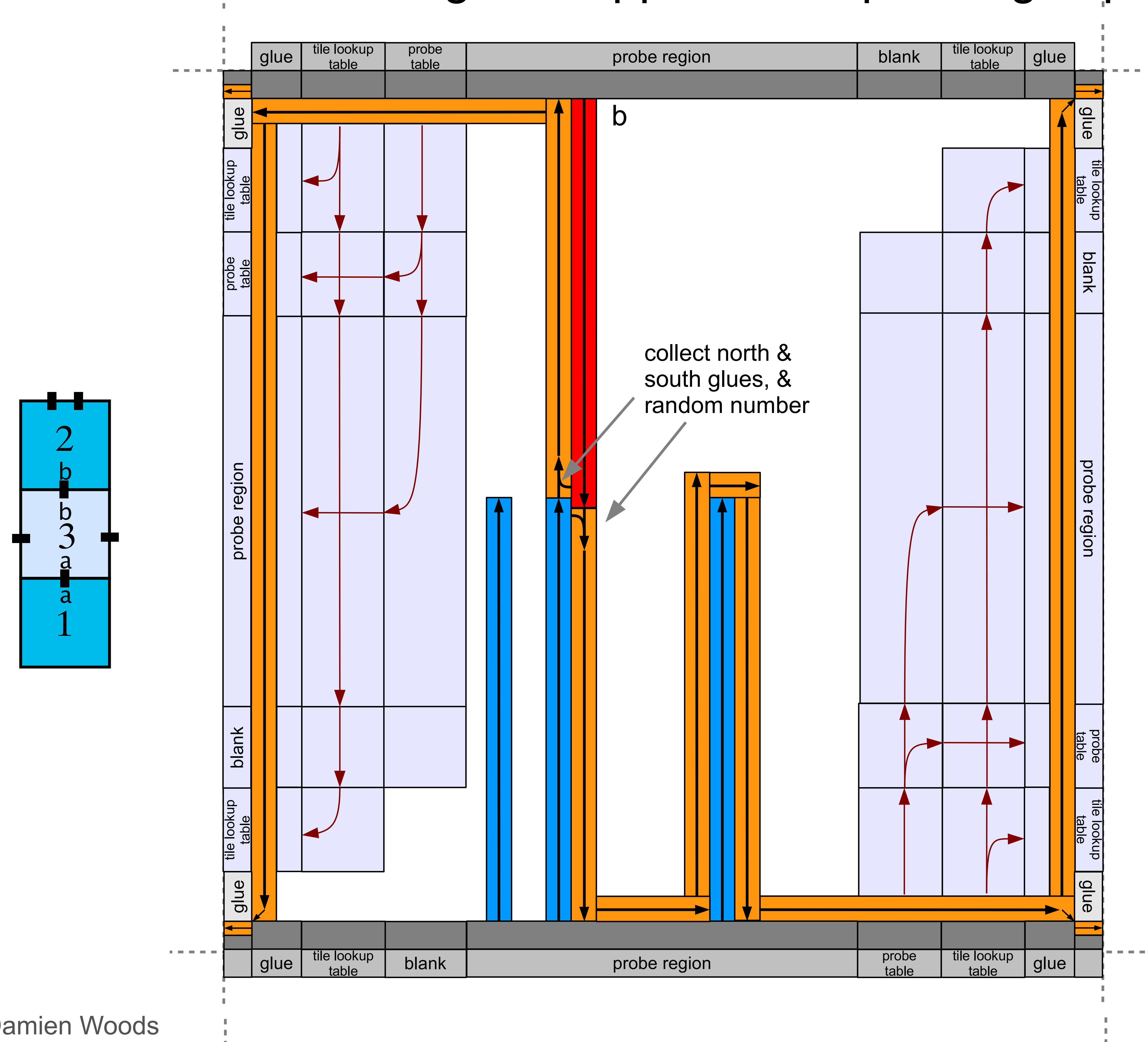


Two-sided binding with opposite cooperating supersides

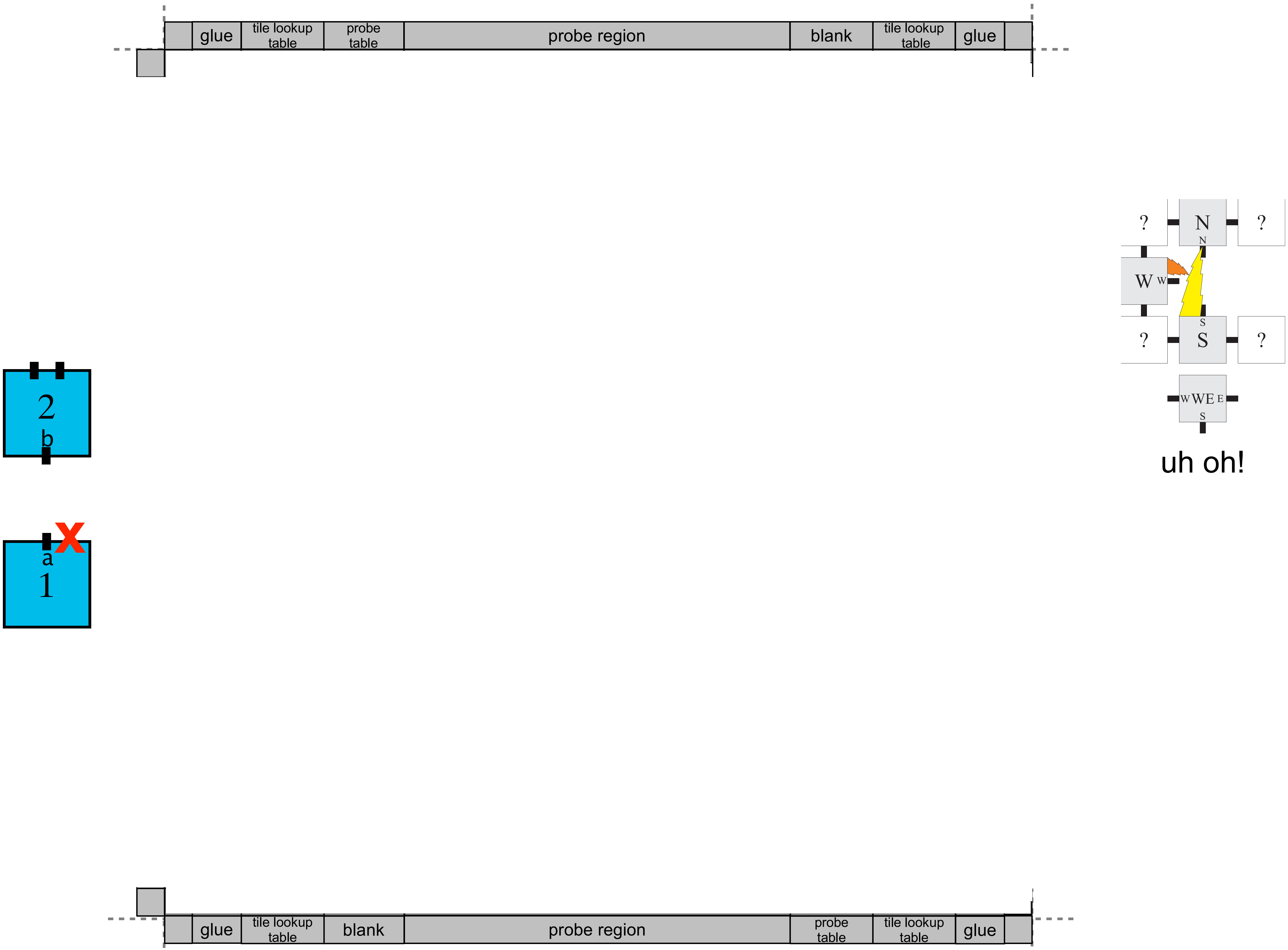




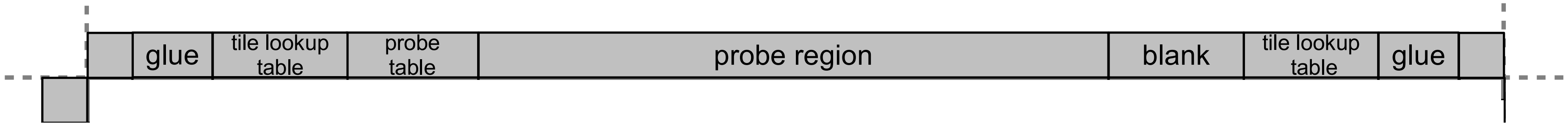
Two-sided binding with opposite cooperating supersides

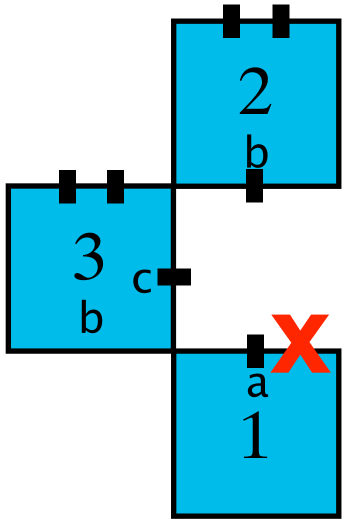


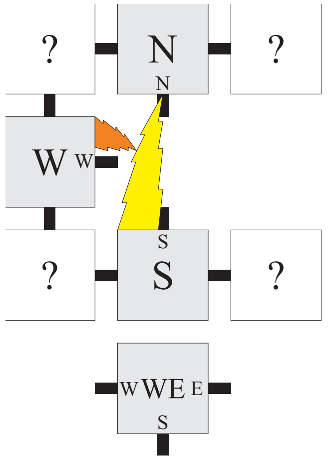
3-sided “uh-oh” example: probes miss each other



3-sided "uh-oh" example: probes miss each other

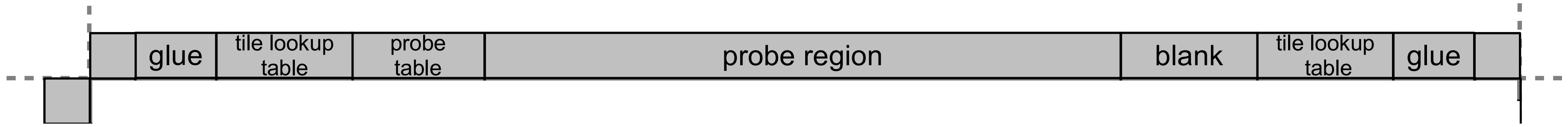


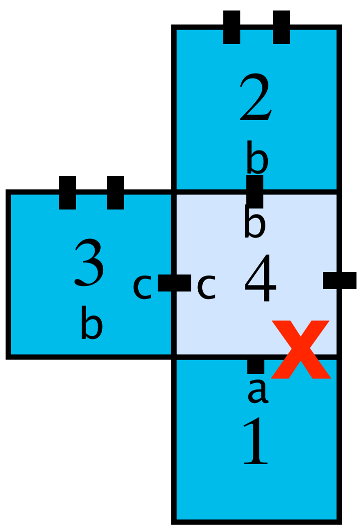


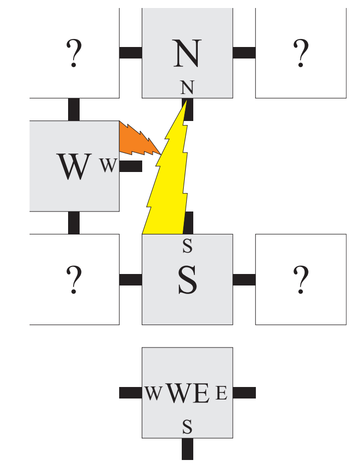


uh oh!

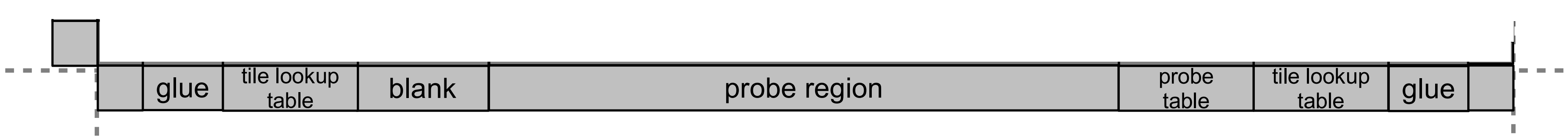
3-sided "uh-oh" example: probes miss each other







uh oh!



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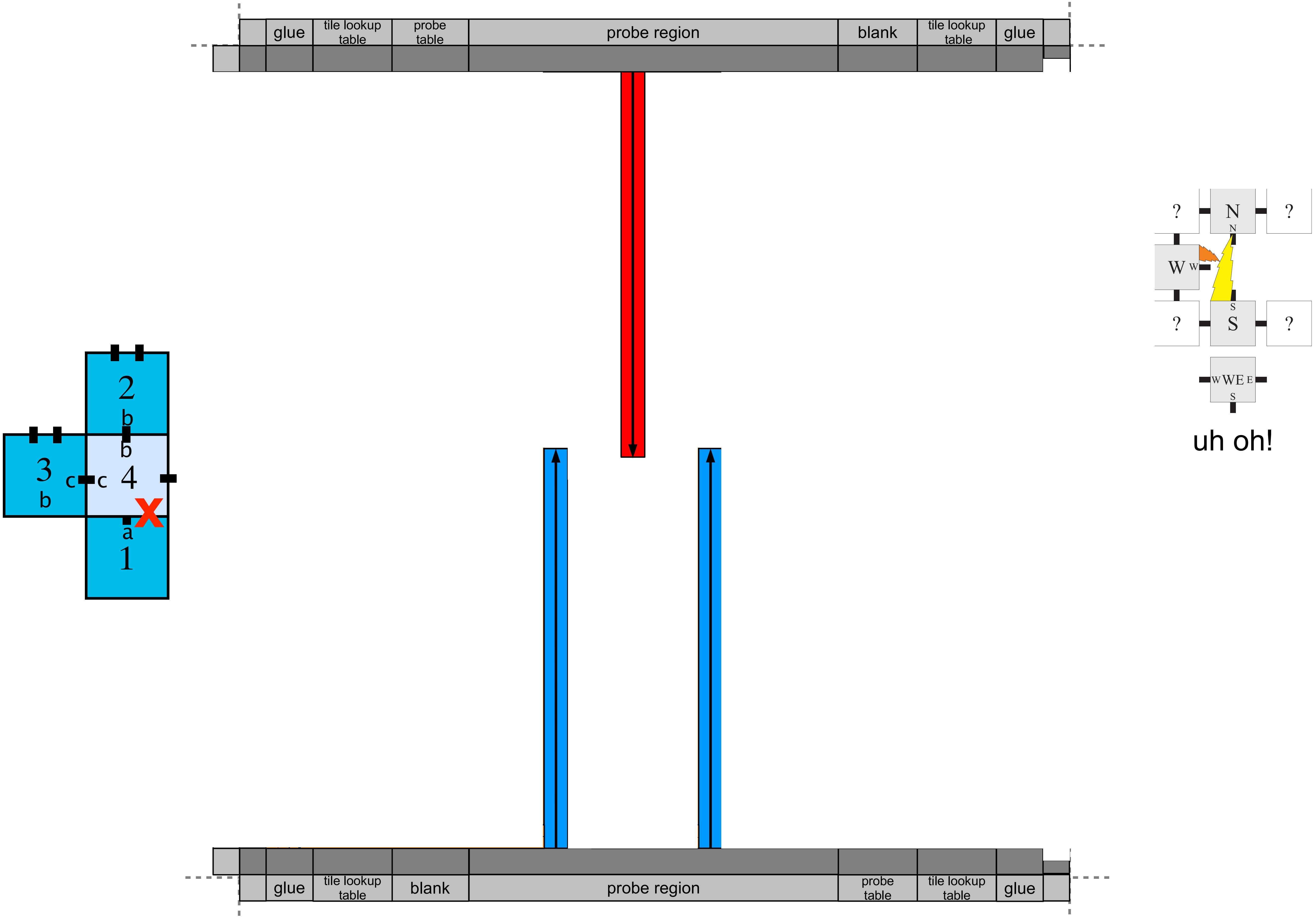

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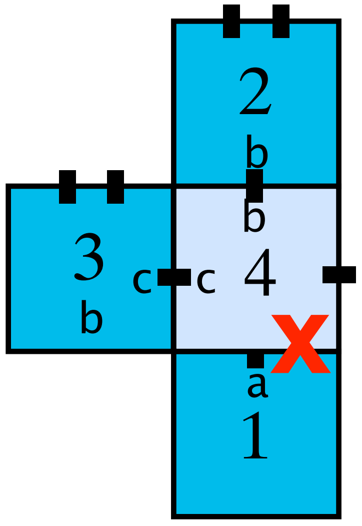
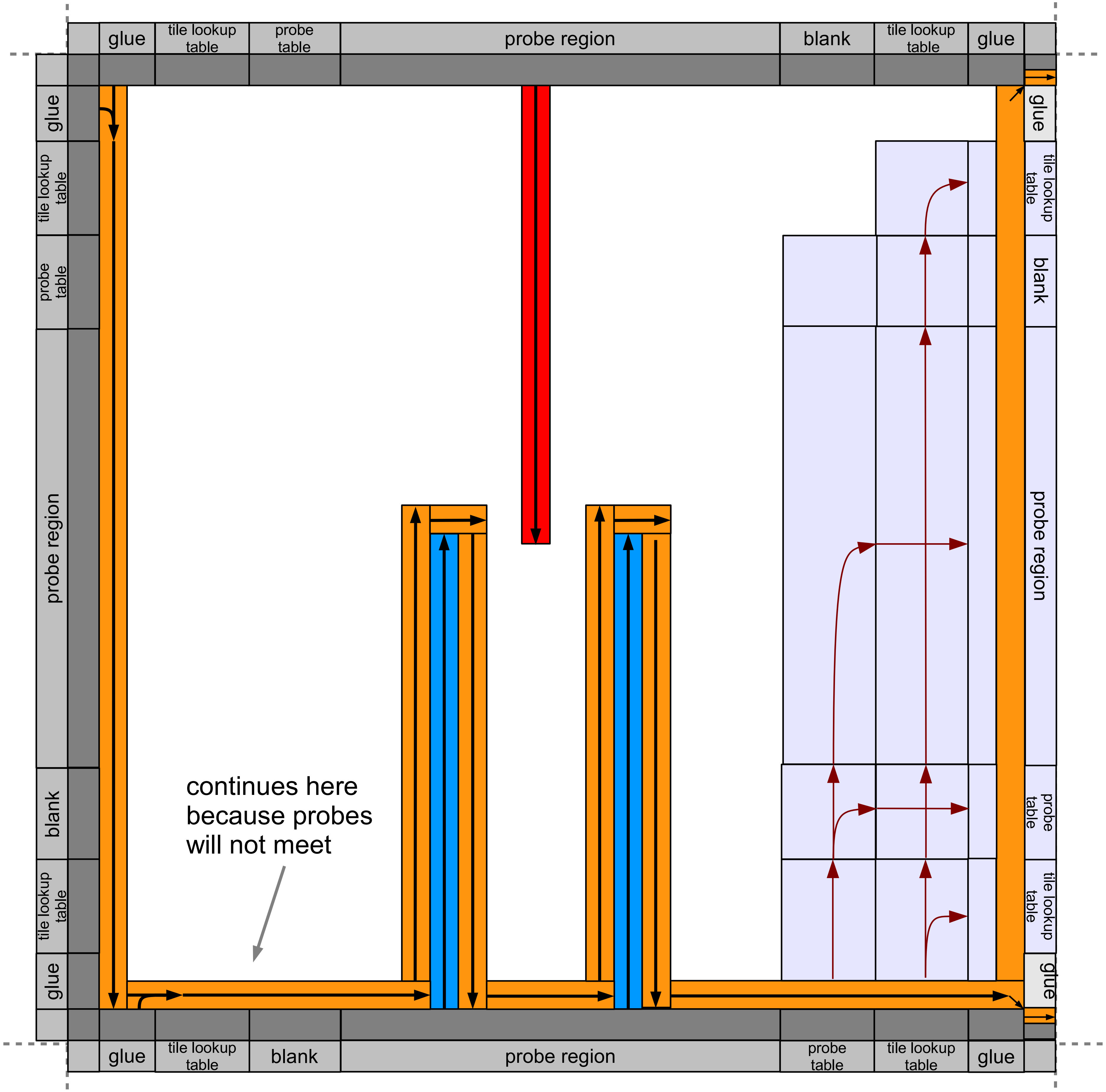
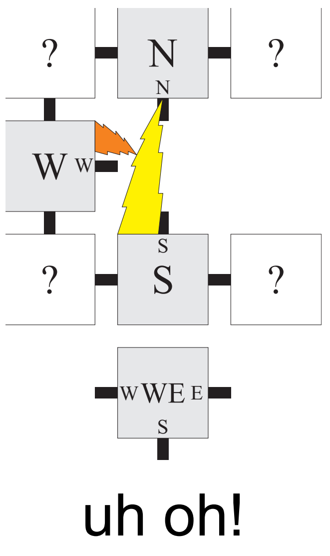
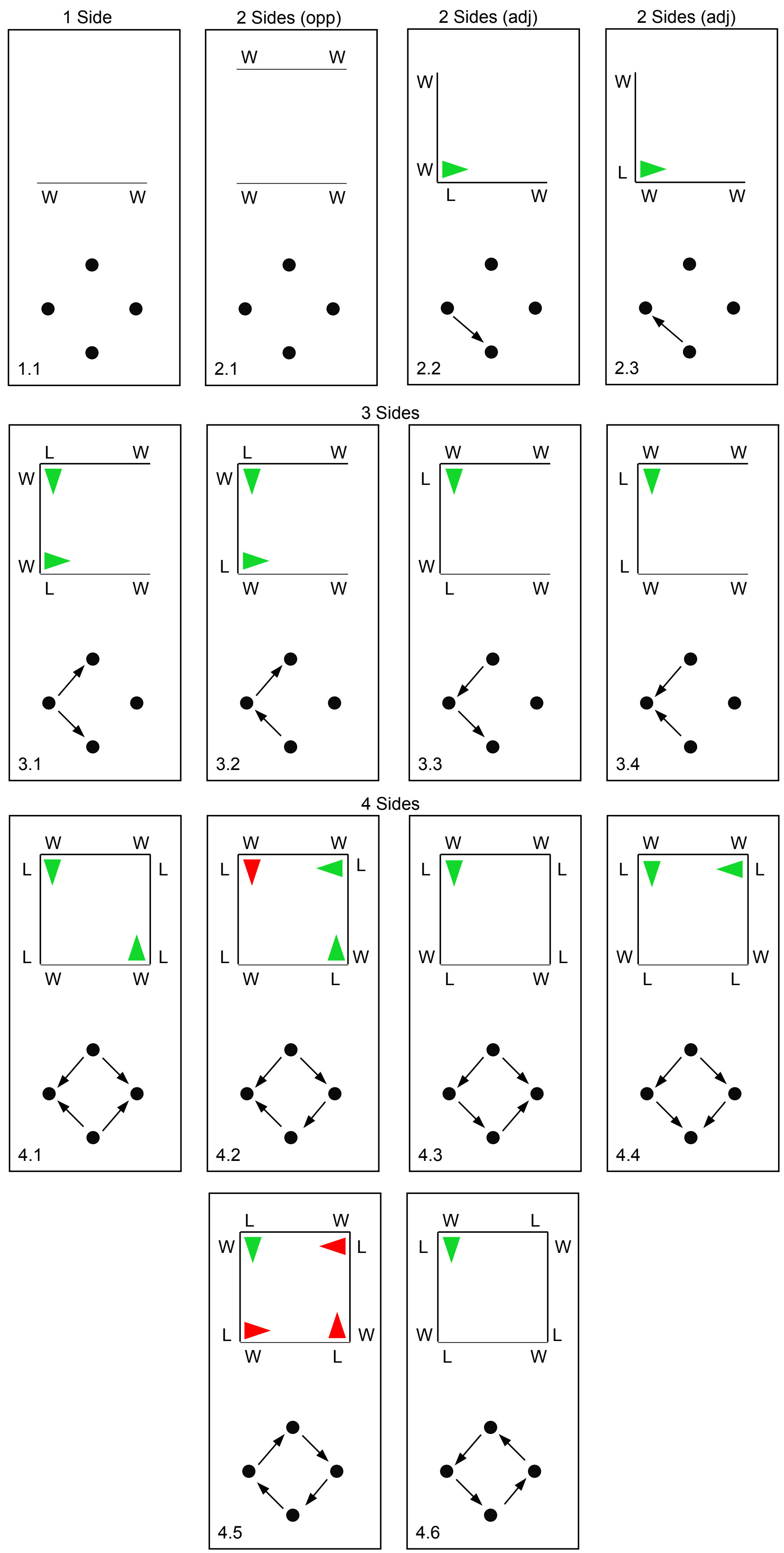




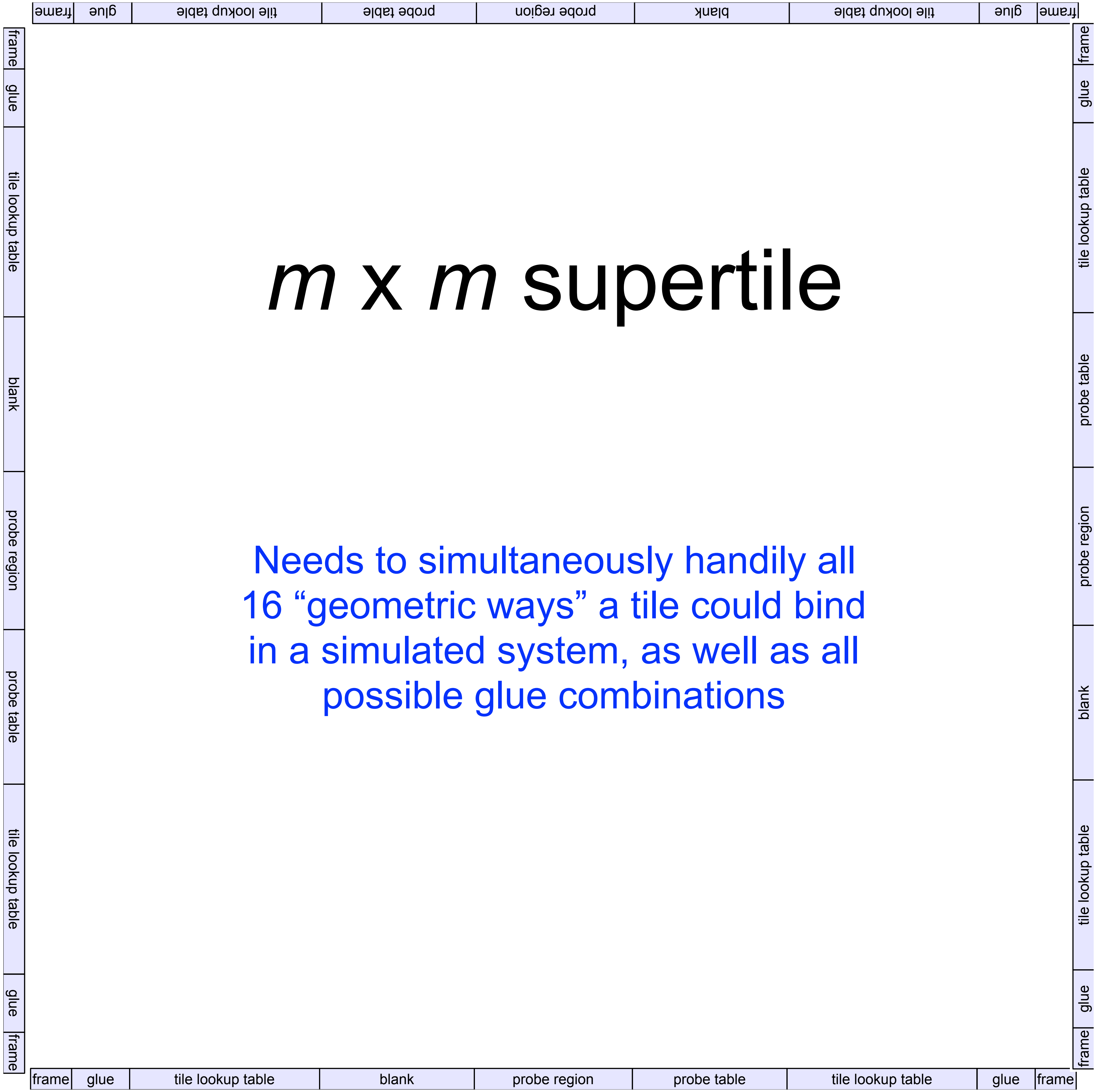


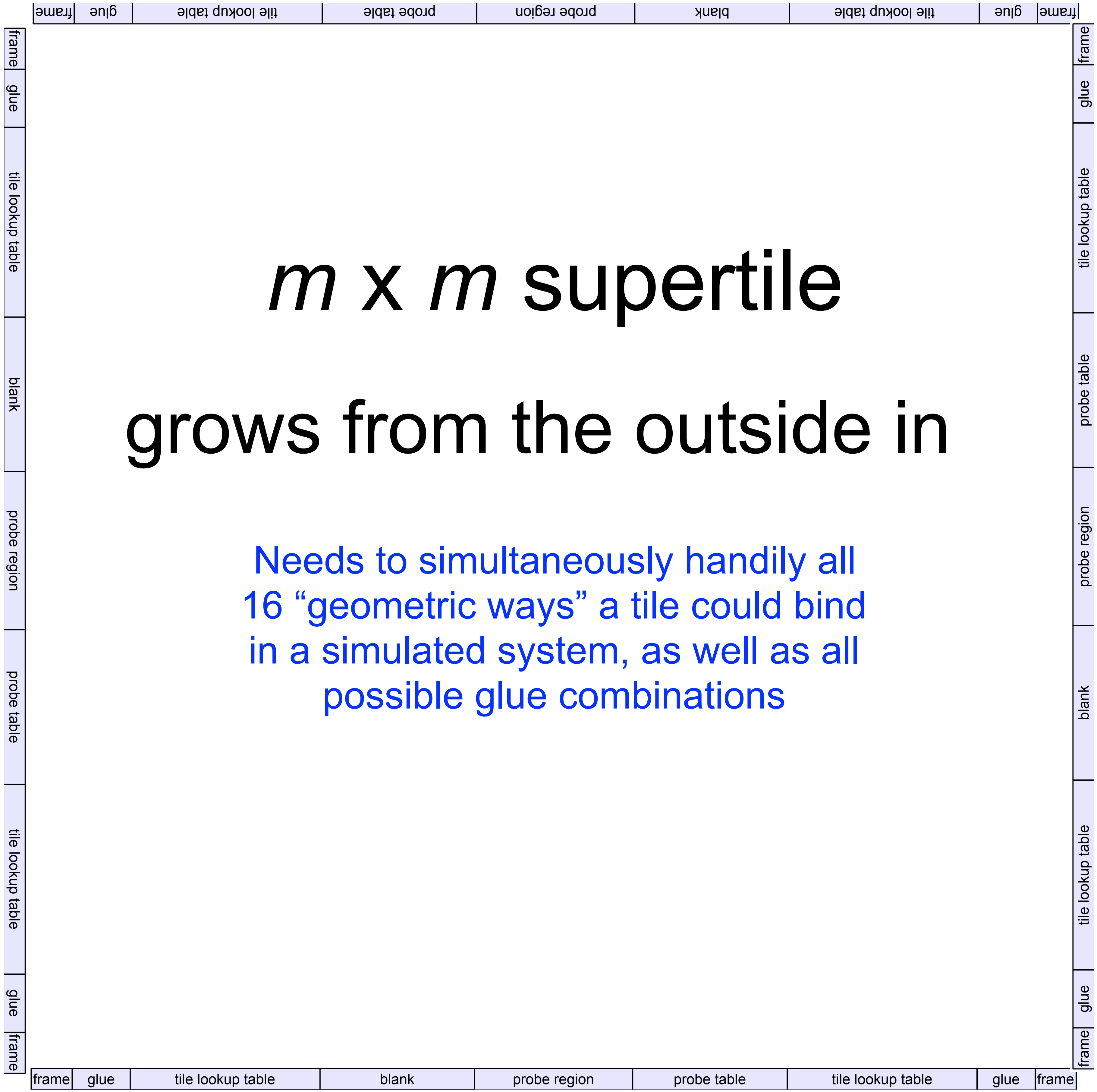
- Variety of cases for different orders of superside arrival
- Superside win/lose configurations and crawler initiation locations (green)
- Proof analogy:
  - Distributed game
  - Computation & geometry
  - Key challenge: make all the tricks work together



m x *m* supertile

Needs to simultaneously handily all 16 “geometric ways” a tile could bind in a simulated system, as well as all possible glue combinations



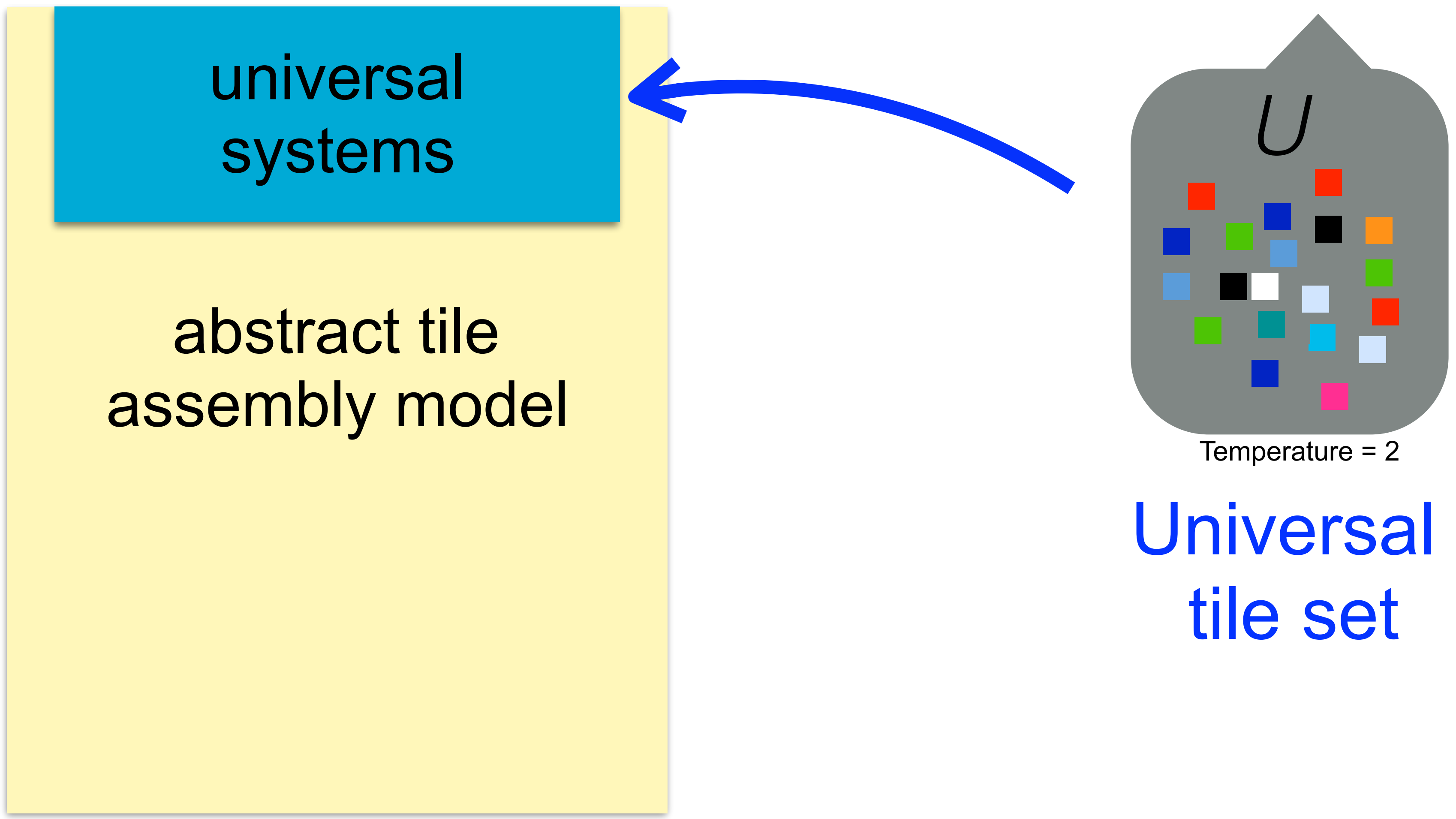


m x *m* supertile

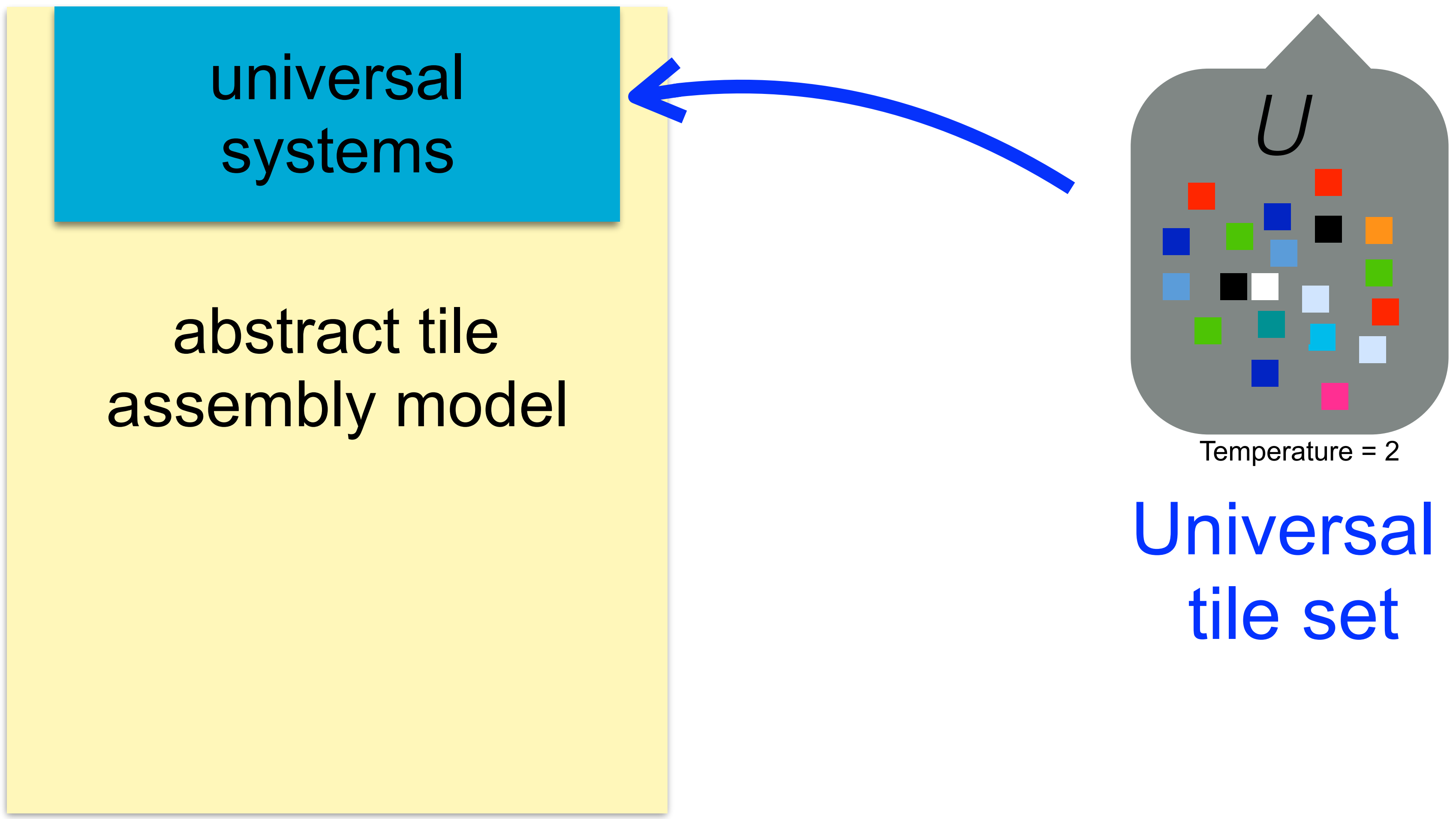
grows from the outside in

Needs to simultaneously handily all 16 “geometric ways” a tile could bind in a simulated system, as well as all possible glue combinations

Is the abstract tile assembly model intrinsically universal?



Is the abstract tile assembly model intrinsically universal? **Yes!**



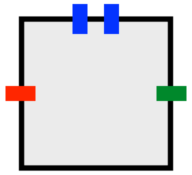
Theorem: There is a single intrinsically universal tile set *U* that simulates *any* tile assembly system

Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

A zoo of self-assembly models

How to compare these models?

abstract tile
assembly model

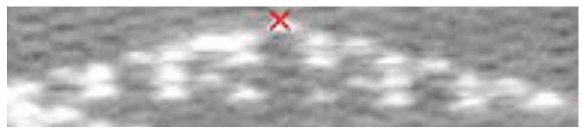


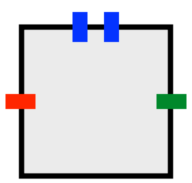
A zoo of self-assembly models

How to compare these models?

abstract tile
assembly model

temperature 2





locally consistent

temperature 1

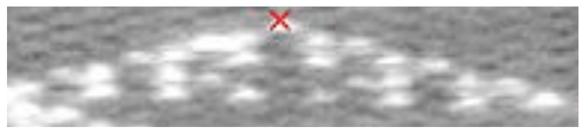
…

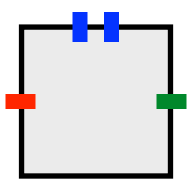
A zoo of self-assembly models

How to compare these models?

abstract tile
assembly model

temperature 2





locally consistent

temperature 1

…

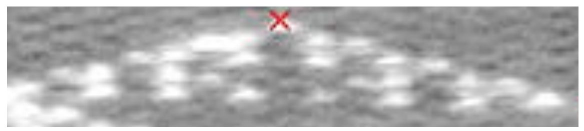
hierarchical

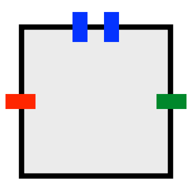
A zoo of self-assembly models

How to compare these models?

abstract tile
assembly model

temperature 2





locally consistent

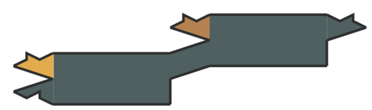
temperature 1

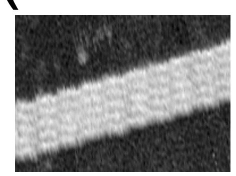
…

hierarchical

polyomino

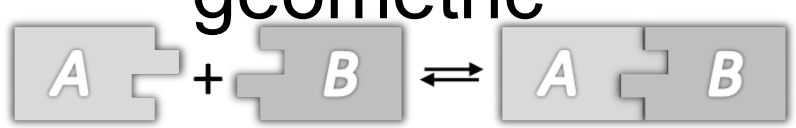
dupled (domino)





polygonal
free-body

geometric



A zoo of self-assembly models

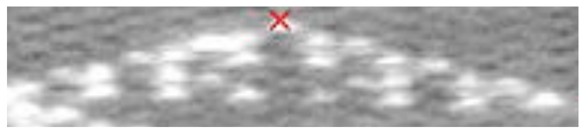
How to compare these models?

abstract tile
assembly model

hierarchical

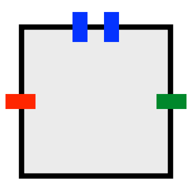
kinetic

temperature 2

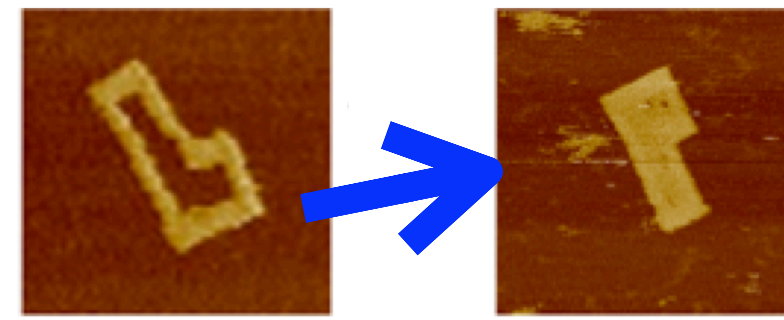


polyomino

staged (mixing)



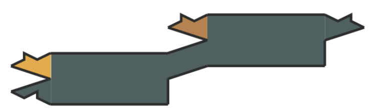
locally consistent

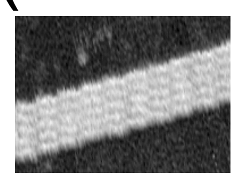


dupled (domino)

temperature 1

…

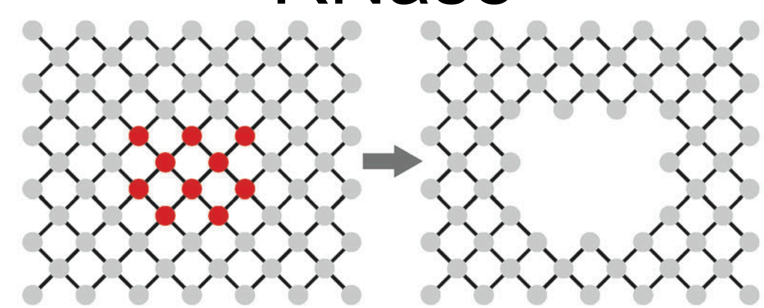


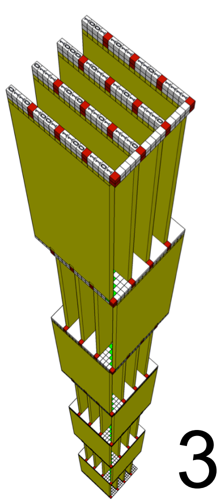


signal-passing

polygonal
free-body

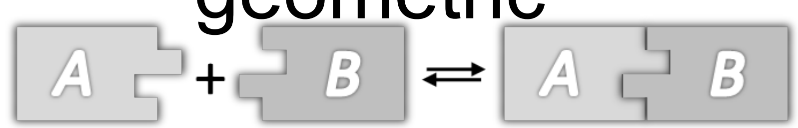
RNase





negative
glue

geometric

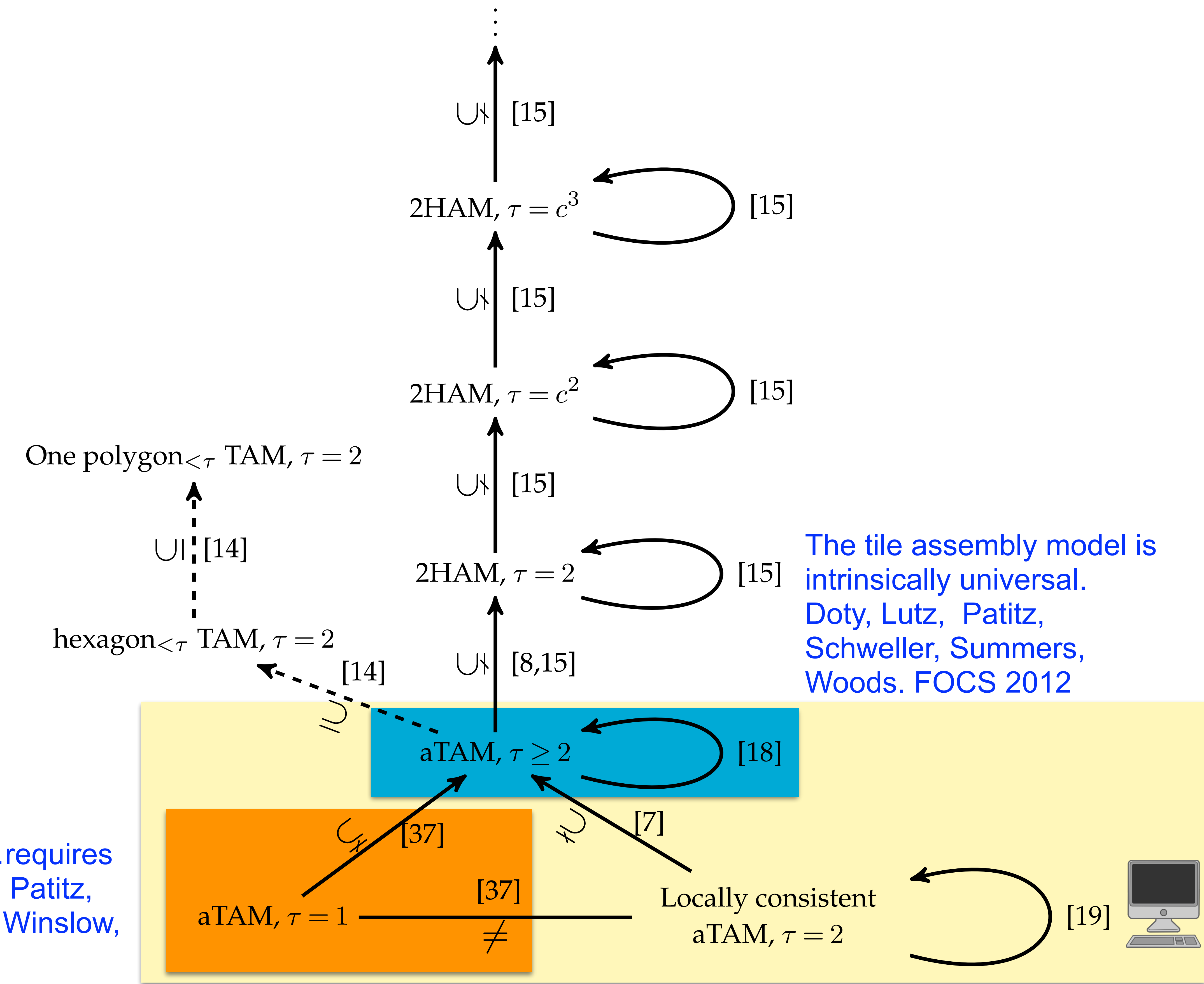


3D

Flexible glue

dupled & restricted glue

A complexity theory for self-assembly



The tile assembly model is intrinsically universal. Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

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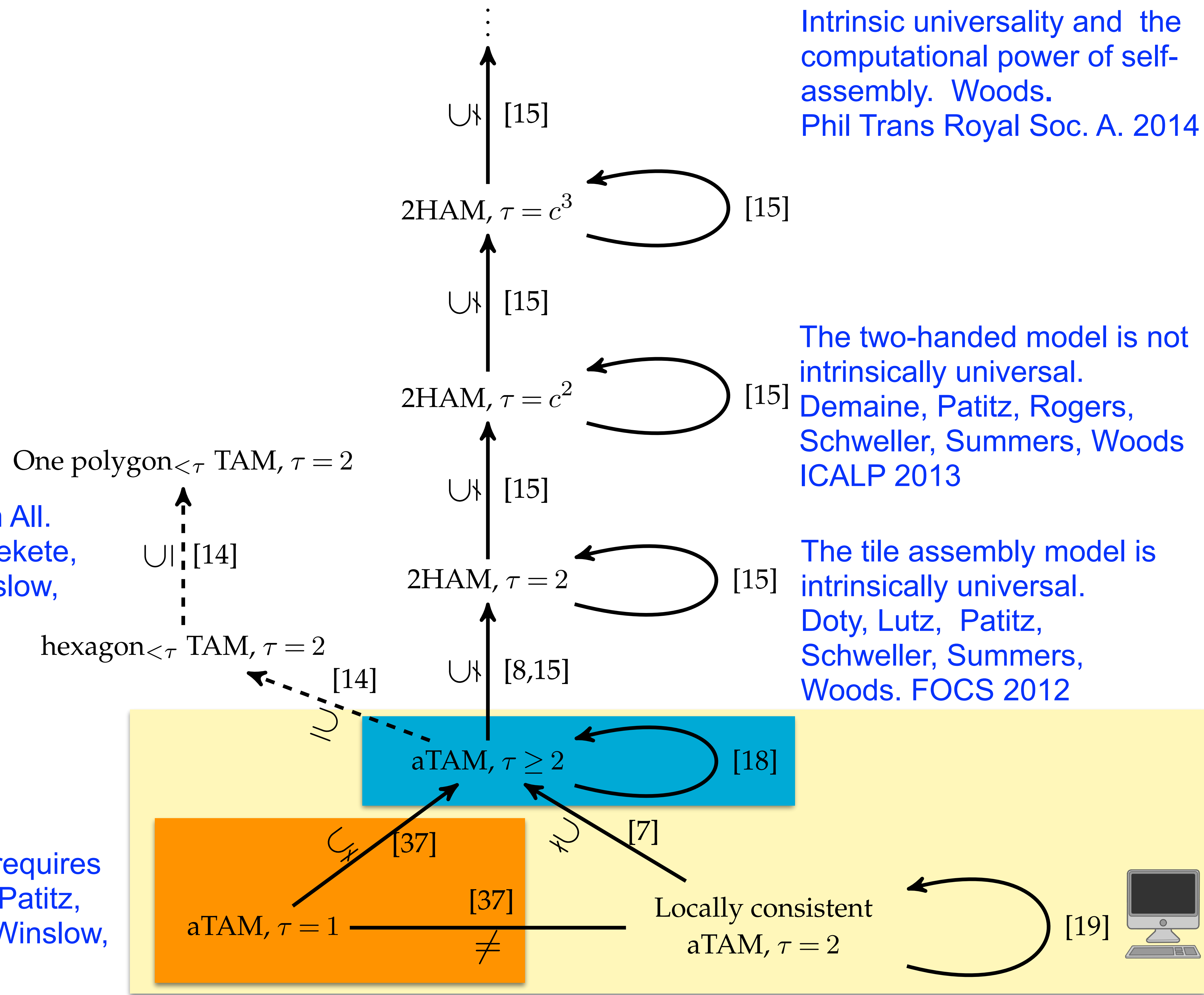

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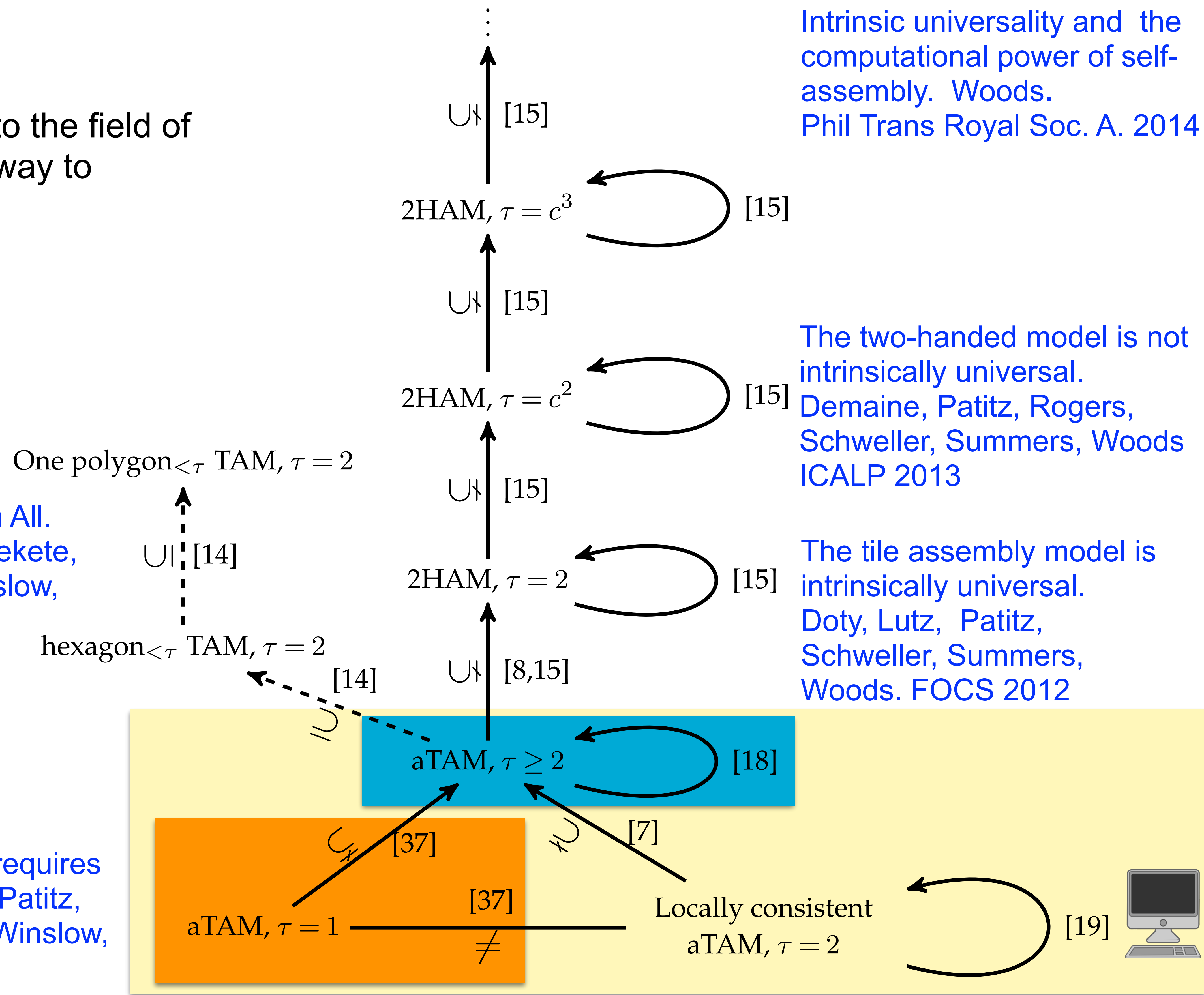
A complexity theory for self-assembly

Gives a structure to the field of self-assembly - a way to compare models



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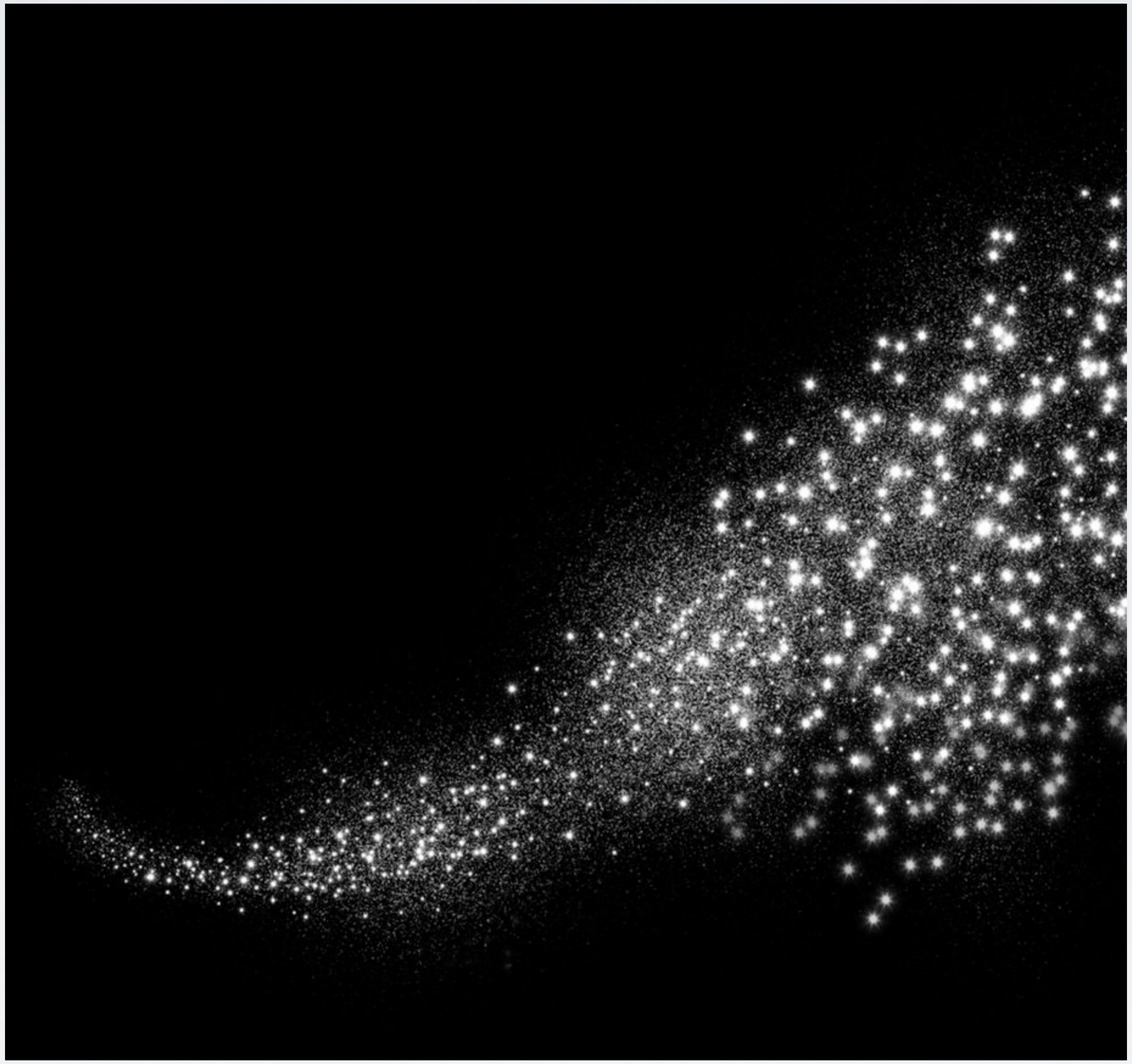
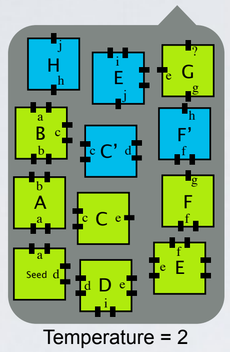
Magic dust

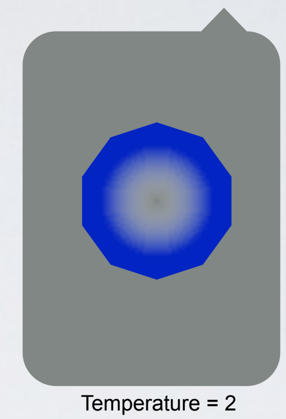


http://nighthawk101stock.deviantart.com/

Result 1: There is a **single** rotatable polygon that simulates **all** tile assembly systems

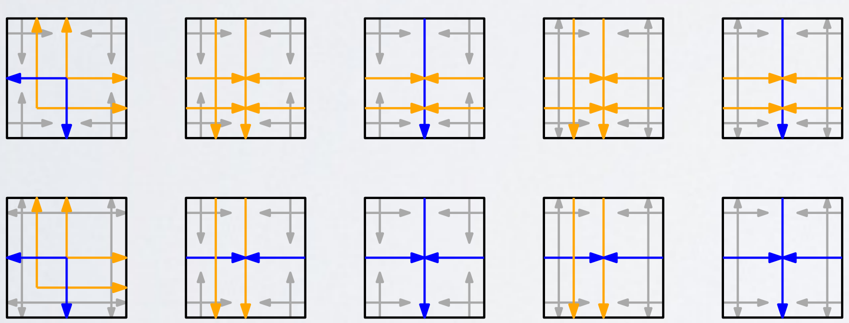
tile assembly system *T*



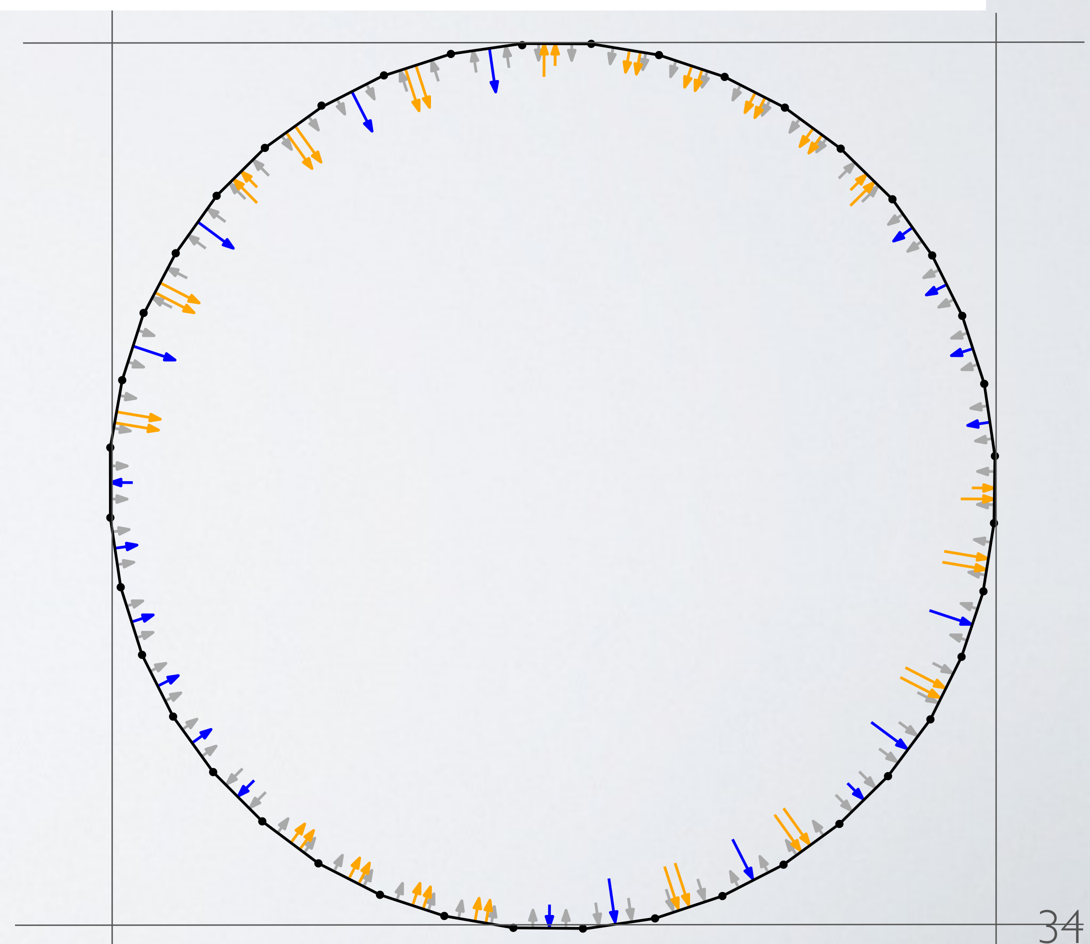


Result 2: For each (e.g. Wang) plane tile system there is one rotatable polygon that simulates it

Theorem 7.1 *Each colored square and hexagon plane tiling system in the families* $(\{\}, c_m)$, $(\{t_r, t_f\}, c_m)$, $(\{\}, c_c)$ *and* $(\{t_r, t_f\}, c_c)$ *is simulated by an* n*-gon nearly-plane tiling system.*

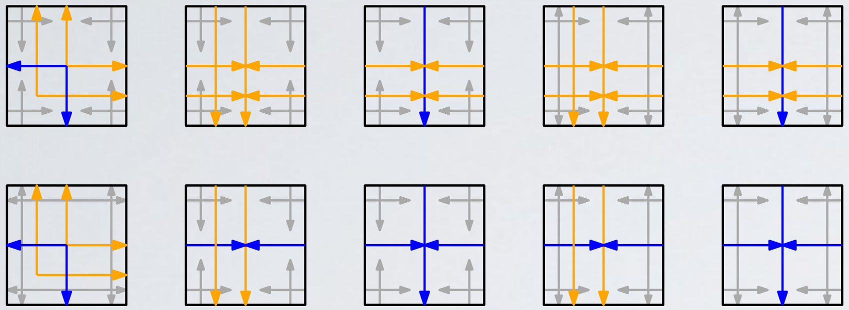


Robinson's 10-tile aperiodic tile set

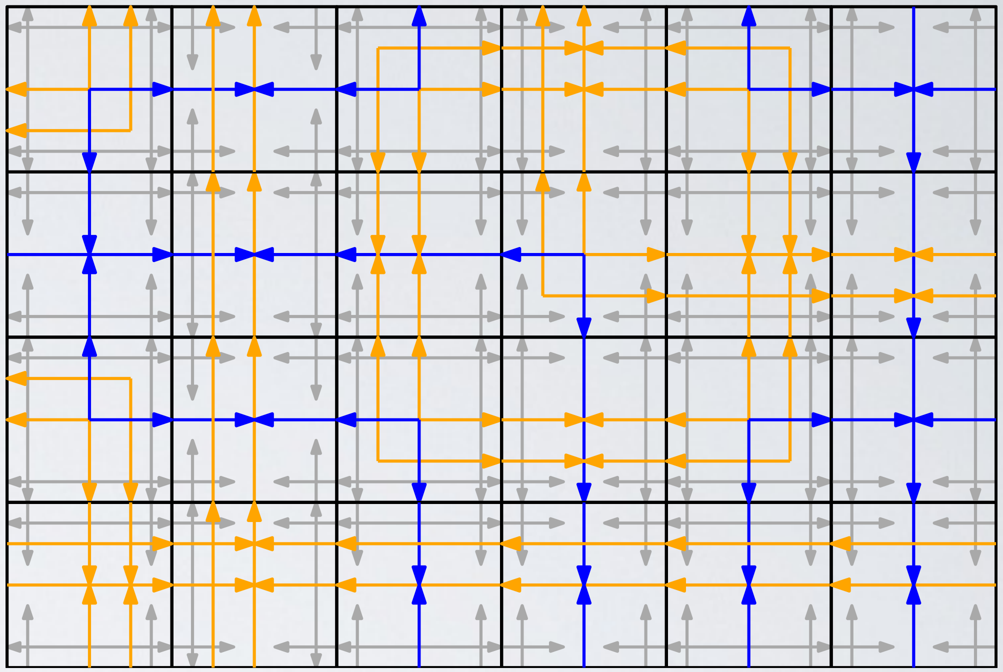


Demaine, Demaine, Fekete, Patitz, Schweller, Winslow, Woods. One Tile to Rule Them All: Simulating Any Turing Machine, Tile Assembly System, or Tiling System with One Rotatable Puzzle Piece. ICALP 2014.

Theorem: For each (Wang) plane tiling system there is one rotatable polygon that simulates it



Robinson's 10-tile aperiodic tile set



Portion of a tiling of Robinson's 10-tile aperiodic square tile set (with rotations)

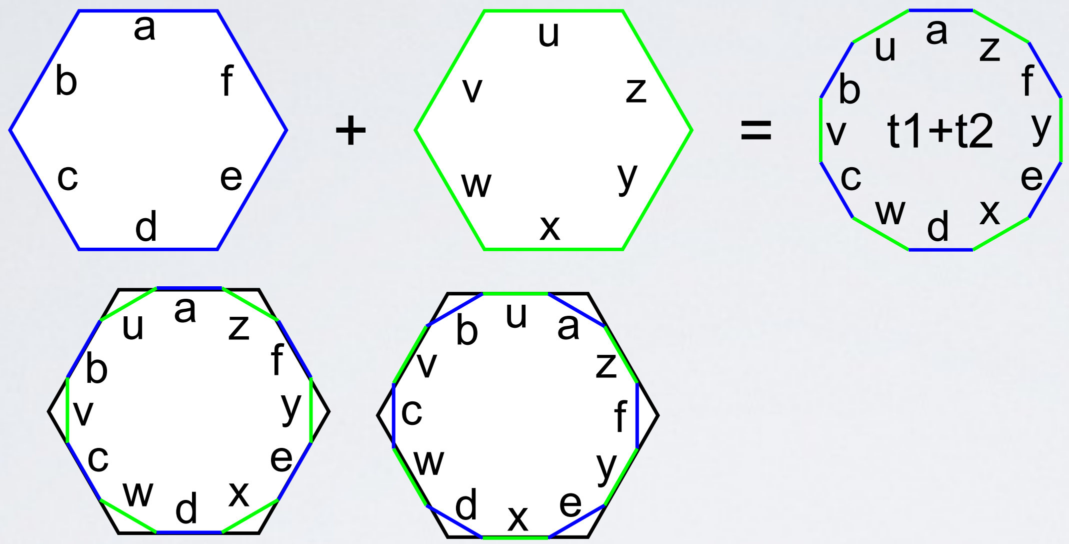
- Wang plane tiling system:
  - Try to fill the plane with tiles.
  - All sides must match.
  - We care about the existence of tilings, but not how we made the tiling

Demaine, Demaine, Fekete, Patitz, Schweller, Winslow, Woods. One Tile to Rule Them All: Simulating Any Turing Machine, Tile Assembly System, or Tiling System with One Rotatable Puzzle Piece. ICALP 2014.

A rotatable polygon that simulates a tile set

- For each set of (possibly rotatable, flipable) Wang square/hexagon tiles there is a single rotatable tile that simulates it

Hexagon example



rotation 1 = t1 rotation 2 = t2

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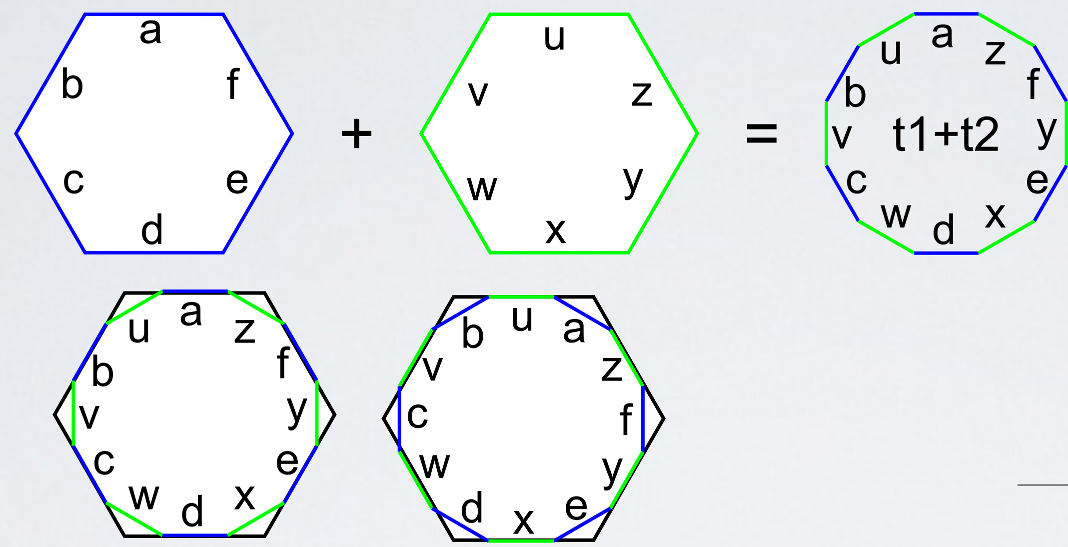

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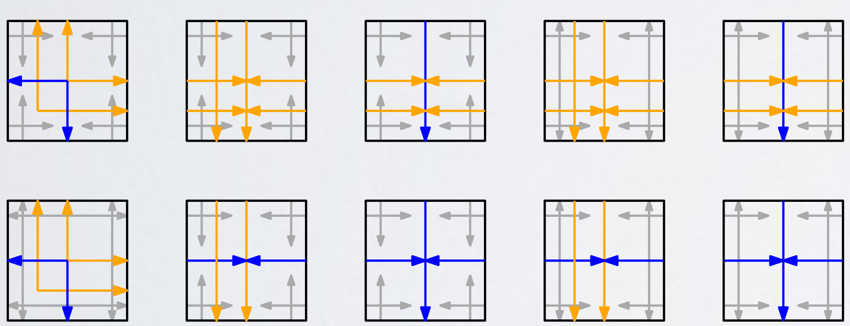
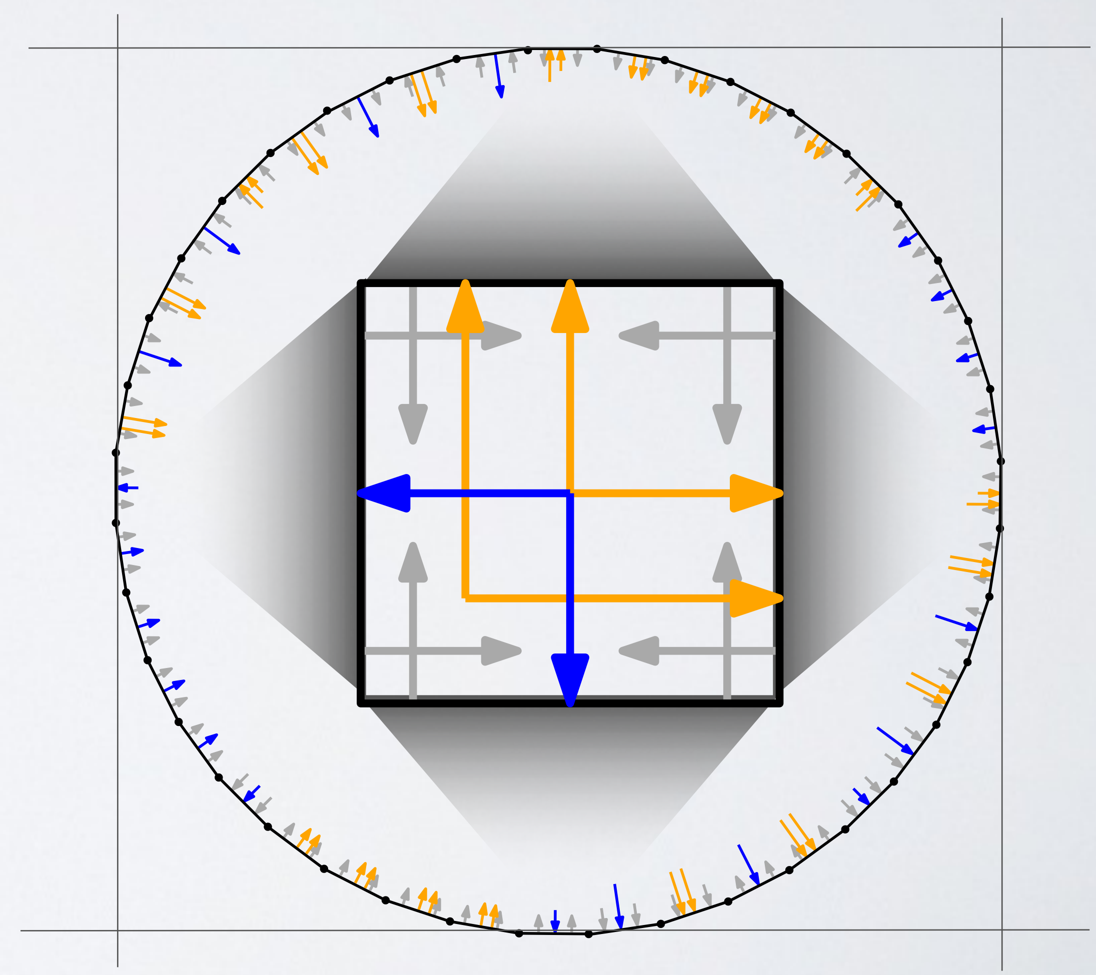


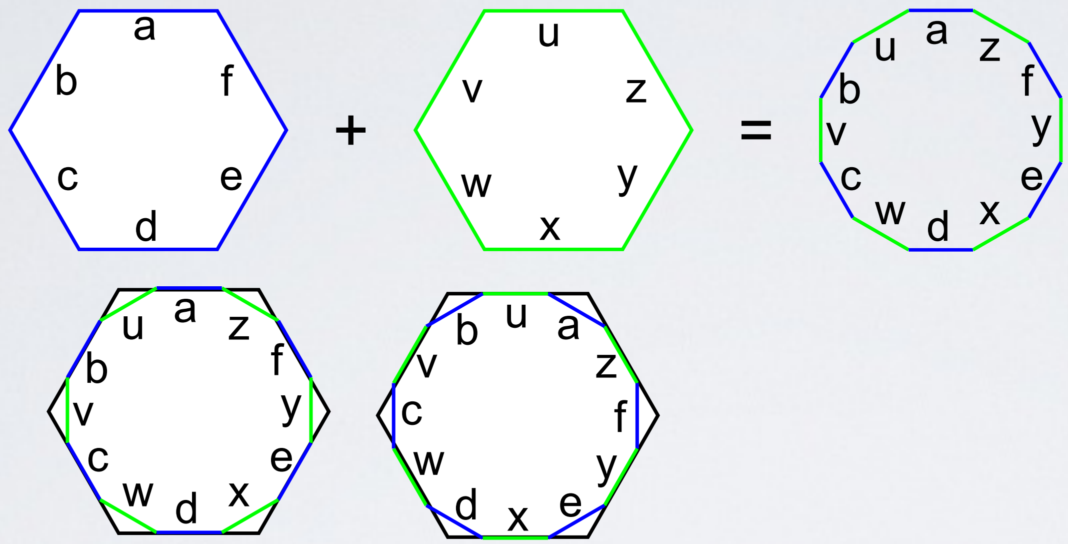
Robinson's 10-tile aperiodic tile set
(complimentary matching constraint)



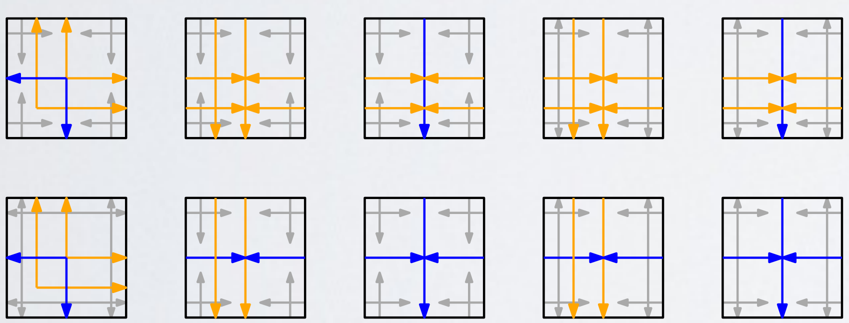
A rotatable polygon that simulates a tile set

- For each set of (possibly rotatable, and/or flipable) square or hexagon tiles there is a single (rotatable, flipable) tile that simulates it

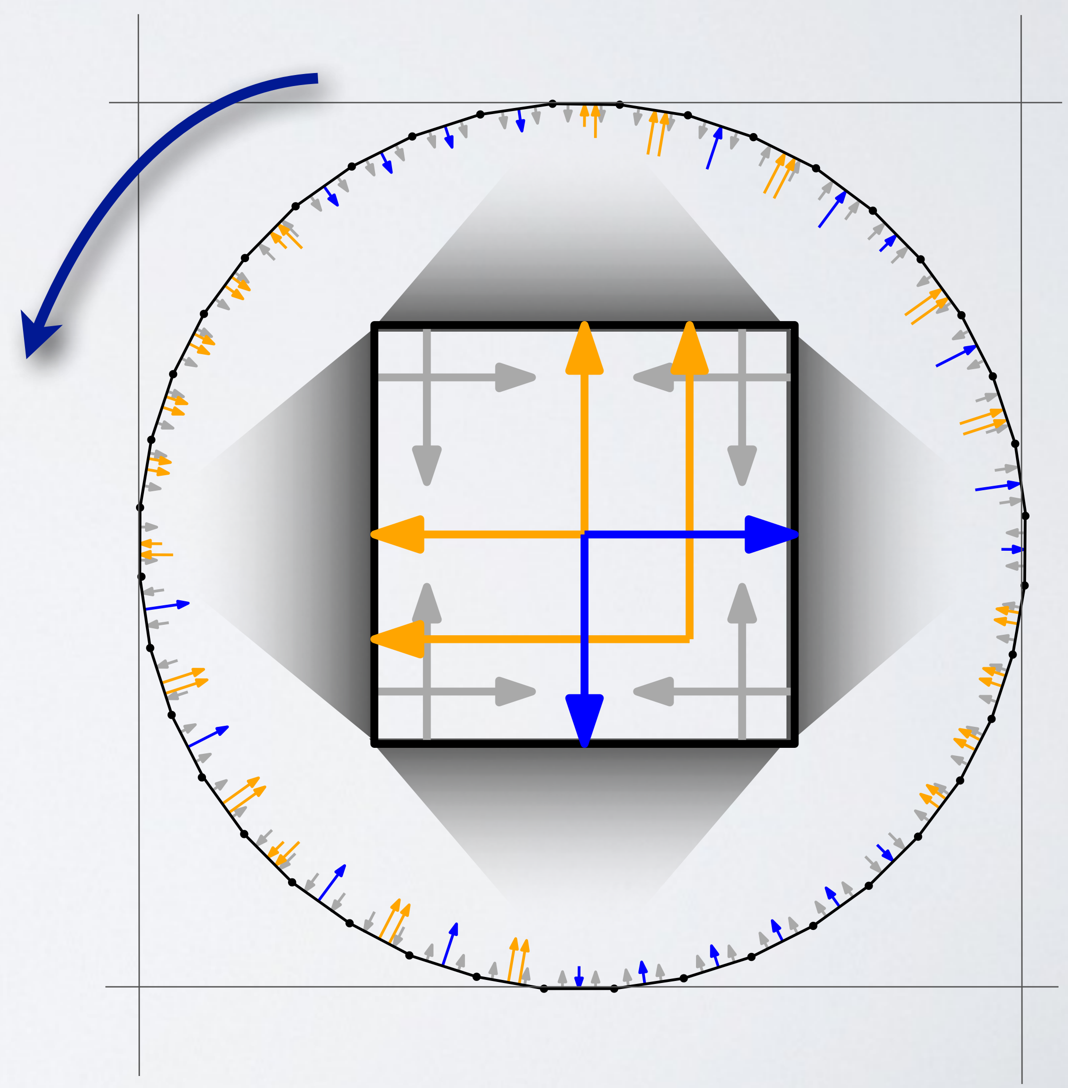
Hexagon example



rotation 1 = t1   rotation 2 = t2
(other rotations may cause problems)

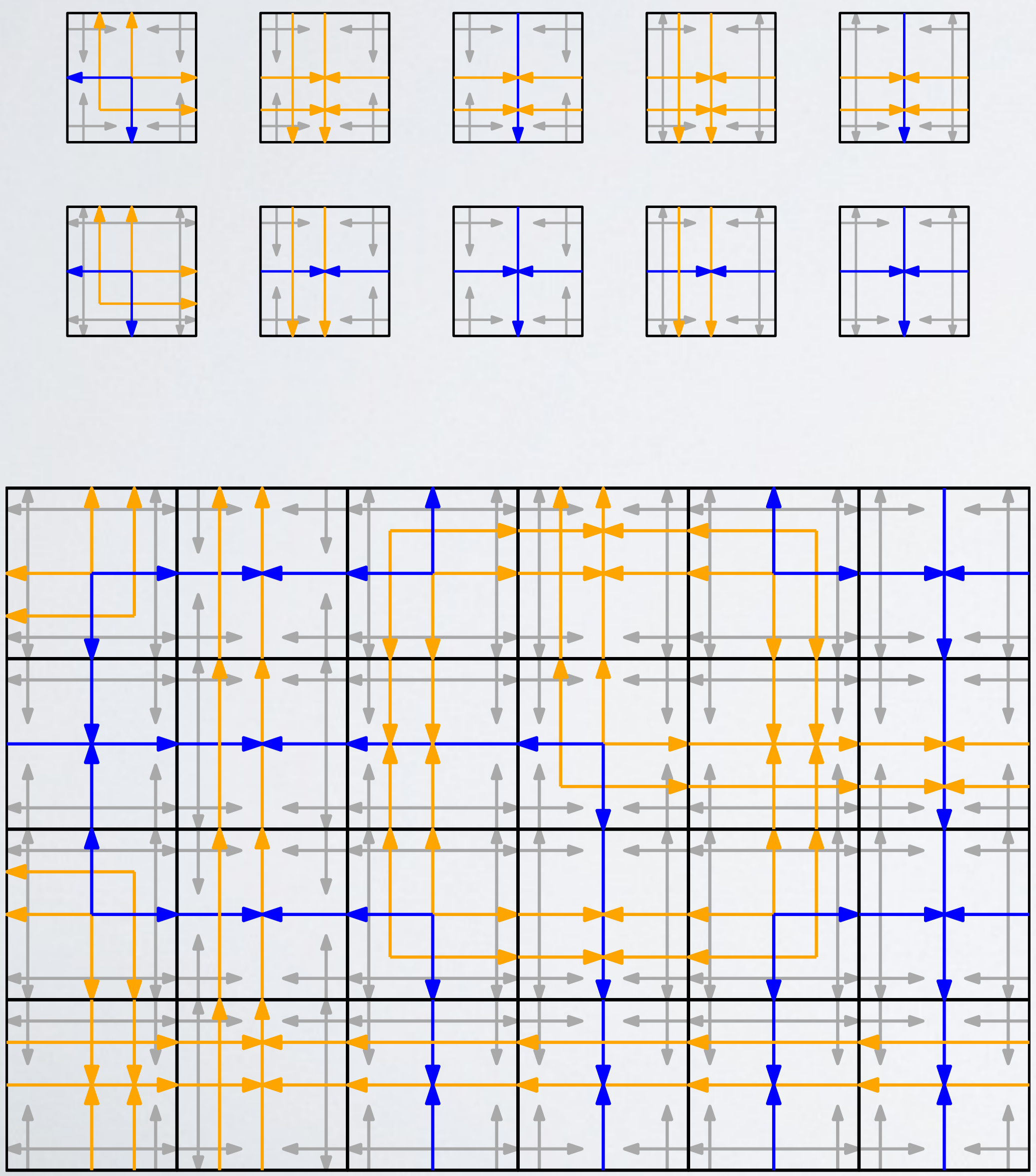


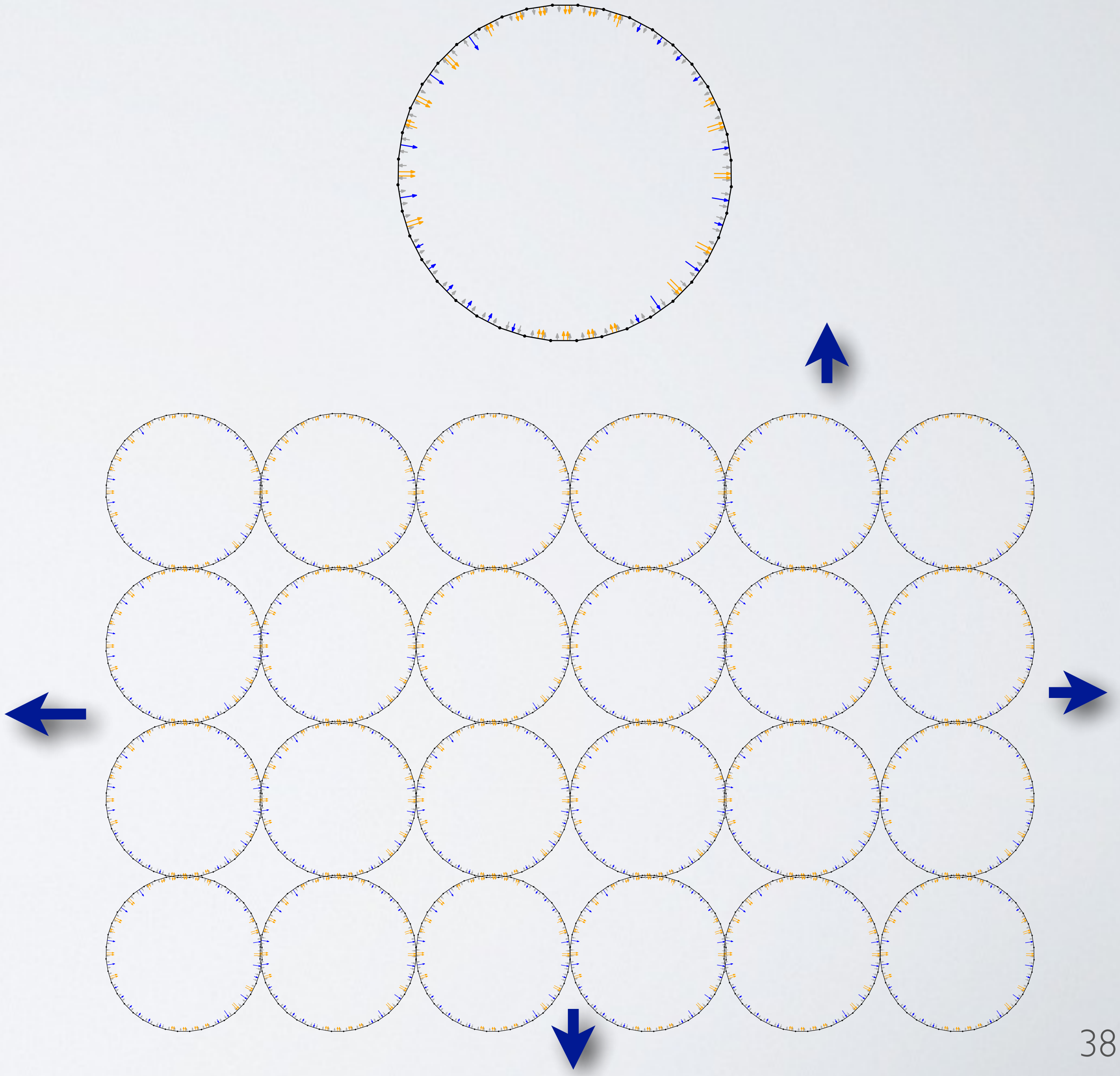
Robinson’s 10-tile aperiodic tile set
(complimentary matching constraint)



1

- An aperiodic tile set with 1 tile!
- Small gaps (< 1 tile in size) in the tilings
- We have given a general method (a compiler) to convert any square/hexagon plane tiling tile set to a single tile that simulates it





1 aperiodic tile

- An aperiodic tile set with 1 tile!
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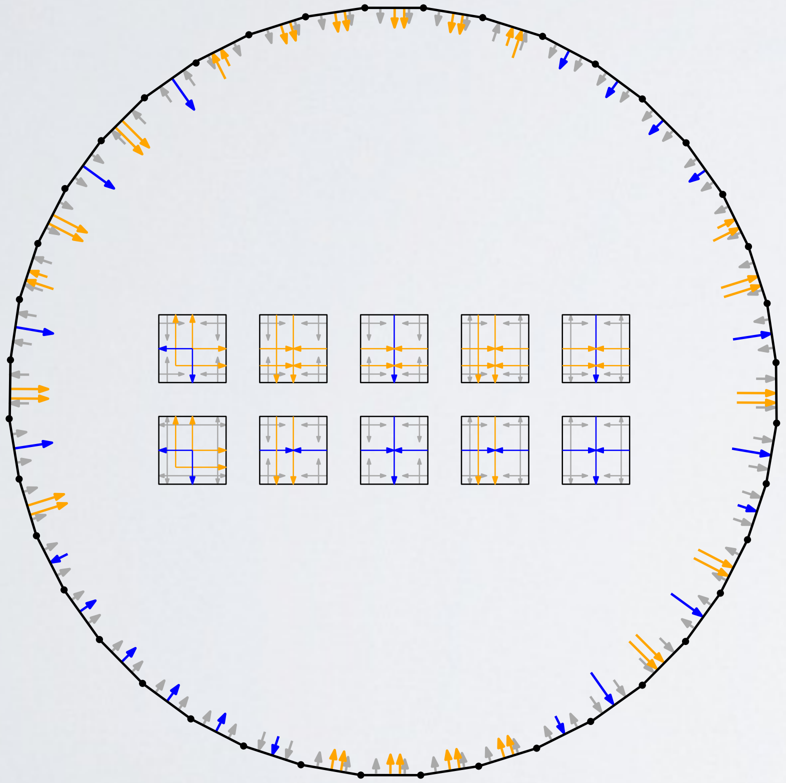
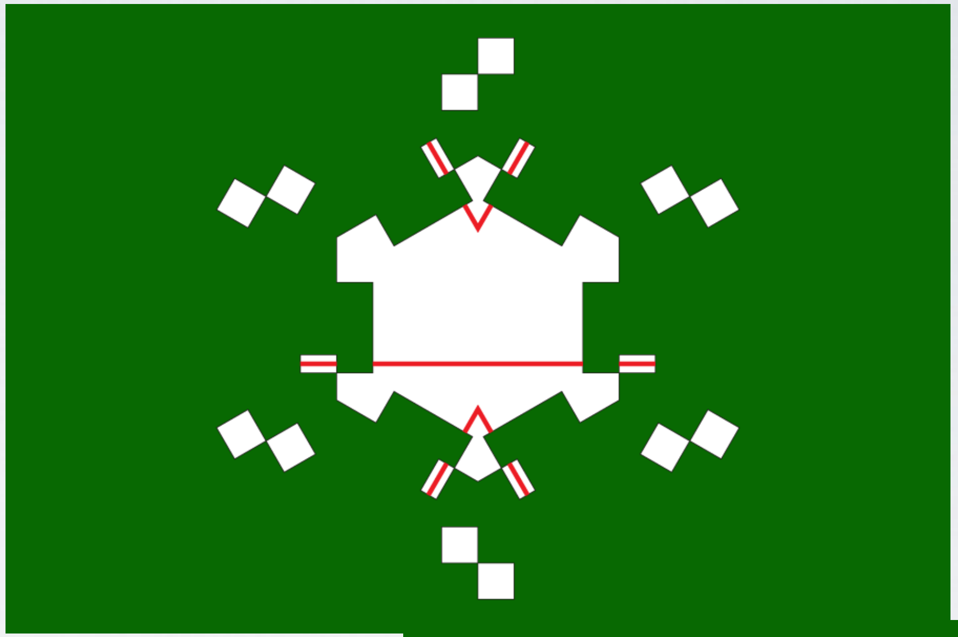
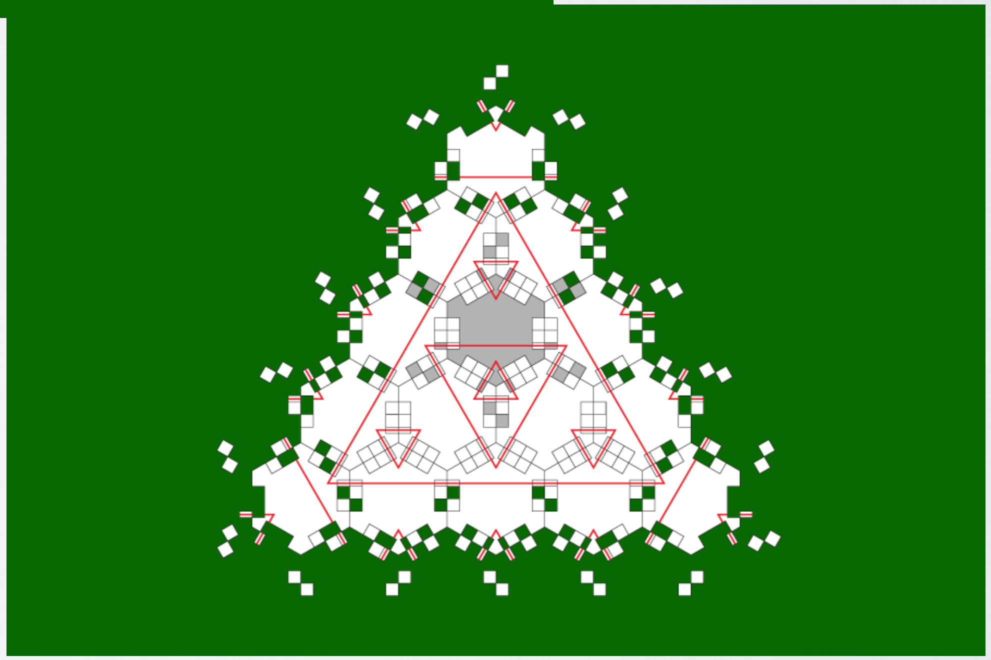


1 aperiodic tile

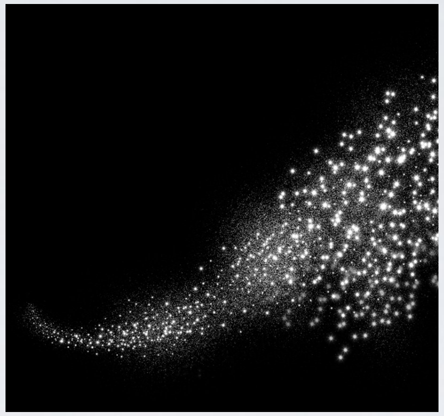
- Socolar–Taylor disconnected tile. 2012
- Aperiodic
- Rotations + flips





- Open question: Is there a single aperiodic connected 2D tile that makes gap-free tilings of the plane?

But this is not (yet) magic dust



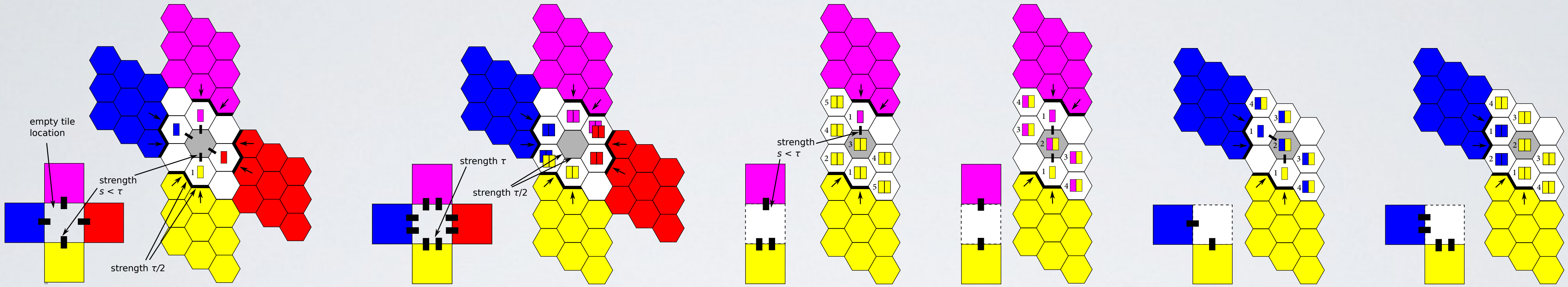
A harder challenge: one tile for all of tile self-assembly

- Simulating tile assembly systems is significantly trickier than plane (Wang) tiling systems
- We want to design a single rotatable, flipable tile that simulates any tile assembly system (Note that as a corollary this gives a single tile that simulates any algorithm)

- Problem 1:
  - Strength tau glues on rotatable tiles => Argh! There's pumpable junk everywhere!
  - Maybe we could find an intrinsically universal square tile set with strength < τ glues? No! Any such tile set with a finite seed can not leave the seed's bounding box
  - Lets try hexagons!

Low strength hexes simulate high-strength squares

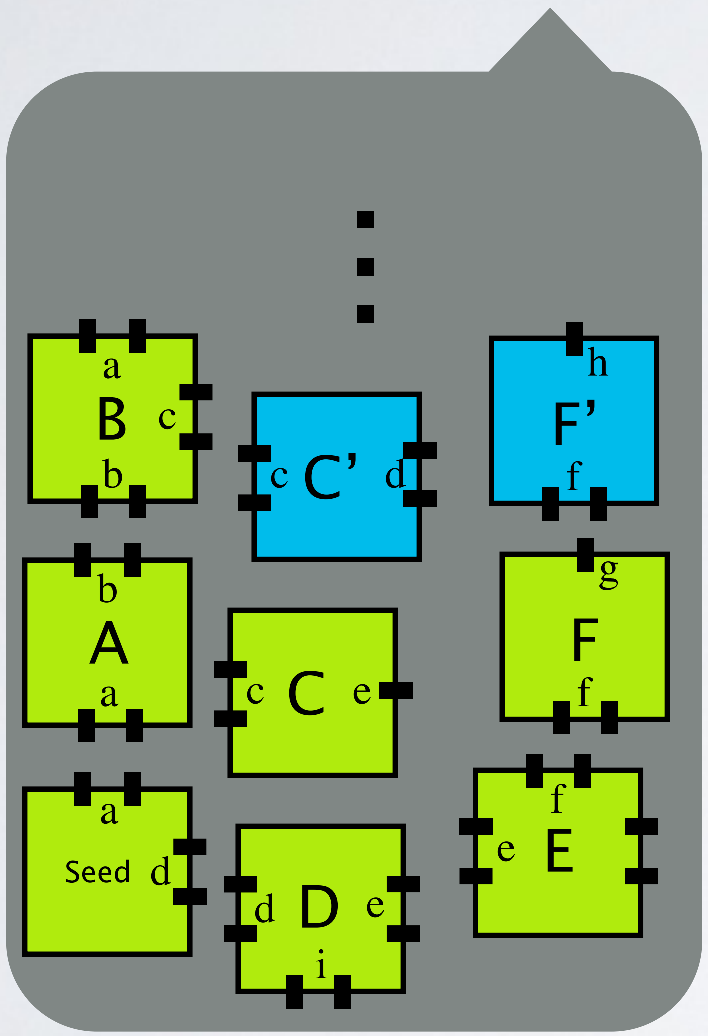
- Strength 1 or 0 hexagon glues, simulating strength 2, 1 or 0 square glues



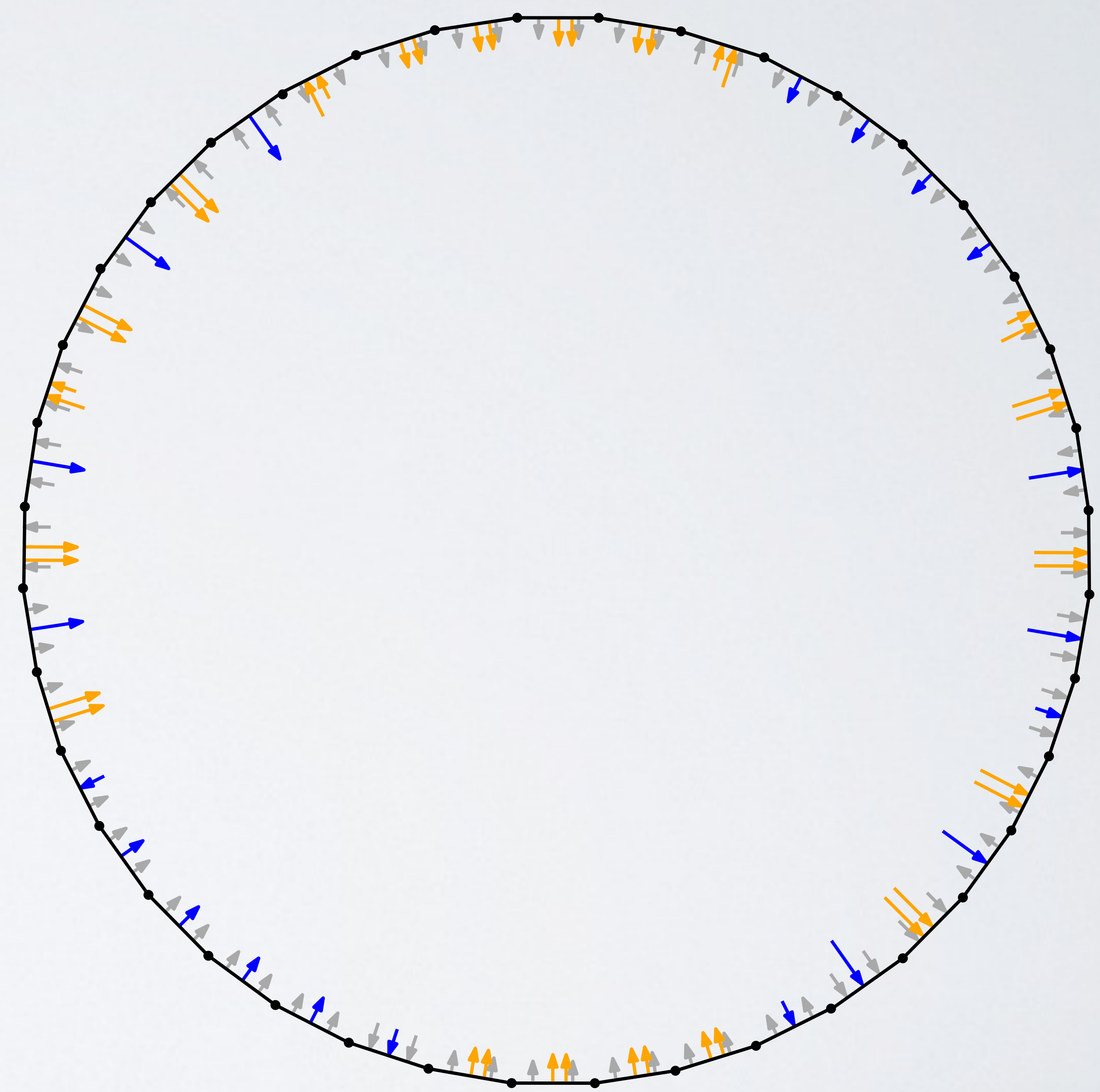
- Then we can simulate a set of low-strength hexagons with a single rotatable polygon
    - Bumps and dents to stop incorrect orientations and incorrect bindings
    - Glues are carefully rearranged on the polygon to allow “self seeding”
    - …
    - Many details omitted!

Construction overview

Tile assembly system T → Hexagonal tile assembly system that simulates T (using low-strength glues) → One polygon that simulates T



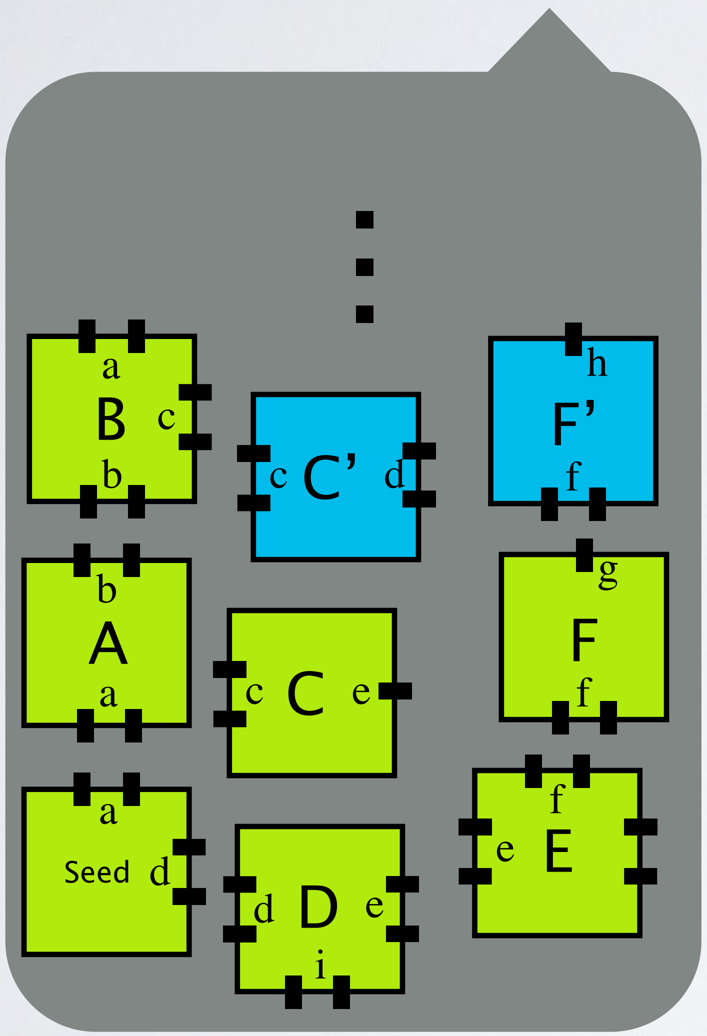
Tile set



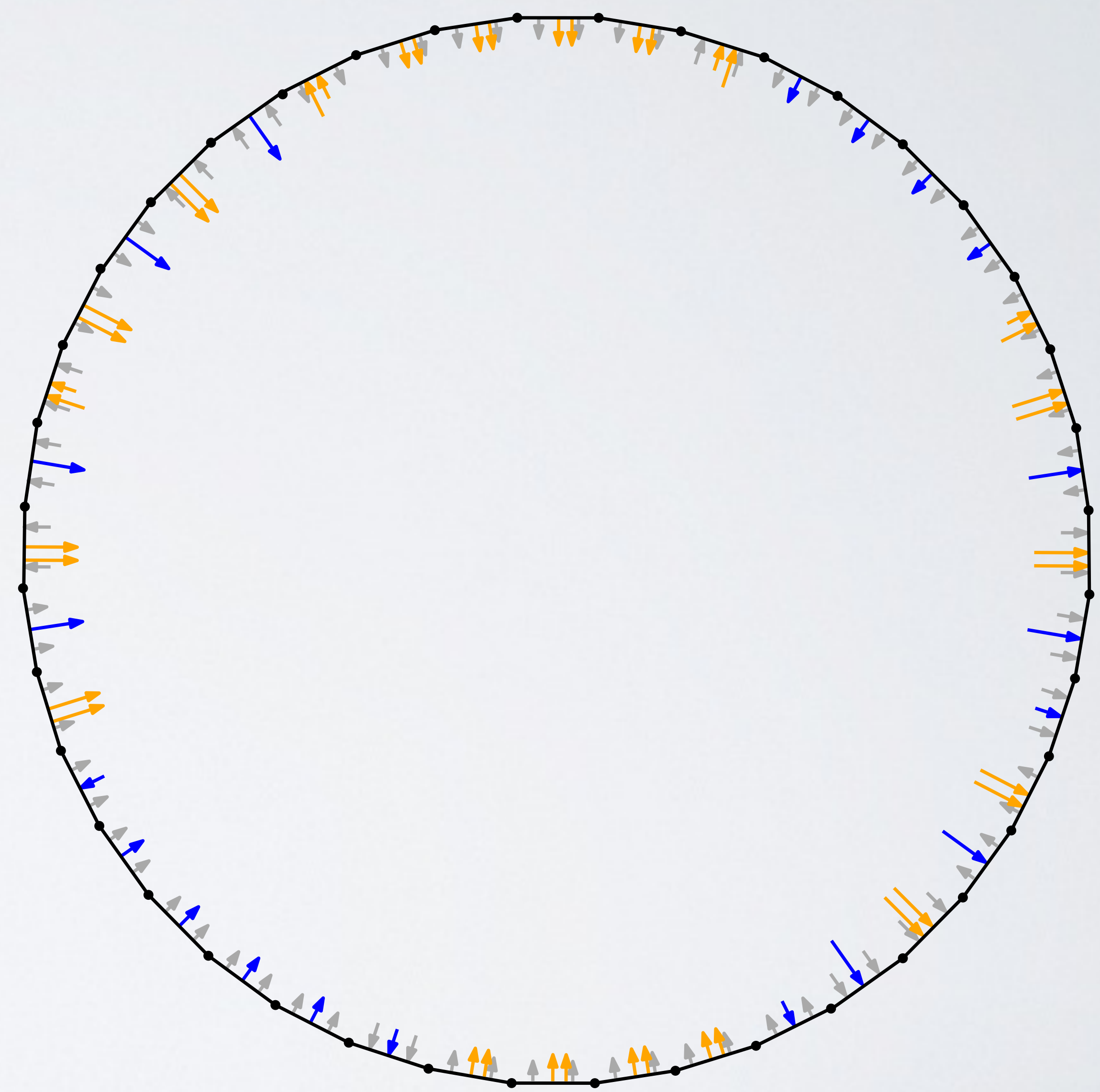
The one

To use The One, simply apply a sequence of tile assembly system simulations:

Tile assembly system T → Tile assembly system U_T over the intrinsically universal tile set U → Hexagonal tile assembly system (with low strength glues) → A tile assembly system that simulates T using the single rotatable flipable polygonal tile



Intrinsically universal tile set



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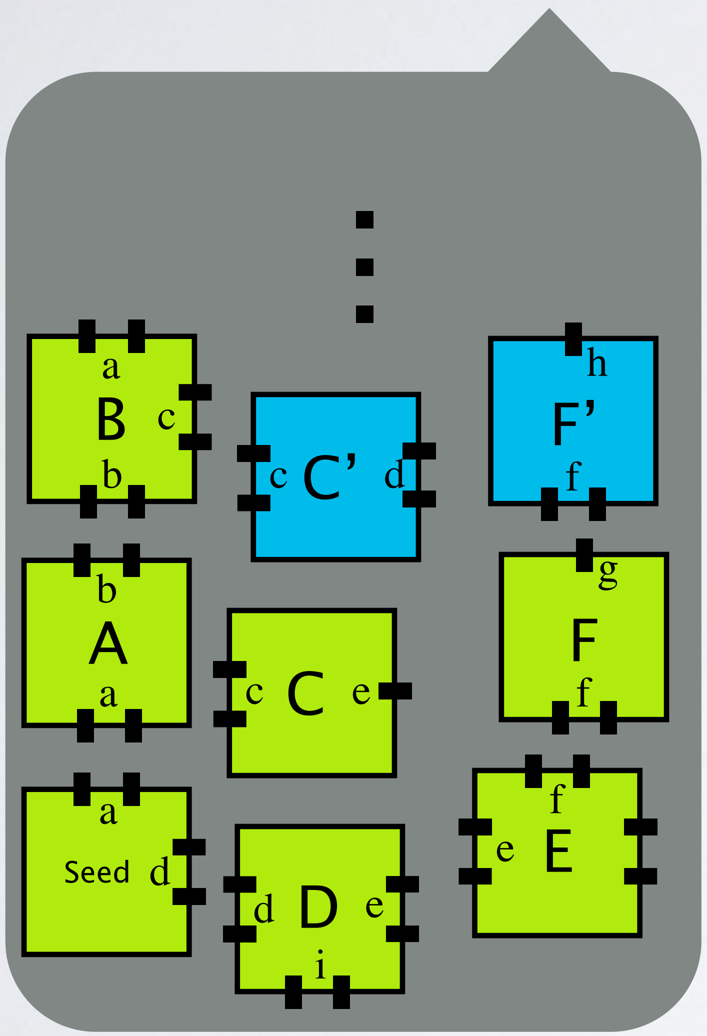
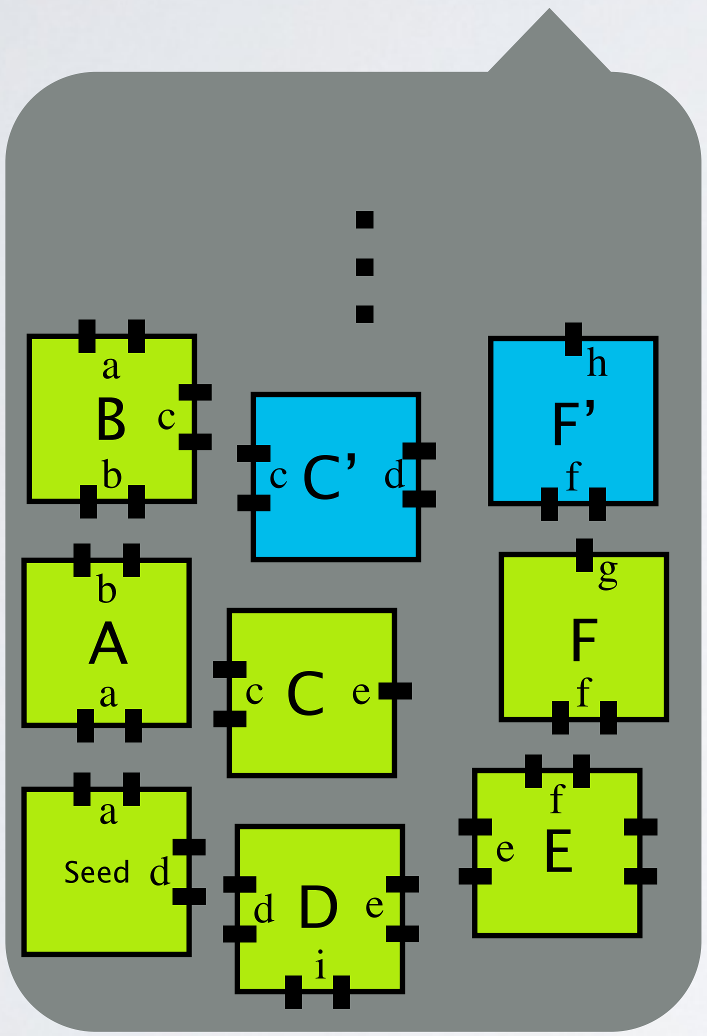


Intrinsically universal tile set

The one

http://oscarfarellano.deviantart.com/

One tile to simulate them all



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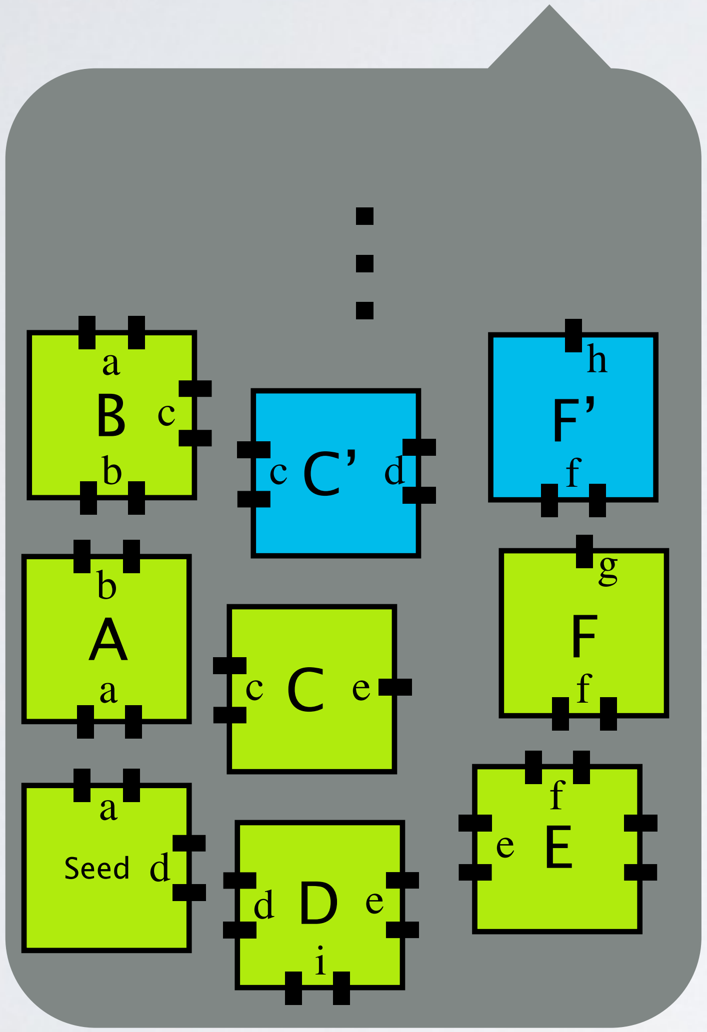

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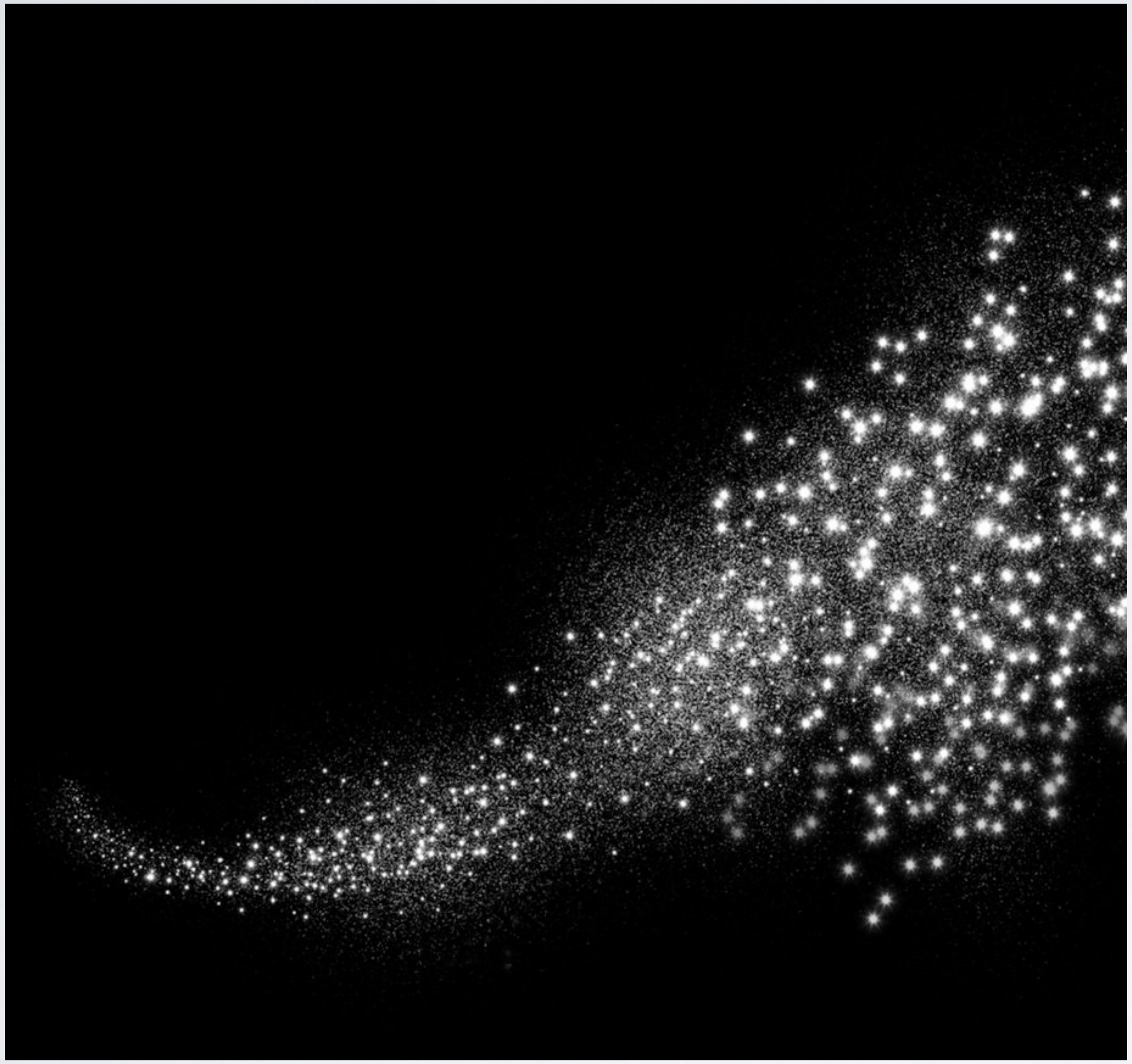
Magic dust



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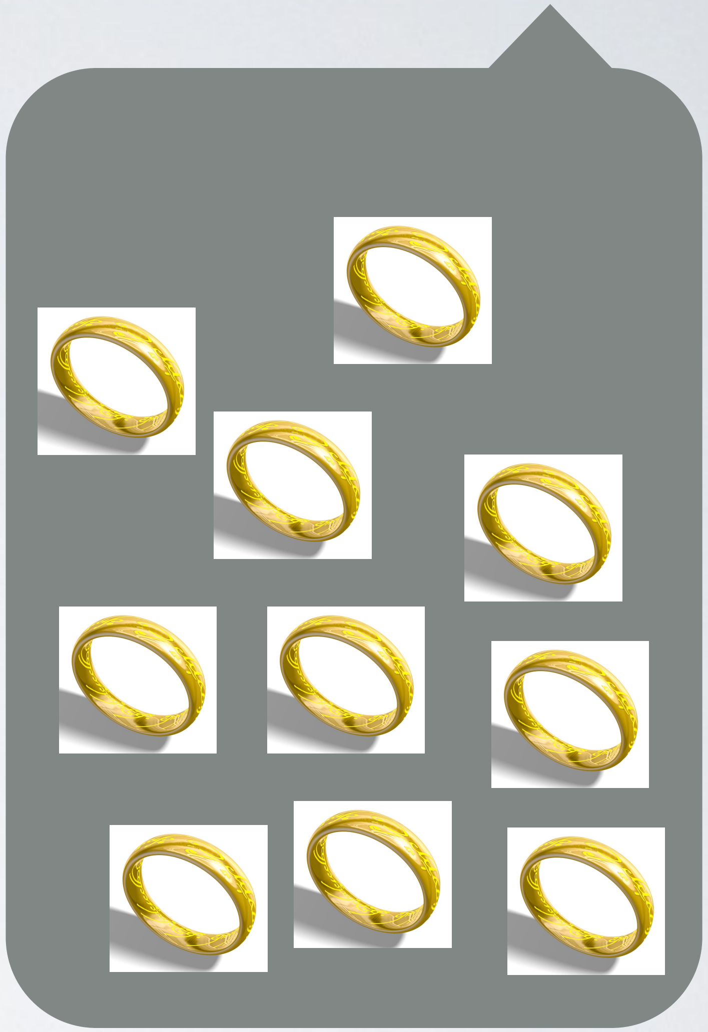
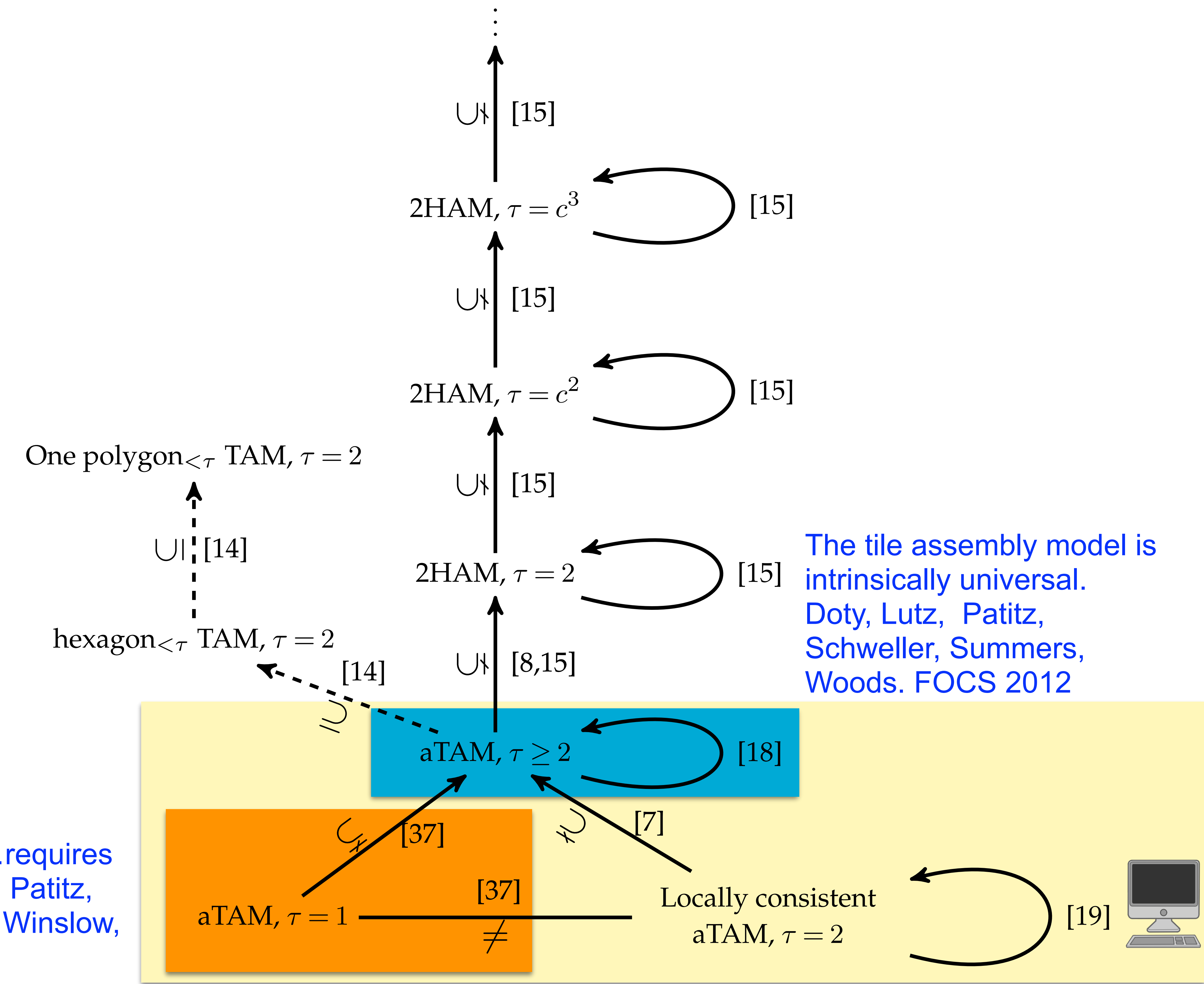


A complexity theory for self-assembly



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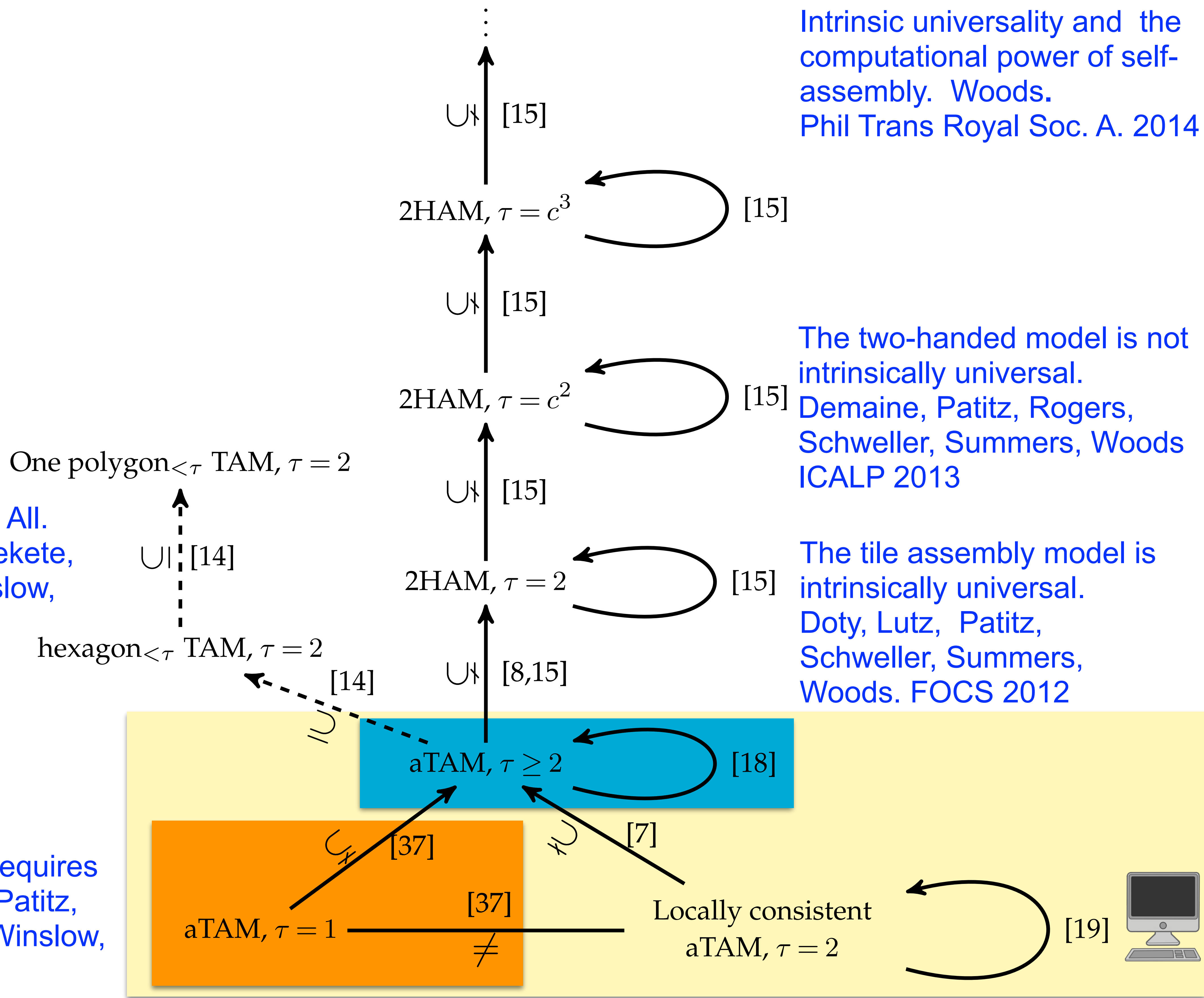

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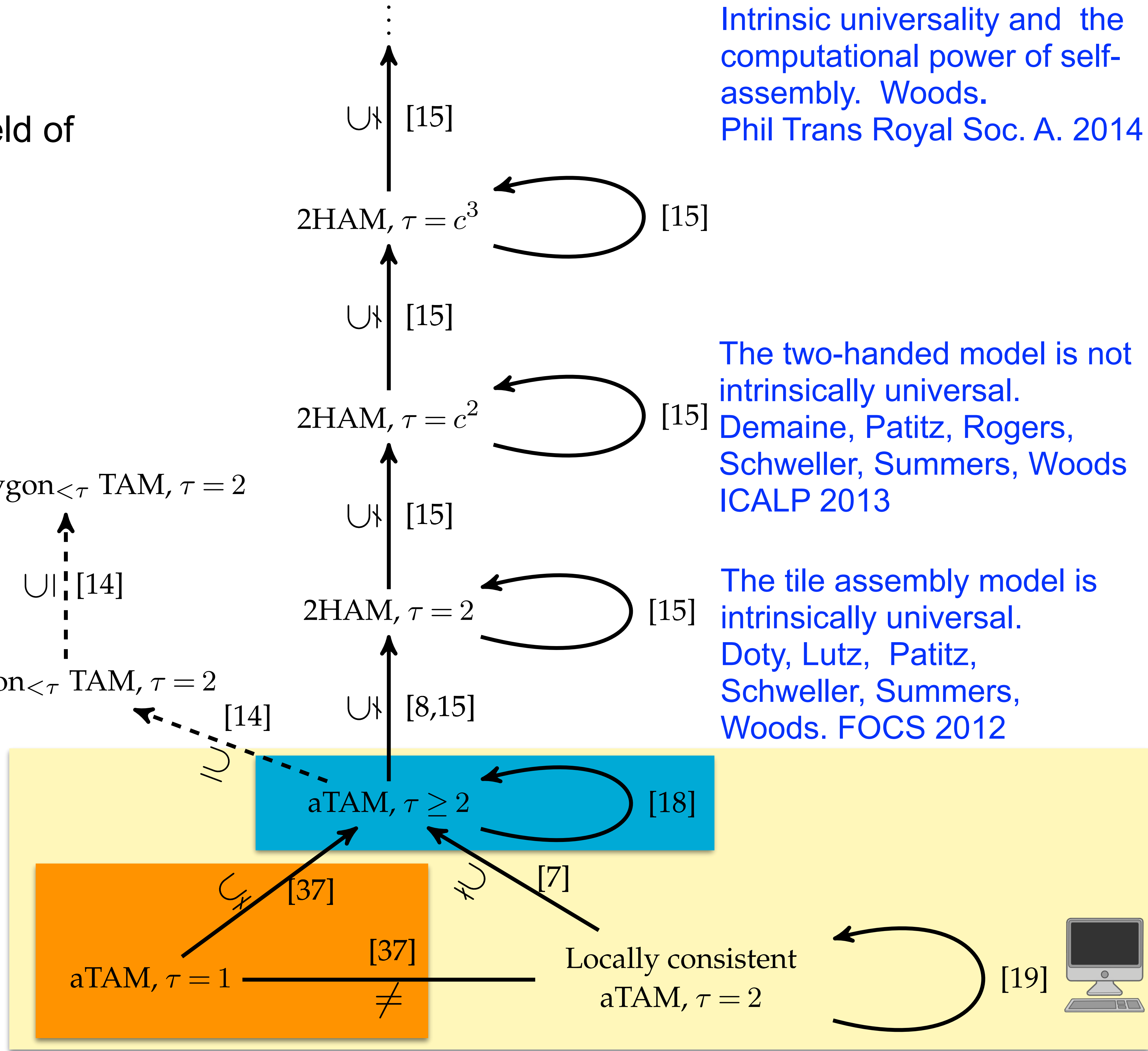

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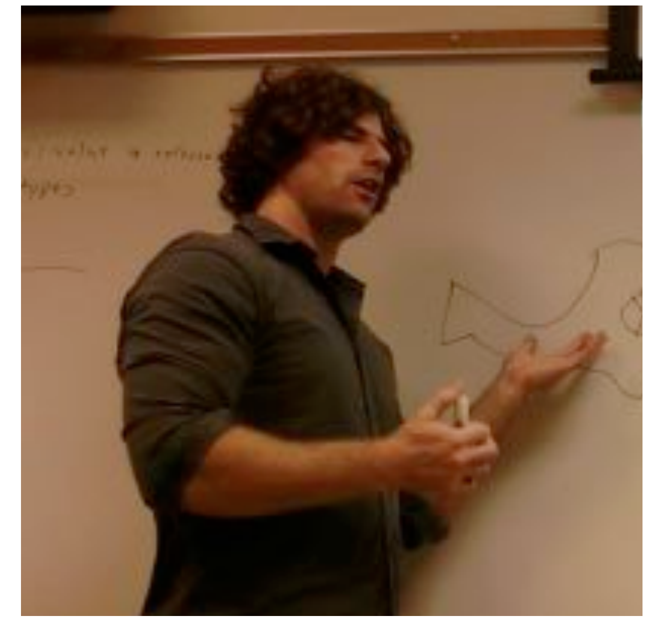


Jack Lutz · Matthew Patitz · Dave Doty · Scott Summers · Robert Schweller

Erik Demaine

Martin Demaine

Sándor Fekete

Pierre-Étienne Meunier

Guillaume Theyssier

Andrew Winslow

And many others ….

Open question: Probabilistically fair intrinsically universal tile set?

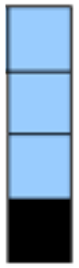
Fin

Structure

- Monday (lecture 1). Some high-level motivations, basic algorithmic self-assembly **models** (definitions) and very recent results on **implementing algorithmic DNA nanotube circuits, a self-assembly model,** in the wet-lab
- Tuesday (lecture 2): DNA sequence design and results on the DNA nanotube circuit model
- Wednesday (lecture 3). Complexity theory for self assembly.
  - **Theorem**: The (cooperative, temperature >= 2) abstract tile assembly model is intrinsic universal
- Thursday (lecture 4). Complexity theory for self assembly.
  - **Theorem**: The noncooperative (temperature 1) abstract tile assembly model does not simulate the cooperative model

Temperature 1 tile assembly

- Temperature 1 tile assembly systems:
  - Tile binds to an assembly if ≥ 1 side match
  - Snakes on a plane



Temperature 1 tile assembly

- Temperature 1 tile assembly systems:
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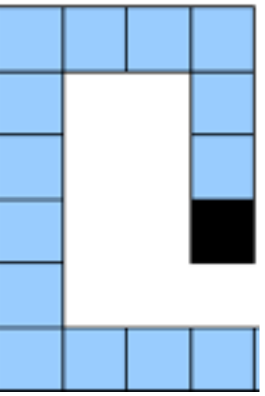

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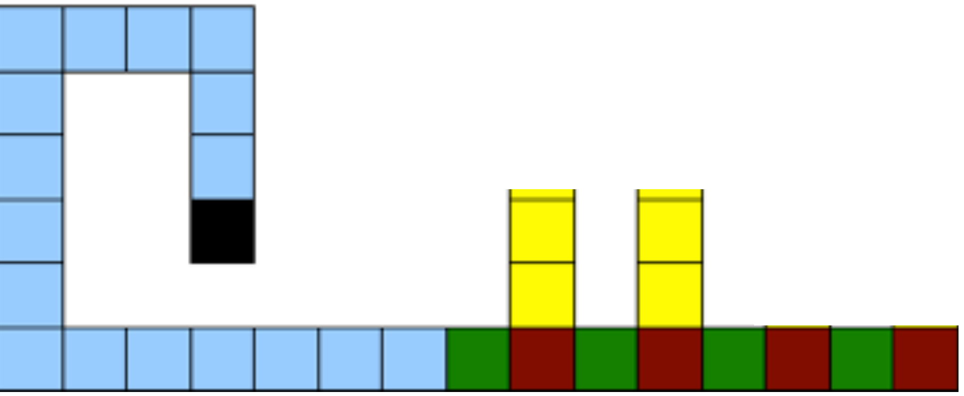

Pic credit: Scott Summers & Dave Doty

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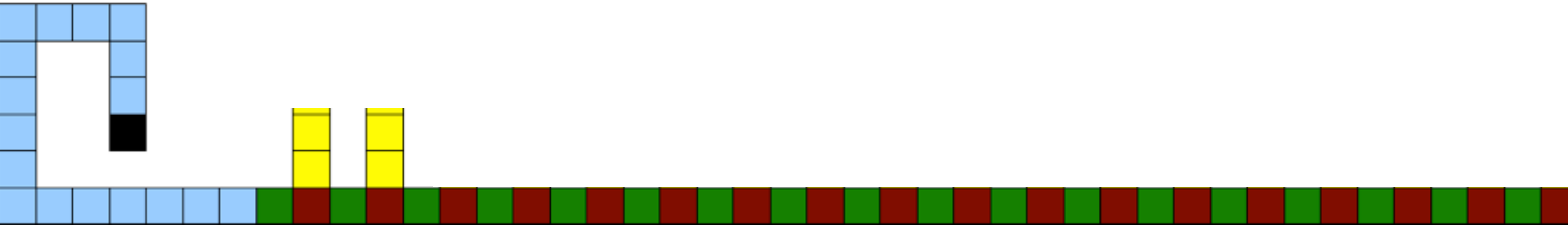

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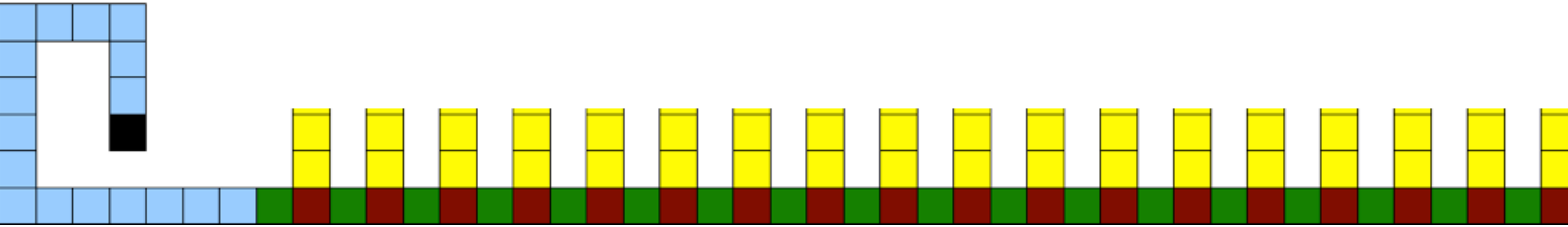

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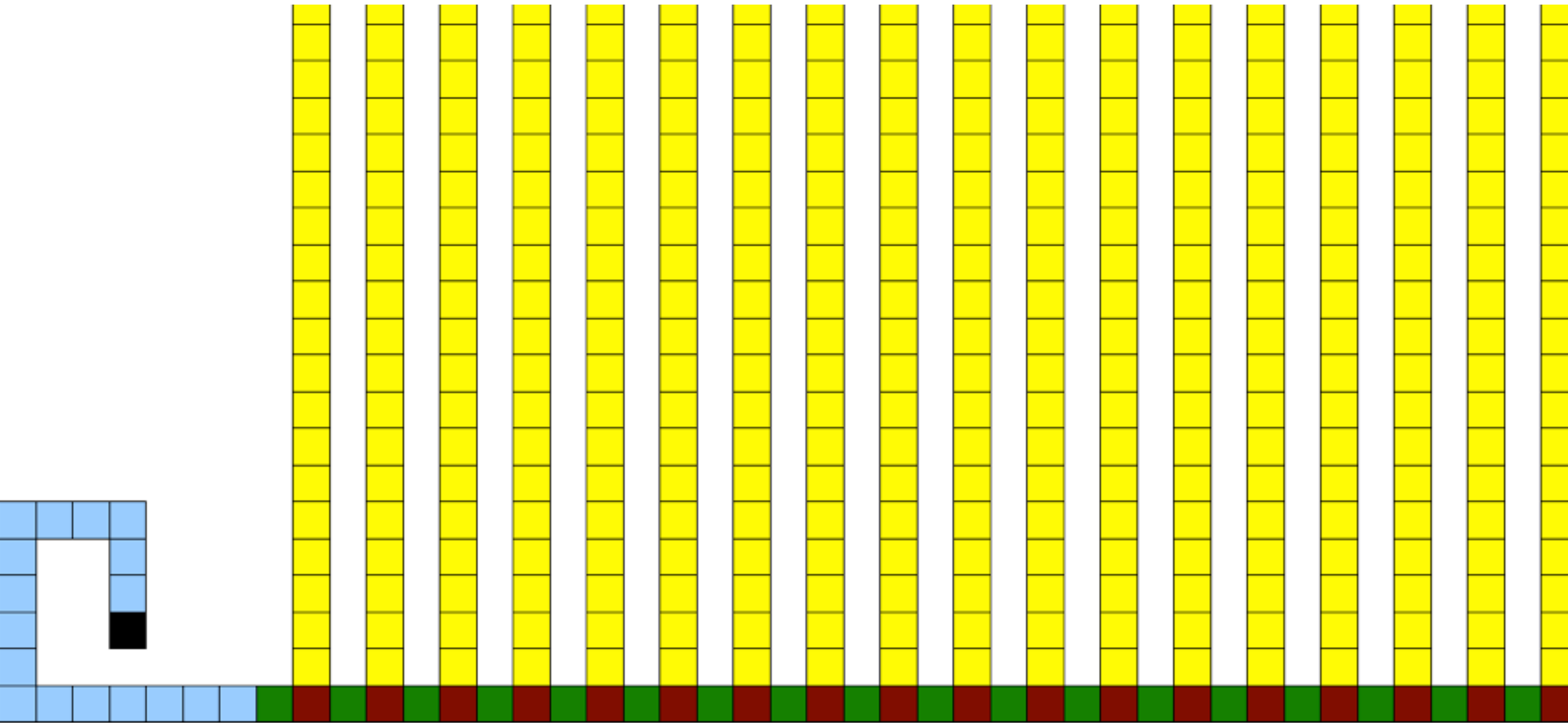

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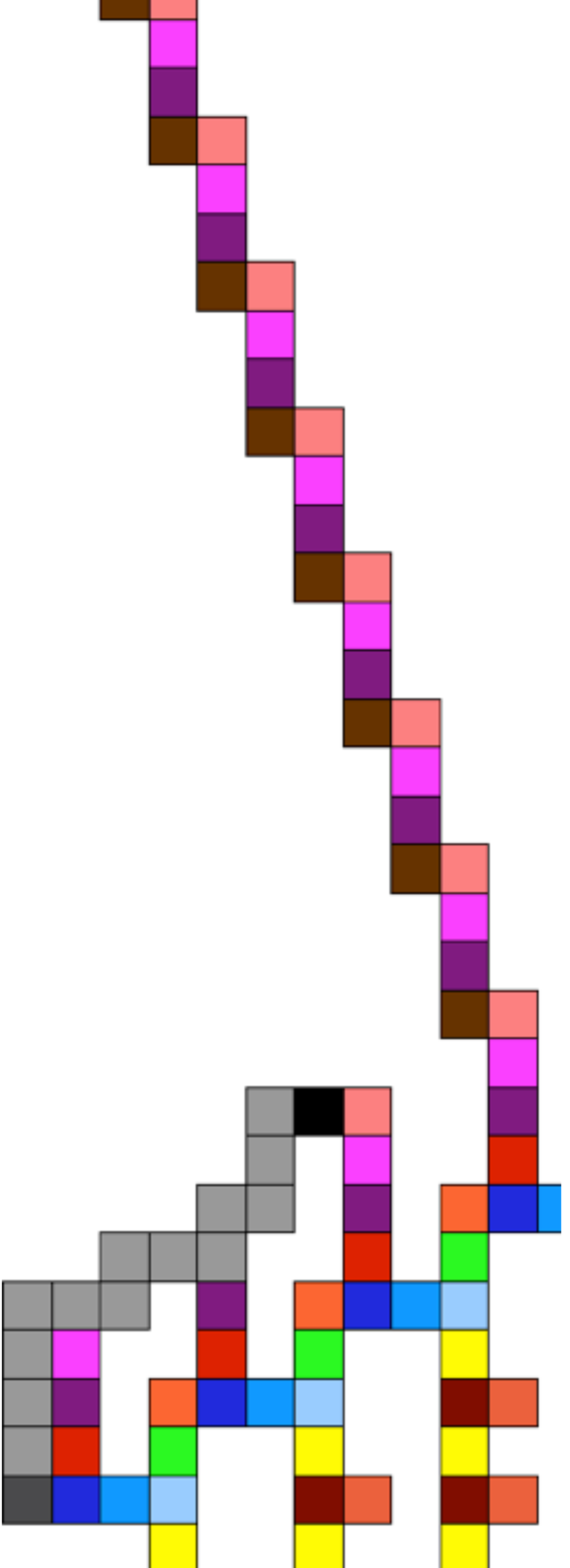

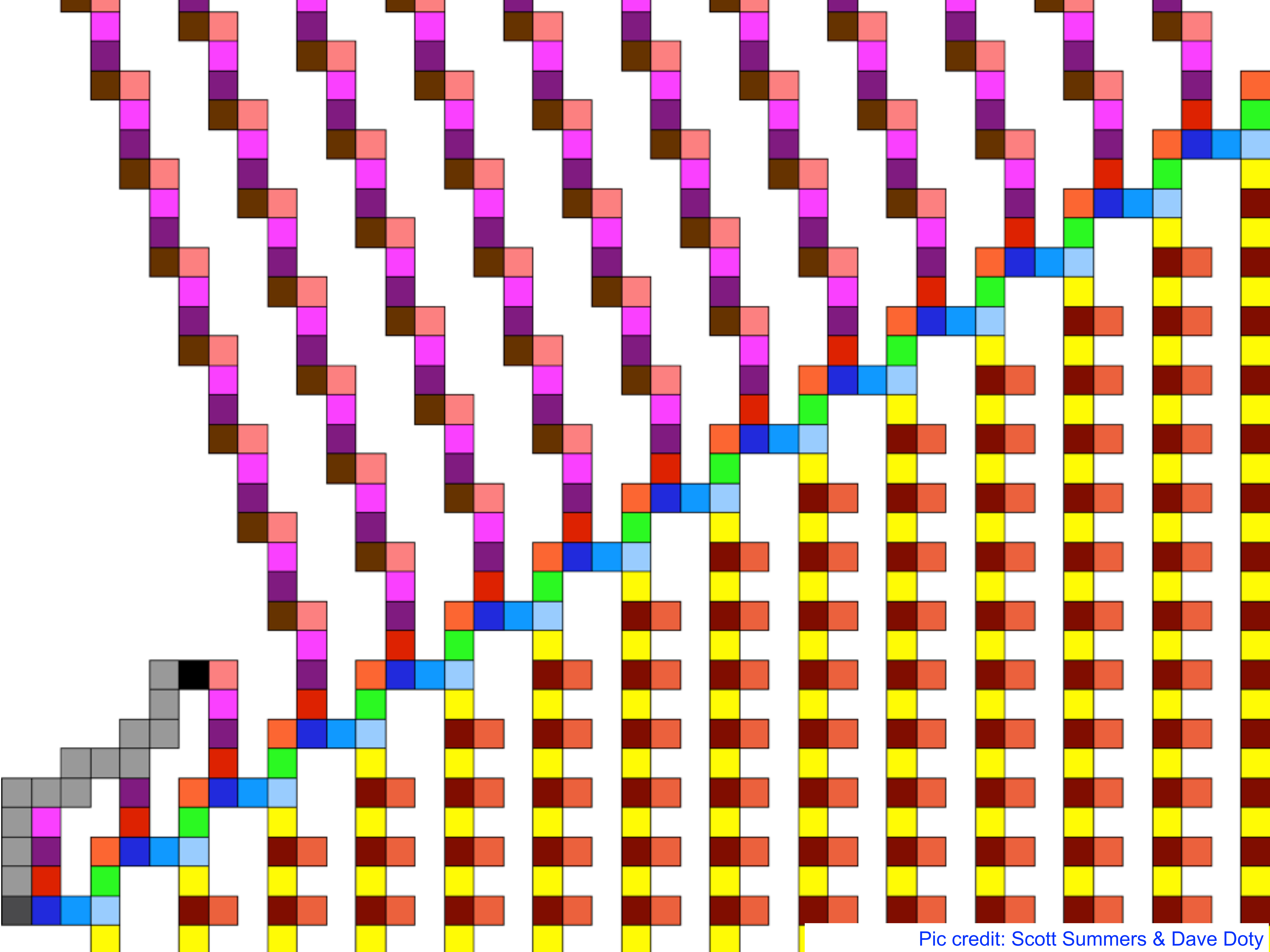

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Is temperature 1 computationally weak?

Binding on 1 side ***seems*** much weaker than binding on 2 sides … right?

- It has been **conjectured** (since 2000) that temperature 1 systems are computationally "weak" Rothemund, Winfree. STOC 2000
- Some partial negative results:
  - Temperature 1 systems that build **fully connected** *n* x *n* squares require at least n^2 tile types Rothemund, Winfree. STOC 2000
  - **Pumpable** temperature 1 systems produce periodic structures Doty, Patitz, Summers. TCS 2011
  - Temperature 1 with **no mismatches** require 2*n*-1 tile types to assemble an *n* x *n* square Manuch, Stacho, Stoll. J Comp. Bio. 2010
- Positive results:
  - 3D deterministic temperature 1 simulates Turing machines Cook, Fu Schweller. SODA 2011
  - 2D temperature 1 simulates Turing machine, but with some error Adleman et al FOCS 2002
  - 2D temperature 1 can grow large(r than tile set size) structures Meunier. In submission. 2015

On the blackboard: fully-connected square result

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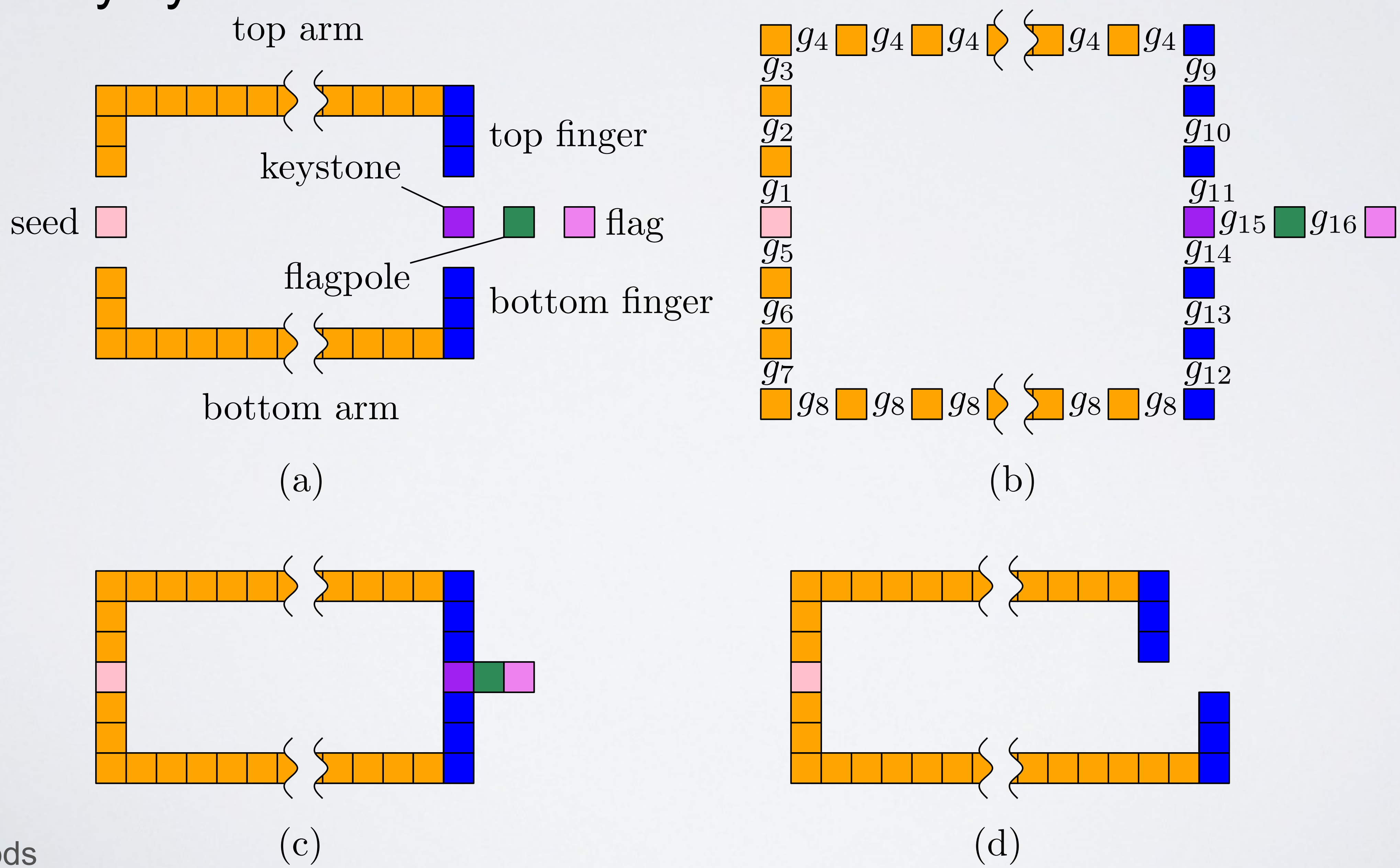
- But can temperature 1 aTAM systems **simulate cooperative tile assembly**?
- Answer: **No!** Meunier, Patitz, Summers, Theyssier, Winslow, **Woods**. SODA 2014

On the blackboard: fully-connected square result

Result

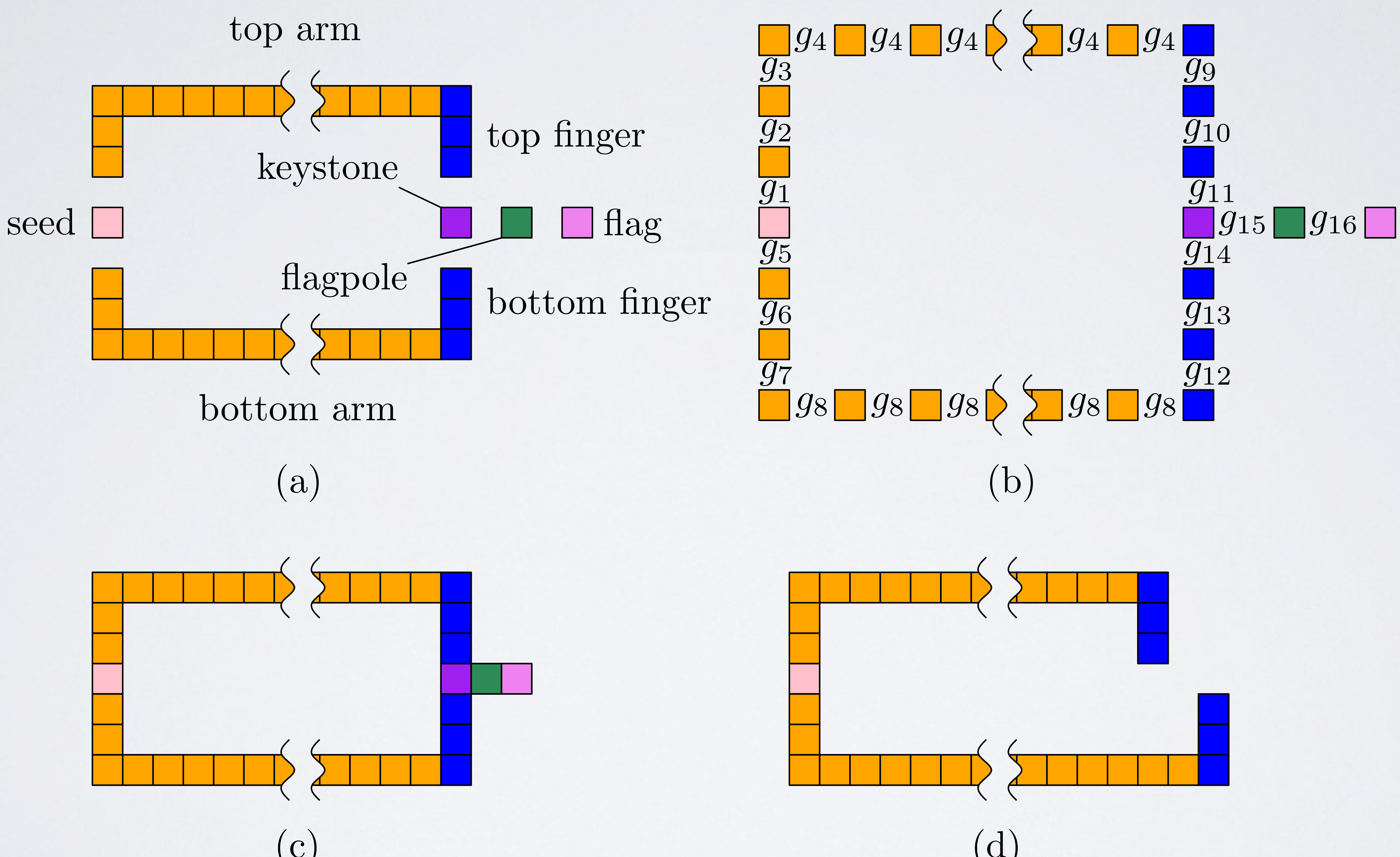
- Theorem 1. There is no tile set U, such that at temperature 1, U simulates all tile assembly systems.

- Theorem 2. There is a 2D temperature 2 tile assembly system T that can not be simulated by any 2D, nor any 3D, temperature 1 tile assembly system.



Temperature 1 can not simulate temperature 2

- We will show that no temperature 1 system simulates the following simple temperature 2 system



Equal arm lengths

Unequal arm lengths

Meunier, Patitz, Summers, Theyssier, Winslow, Woods. SODA 2014

Warm-up: two-seeded system

On the blackboard: a much easier warm-up result

Simulation definitions

Follows:

Definition 3.1 ($\mathcal{T}$ *follows* $\mathcal{U}$). We say that $\mathcal{T}$ *follows* $\mathcal{U}$ (under R) if for all $\alpha', \beta' \in \mathcal{A}[\mathcal{U}]$ where $\alpha' \rightarrow^{\mathcal{U}} \beta'$, it is the case that $R^*(\alpha') \rightarrow^{\mathcal{T}} R^*(\beta')$, and $\alpha' \in \mathcal{A}_{\Box}[\mathcal{U}] \implies R^*(\alpha') \in \mathcal{A}_{\Box}[\mathcal{T}]$.

Models:

Definition 3.2 (*nicely fuzzy*). We say that $\mathcal{U}$ is *nicely fuzzy* with respect to T if for all $\alpha'' \in \mathcal{A}[\mathcal{U}]$ there exists $\alpha' \in \mathcal{A}[\mathcal{U}]_{\text{fuzz-free}}$ such that $\alpha' \rightarrow^{\mathcal{U}} \alpha''$, where $R^*(\alpha'') = R^*(\alpha') = \alpha \in \mathcal{A}^T$.

Definition 3.4 ($\mathcal{U}$ *models* $\mathcal{T}$). We say that $\mathcal{U} = (U, \sigma_{\mathcal{U}}, \tau_{\mathcal{U}})$ models $\mathcal{T} = (T, \sigma_{\mathcal{T}}, \tau_{\mathcal{T}})$ (under R) if:

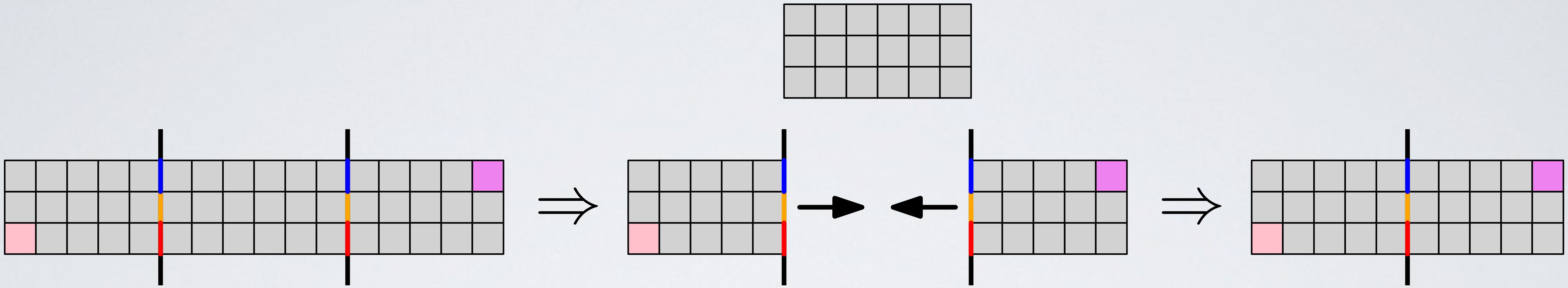
(1) $\mathcal{U}$ is nicely fuzzy with respect to T, and
(2) $R^*(\sigma_{\mathcal{U}}) = \sigma_{\mathcal{T}}$, and
(3) for all $\alpha, \beta \in \mathcal{A}[\mathcal{T}]$ such that $\alpha \rightarrow^{\mathcal{T}} \beta$ it is the case that for all $\alpha' \in \mathcal{A}[\mathcal{U}]_{\text{fuzz-free}}$ where $R^*(\alpha') = \alpha$, there exists $\beta' \in \mathcal{A}[\mathcal{U}]_{\text{fuzz-free}}$ such that $\alpha' \rightarrow^{\mathcal{U}} \beta'$ and $R^*(\beta') = \beta$.

Simulates:

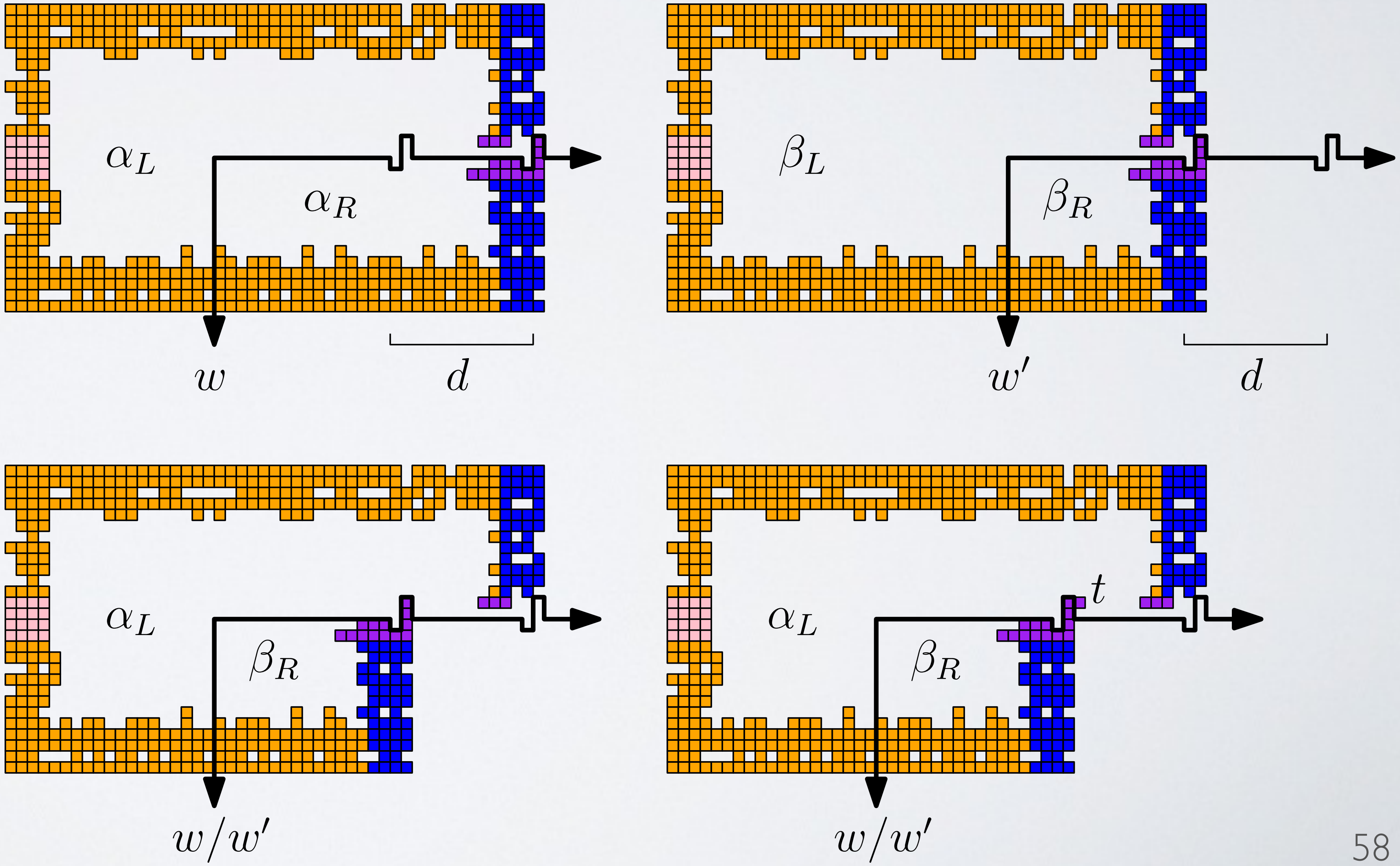
Definition 3.5. We say that $\mathcal{U}$ *simulates* $\mathcal{T}$ (under R) if $\mathcal{T}$ follows $\mathcal{U}$ (under R) and $\mathcal{U}$ models $\mathcal{T}$ (under R).

Temperature 1 is not IU for the aTAM

- First we prove a simple and general pumping lemma for tile assembly at any temperature (the window movie lemma)



- We then use this pumping lemma to “fool” any claimed temperature 1 simulator into exposing its inability to simulate cooperation



Temperature 1 is not IU for the aTAM

- There are simple temperature 2 systems that can not be simulated by any temperature 1 system Meunier, Patitz, Summers, Theyssier, Winslow, Woods. SODA 2014

- First fully-general negative result on temperature 1 (i.e. no restrictions on the model)

- This negative result holds in 2D and 3D Cook, Fu, Schweller. SODA 2011
  - Recall: Deterministic 3D temperature 1 systems can simulate Turing machines!
  - So these Turing-universal (powerful!) tile assembly systems can not simulate tile assembly

- Turing universal algorithmic behaviour in self-assembly provably does not imply the ability to simulate arbitrary algorithmic self-assembly processes. Temp 1 3D can compute, but can't handle geometry

- The proof had almost zero "temperature 1 craziness"
  - Ongoing work: with Pierre-Étienne Meunier, & by Damien Regnault and Pierre-Étienne Meunier towards showing other negative results on temperature 1

Acknowledgements

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Patitz

Scott
Summers

Andrew Winslow

Guillaume
Theyssier

Meunier, Patitz, Summers, Theyssier, Winslow, Woods. SODA 2014

Fin