

The tile assembly model is intrinsically universal

David Doty, Jack H. Lutz, Matthew J. Patitz, Robert T.
Schweller, Scott M. Summers, **Damien Woods**



Caltech



Iowa State

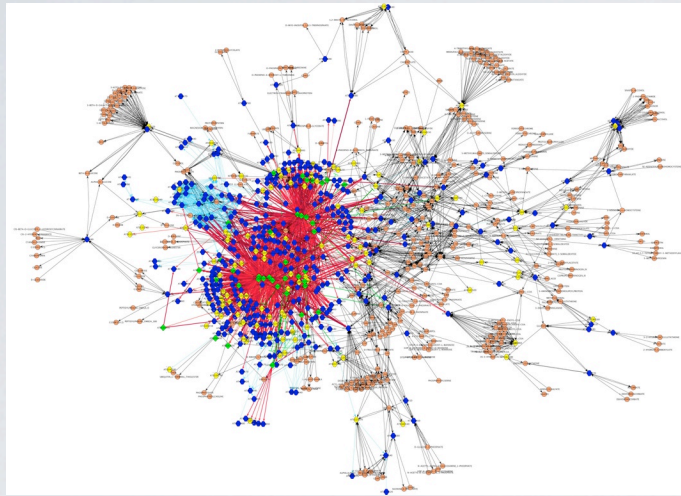


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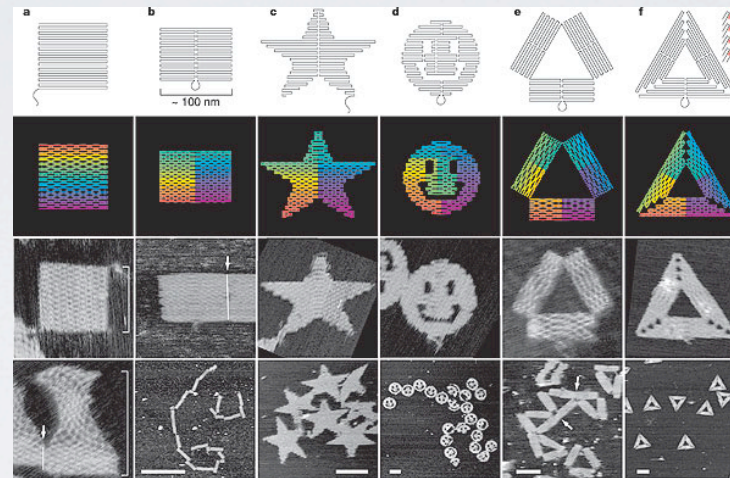


Motivation

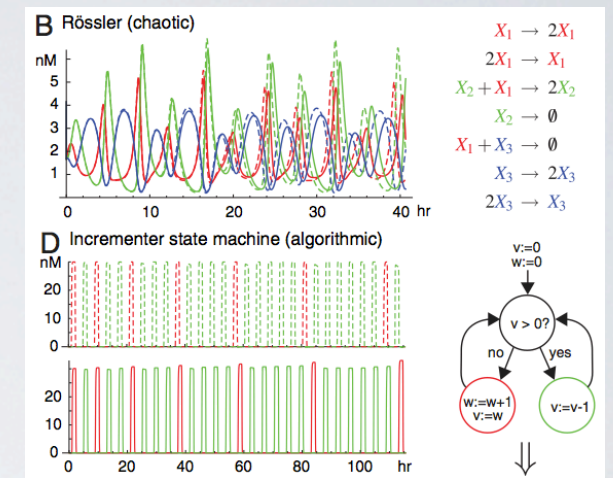
- Nature computes
- Engineers are building nanoscale molecular (chemical) computers



Sumazin et al. Cell 147(2). 2011

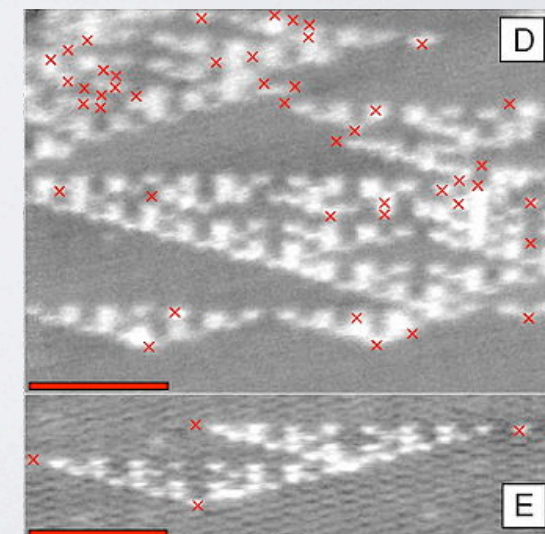


Rothemund, PWK. Folding DNA to create nanoscale shapes and patterns. Nature 2006



Soloveichik, Seelig, Winfree. DNA as a Universal Substrate for Chemical Kinetics. PNAS 2010

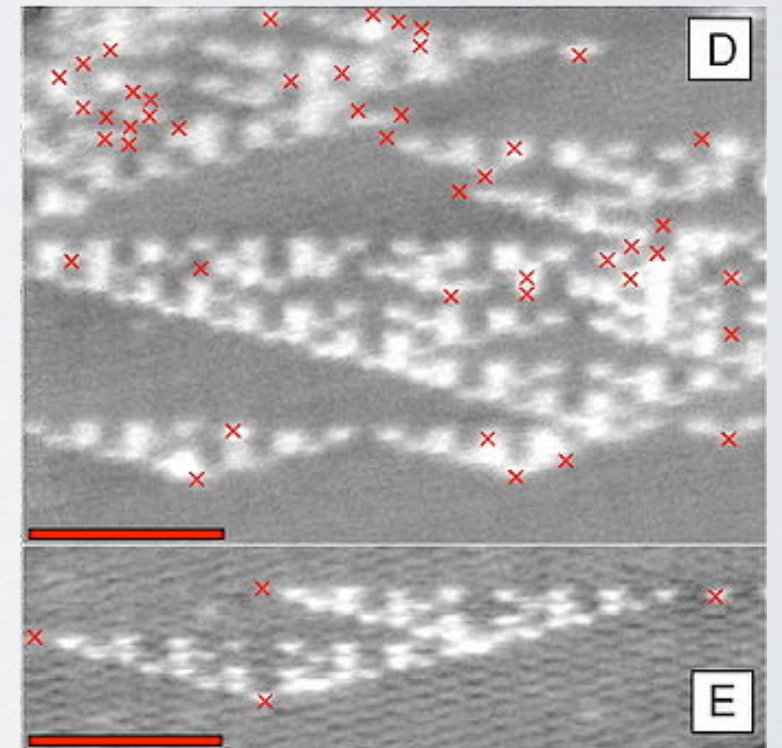
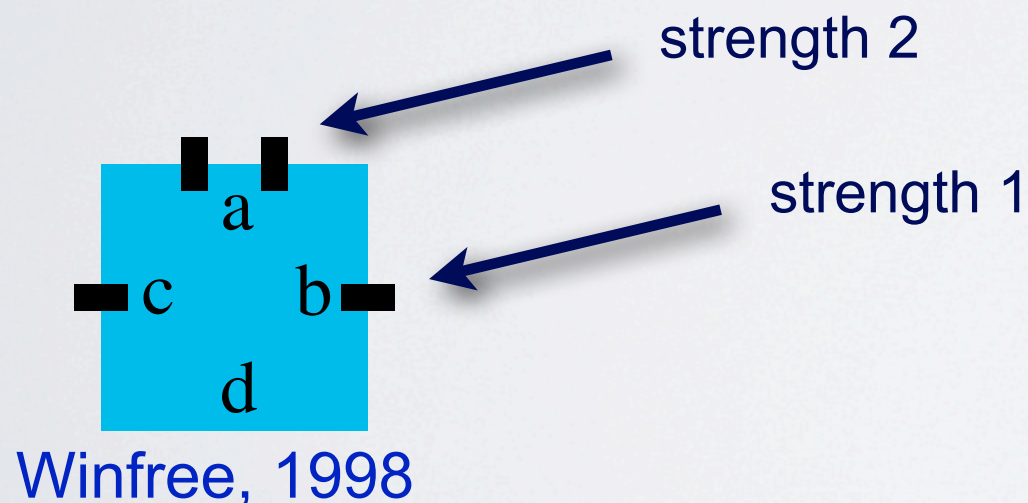
- We need a computational theory of self assembly and molecular interactions
- Perhaps more importantly (for us), we can find interesting theoretical problems: computation, geometry, asynchronousity, kinetics, thermodynamics



Rothemund, Papadakis, Winfree 2004

Abstract tile assembly model

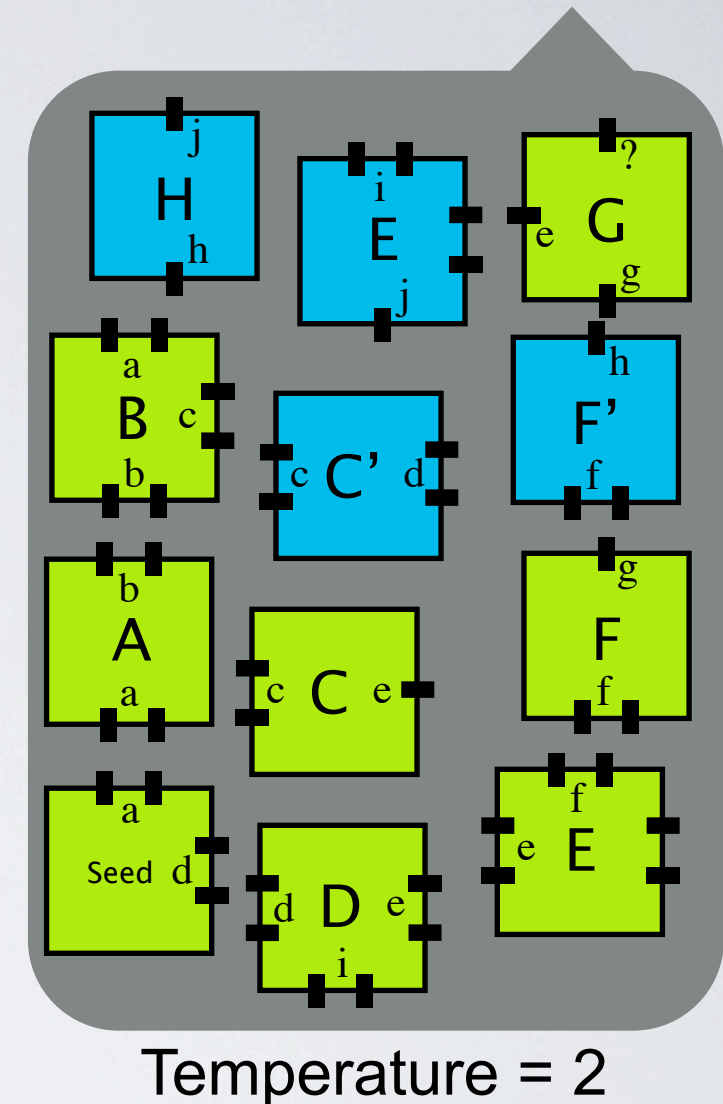
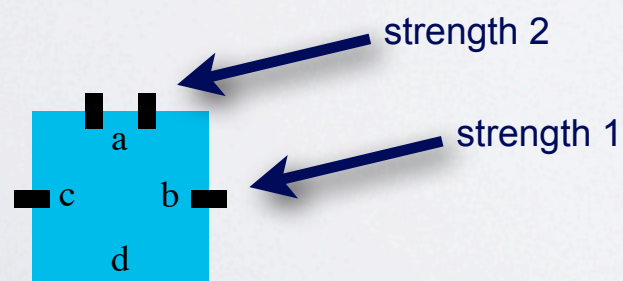
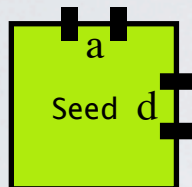
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- Each side has a **glue** (a,b,c,d,...)
- Each glue has a **strength** (0,1,2,3,...)
- **Finite** set of tile types, unlimited supply of each tile type
- Growth begins from a **seed** tile
- Tiles **stick** to the growing assembly if their glues match, and are strong enough (**sum of strengths** is at least the **temperature**)
- Asynchronous



Rothemund, Papadakis, Winfree 2004

Defn. Tile assembly system: tile set, seed tile, temperature

Abstract tile assembly model

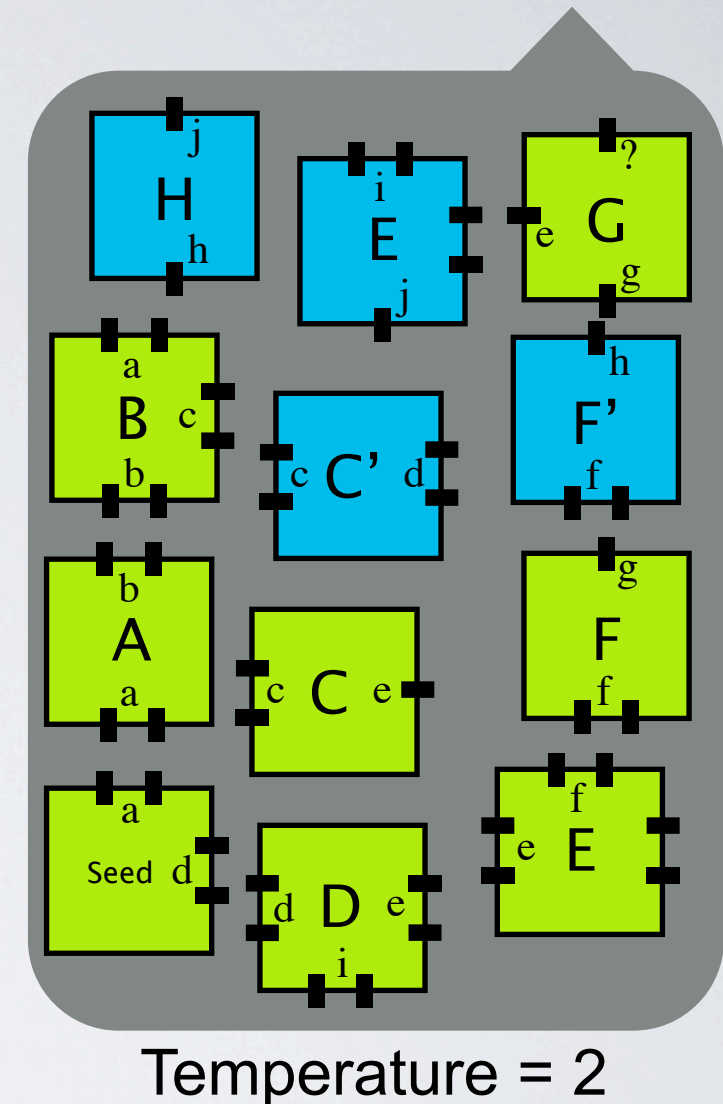
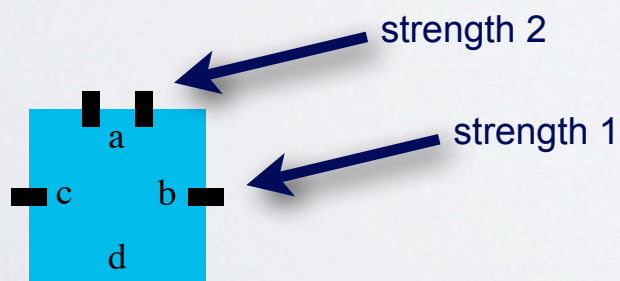
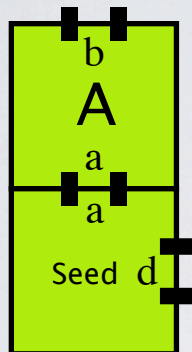


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Winfree, 1998

Abstract tile assembly model



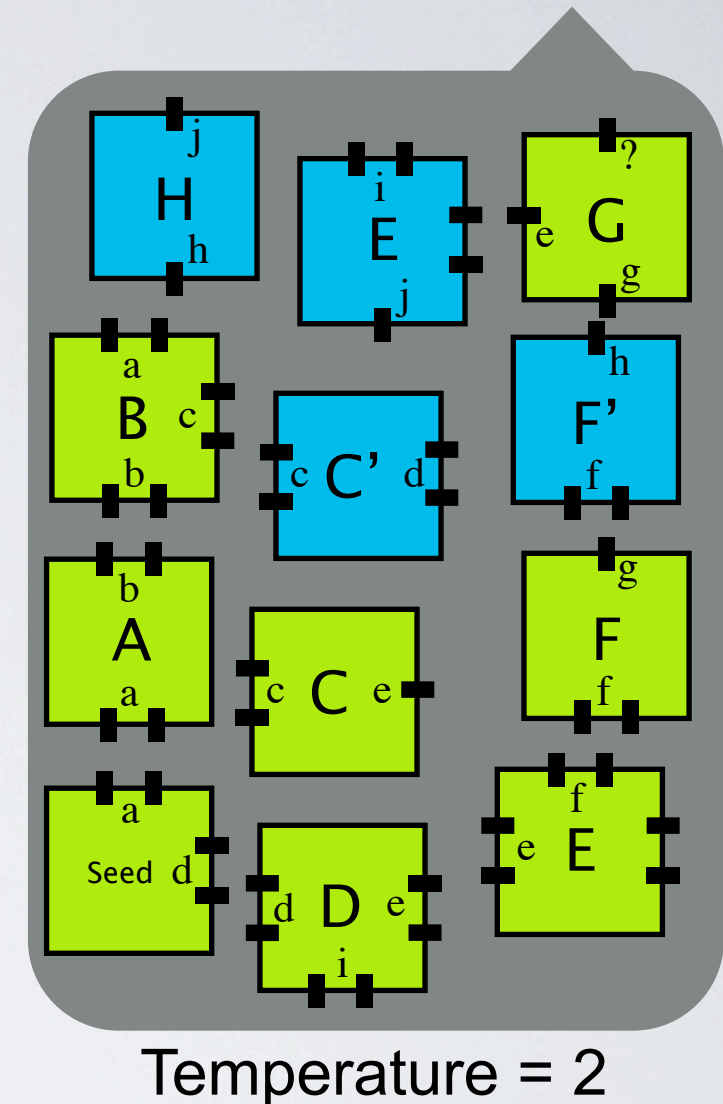
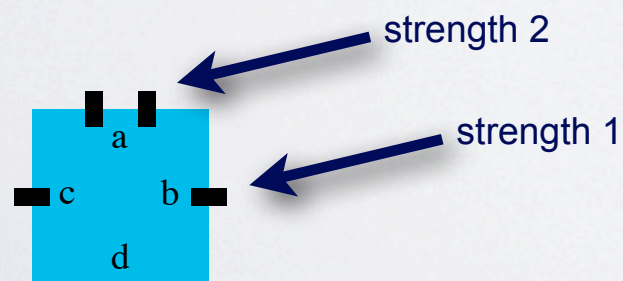
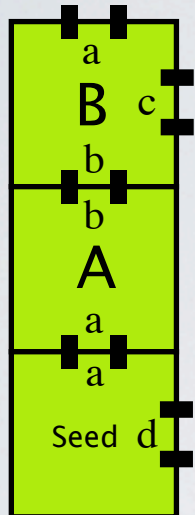
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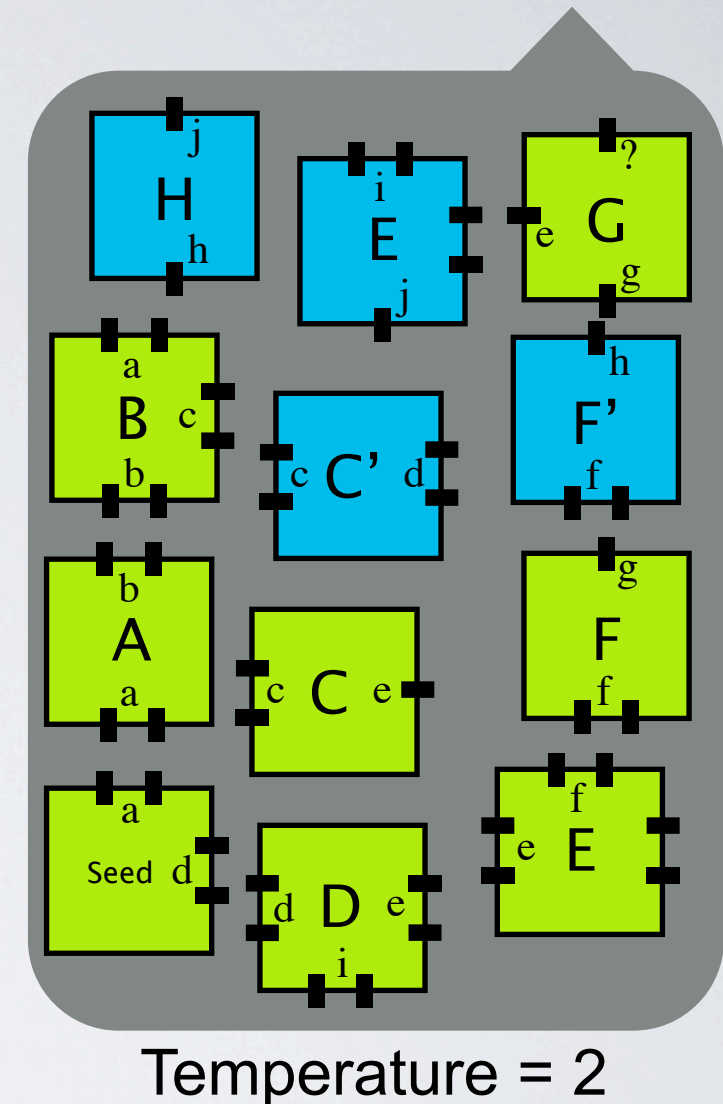
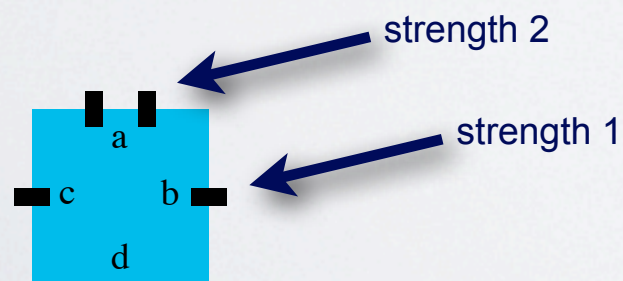
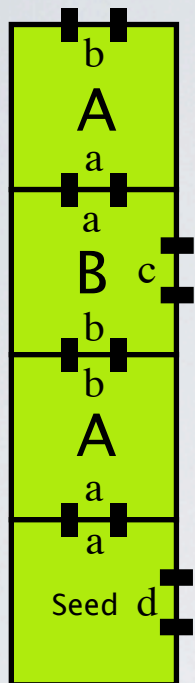
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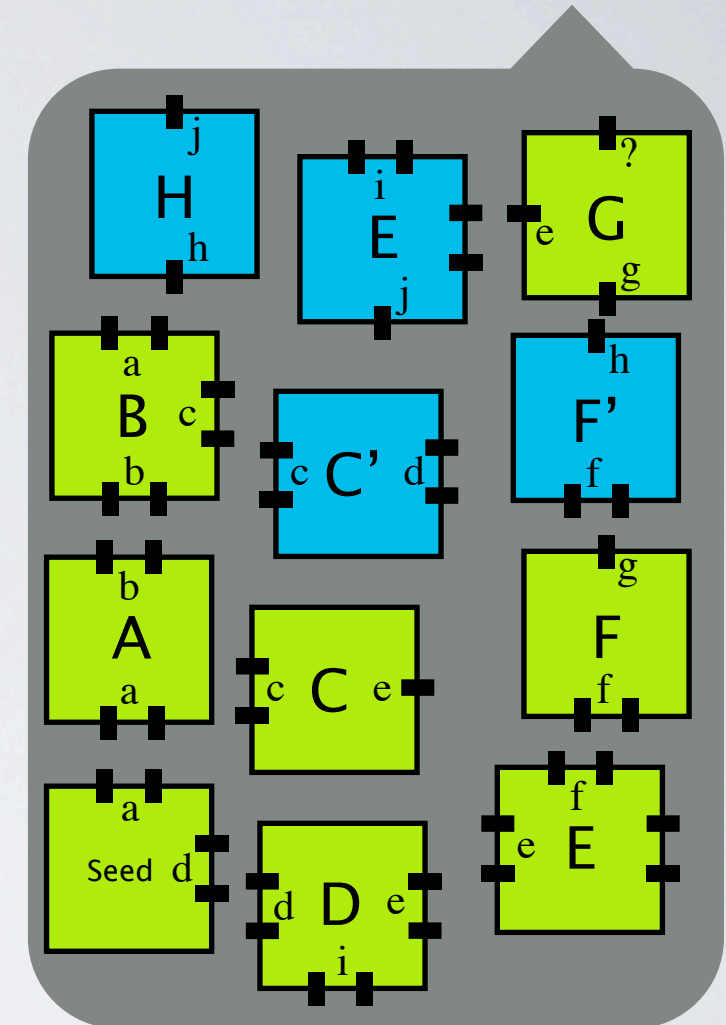
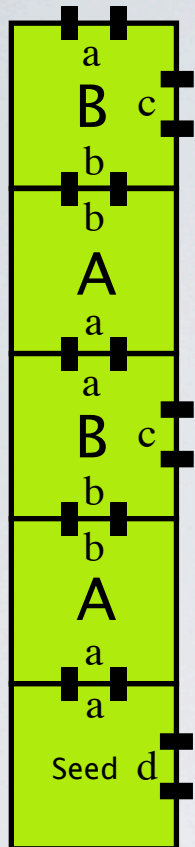
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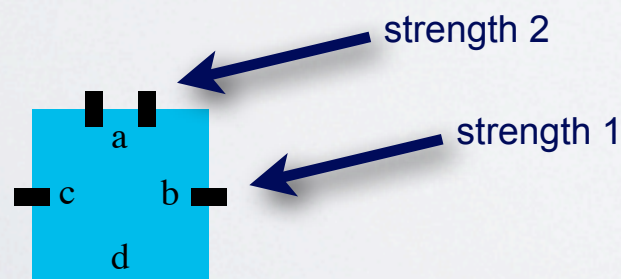
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Abstract tile assembly model



Temperature = 2

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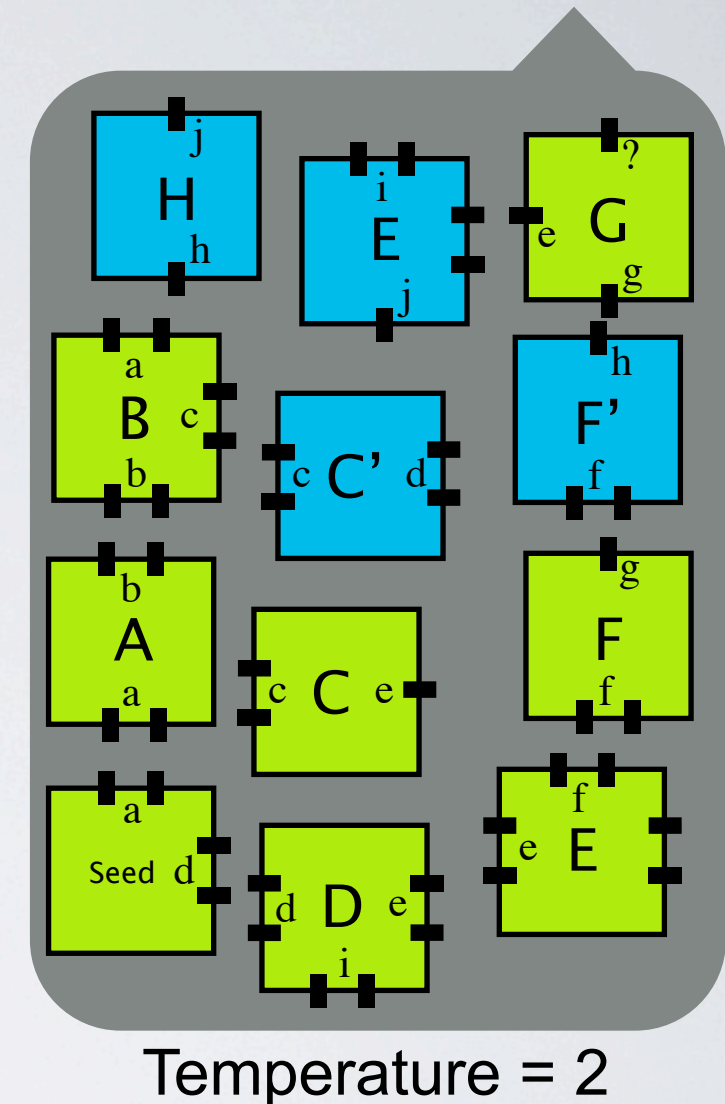
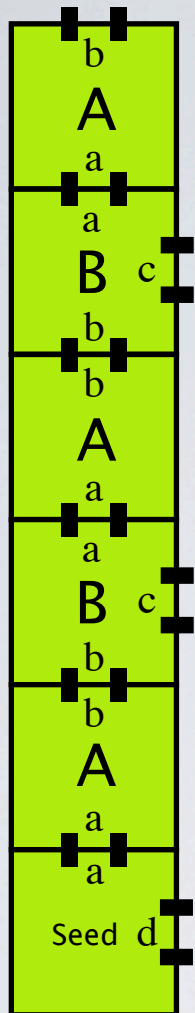


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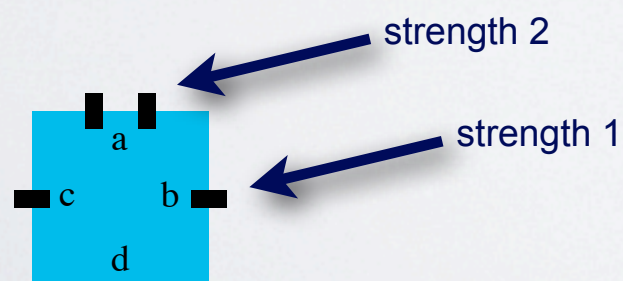
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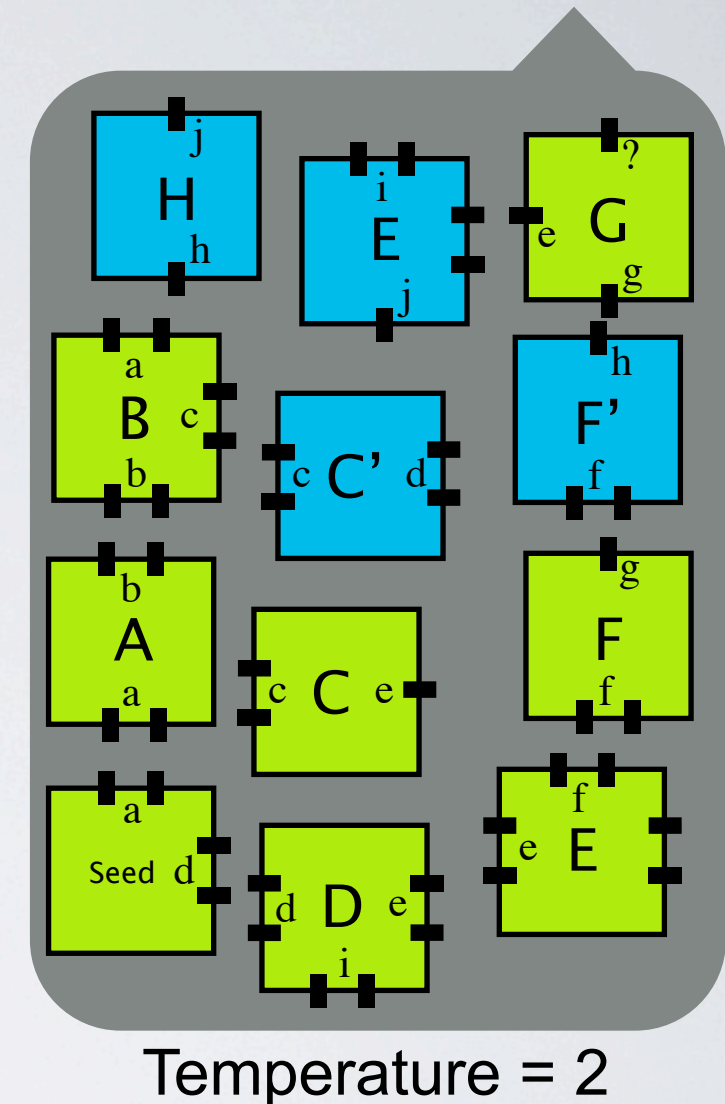
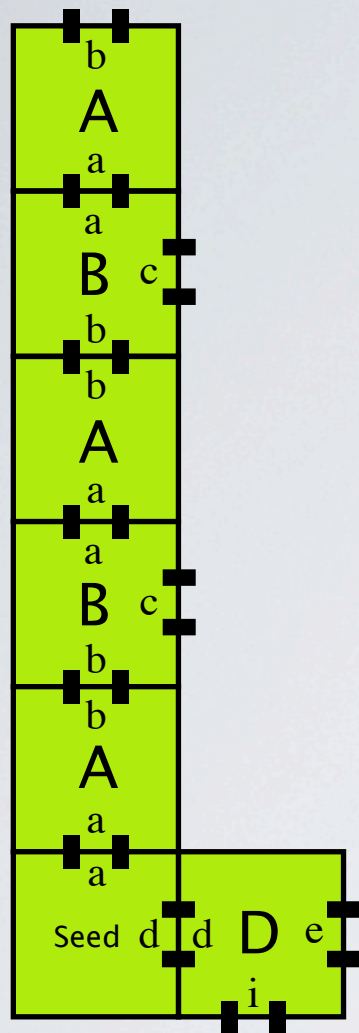


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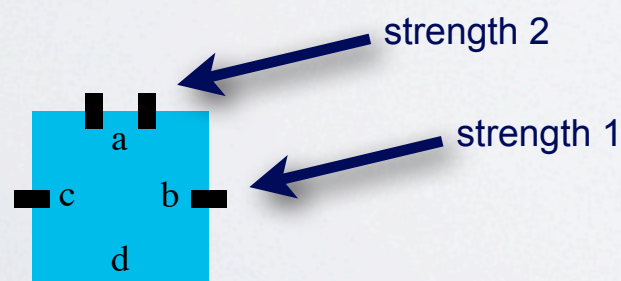
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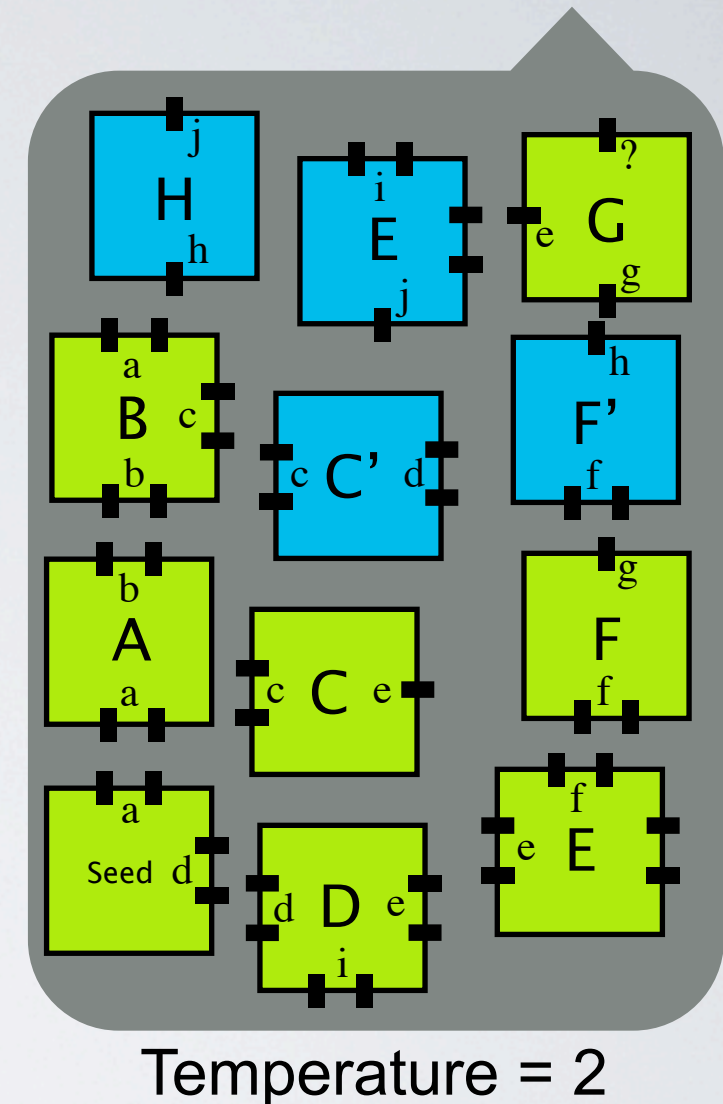
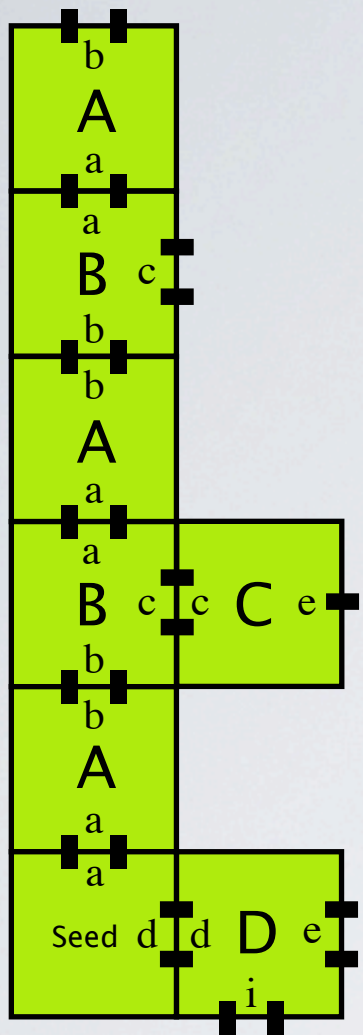


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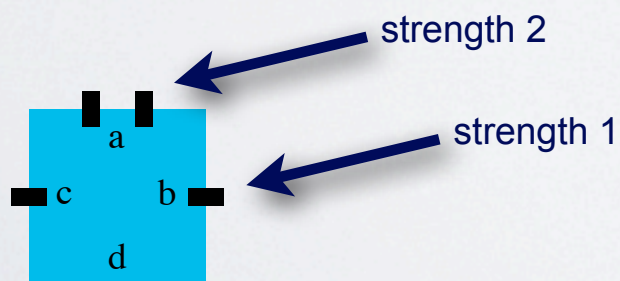
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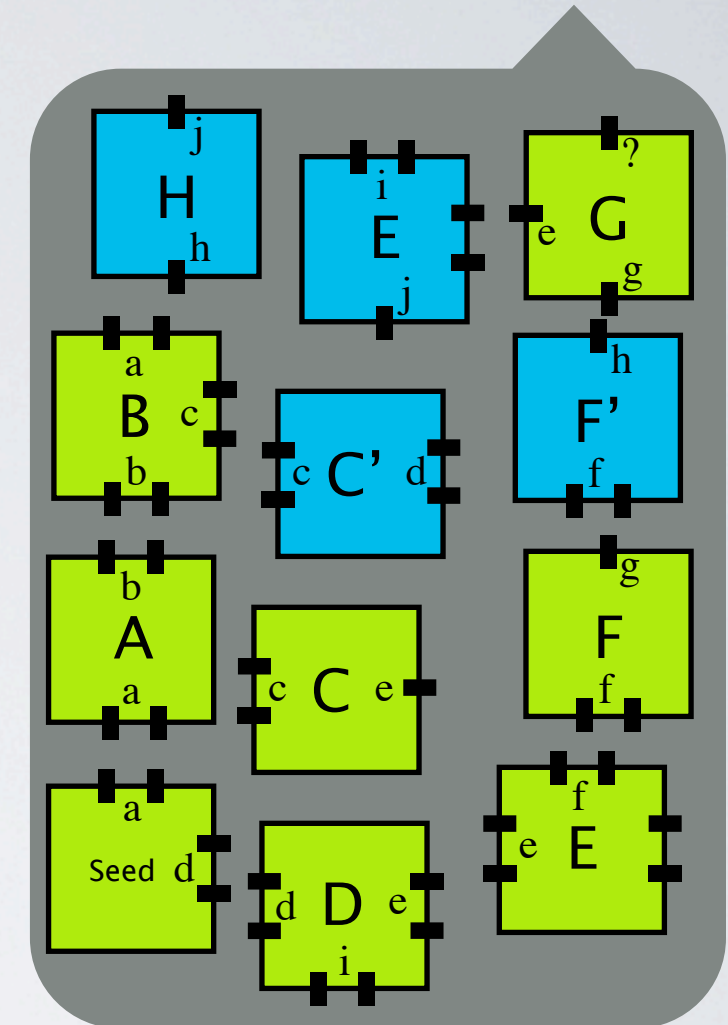
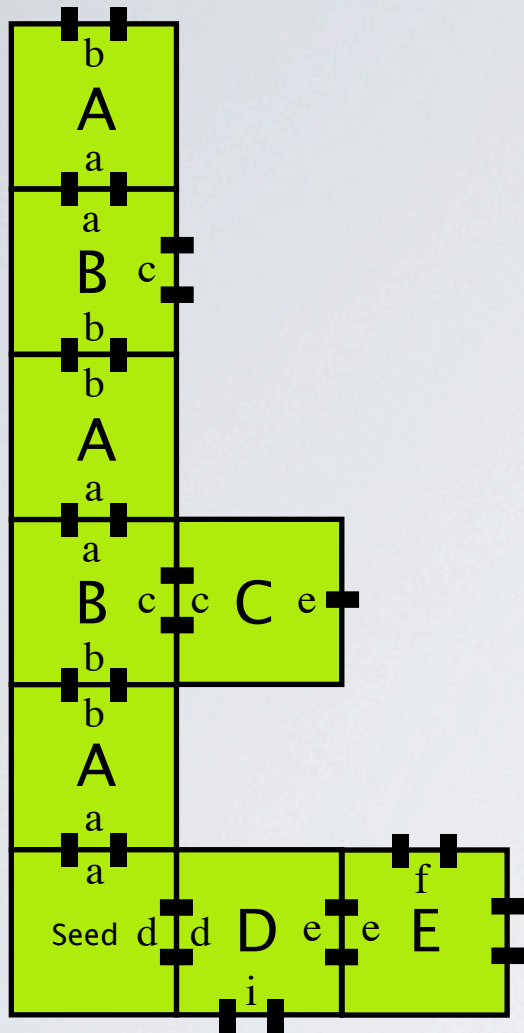


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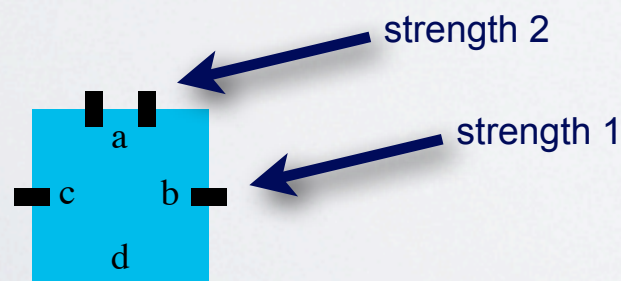
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Temperature = 2

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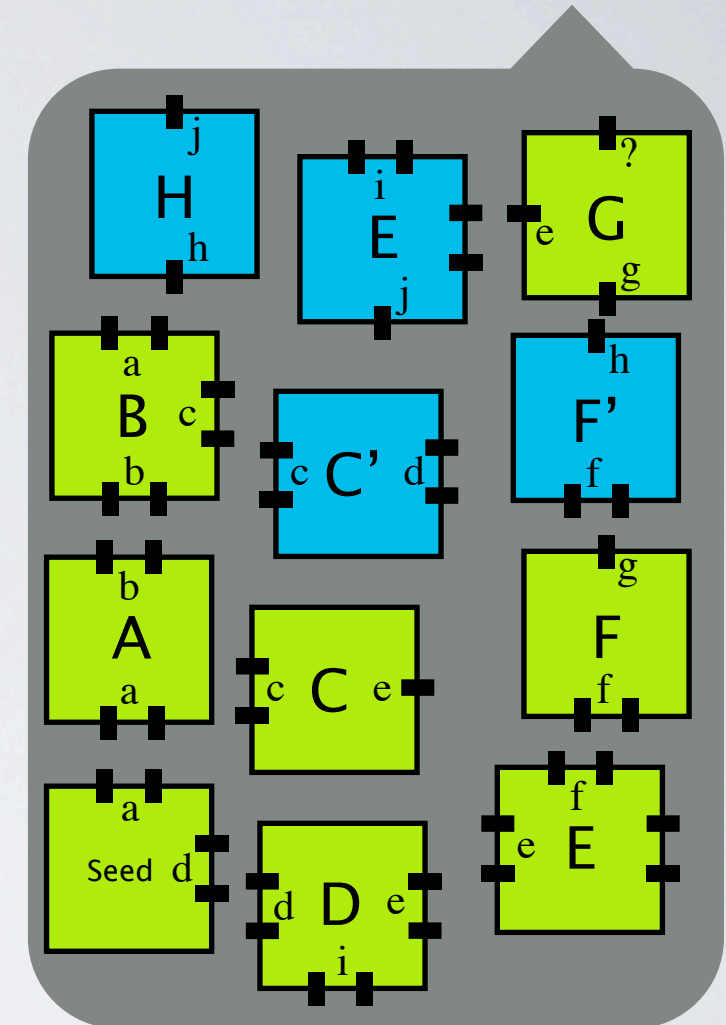
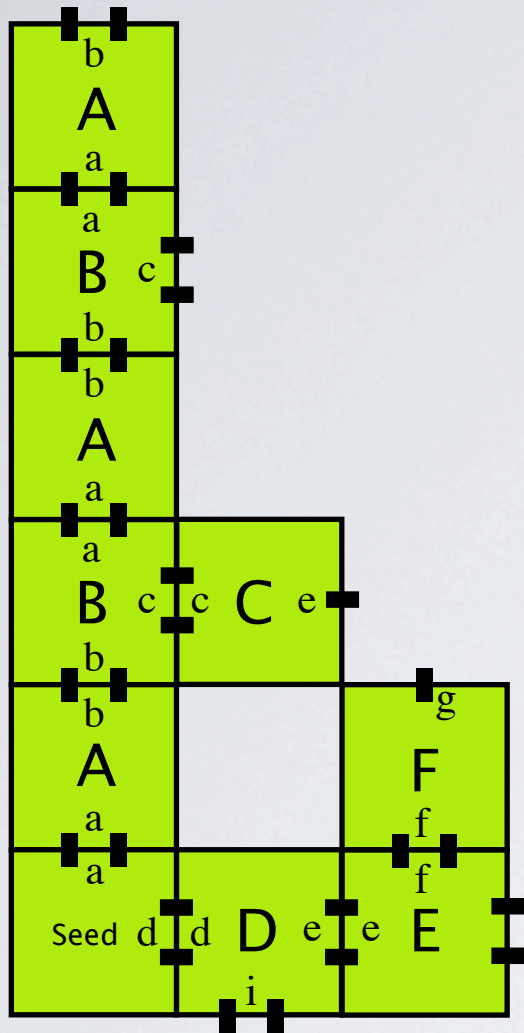


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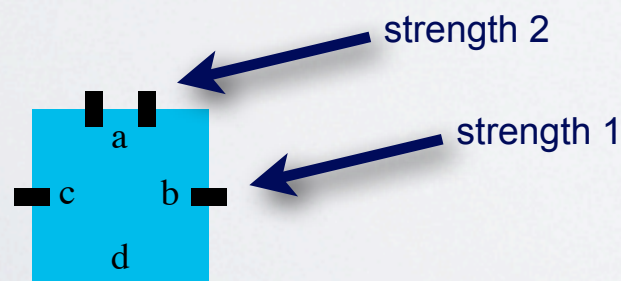
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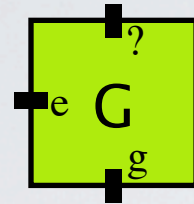
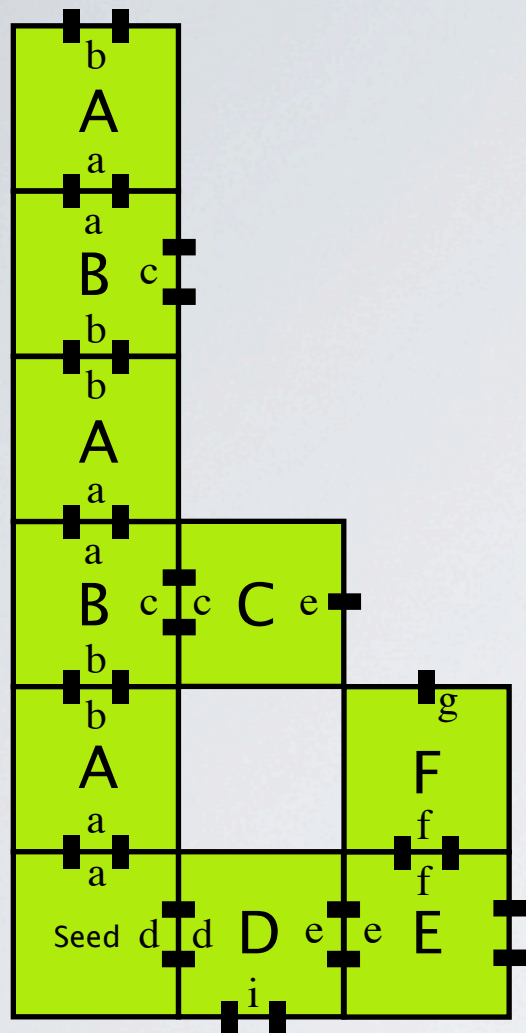


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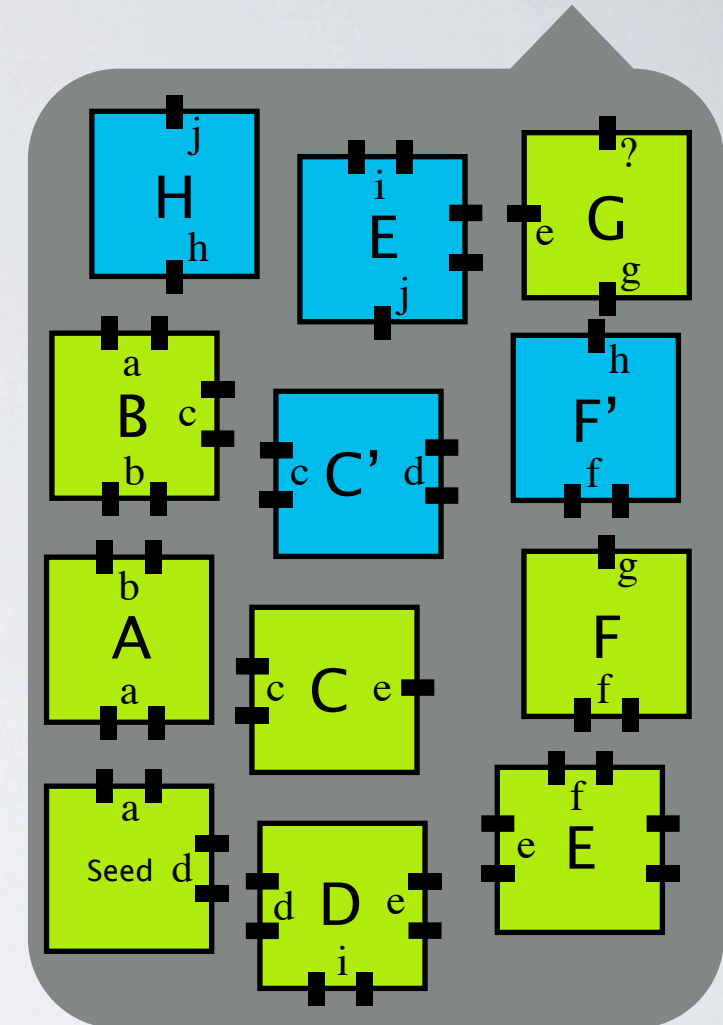
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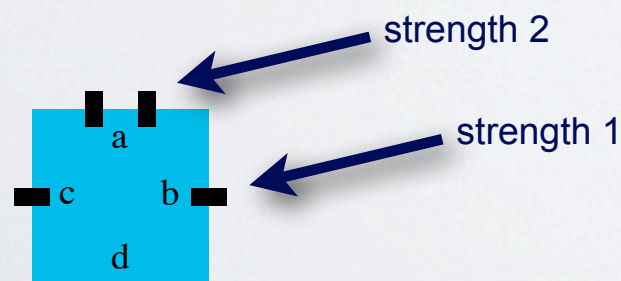
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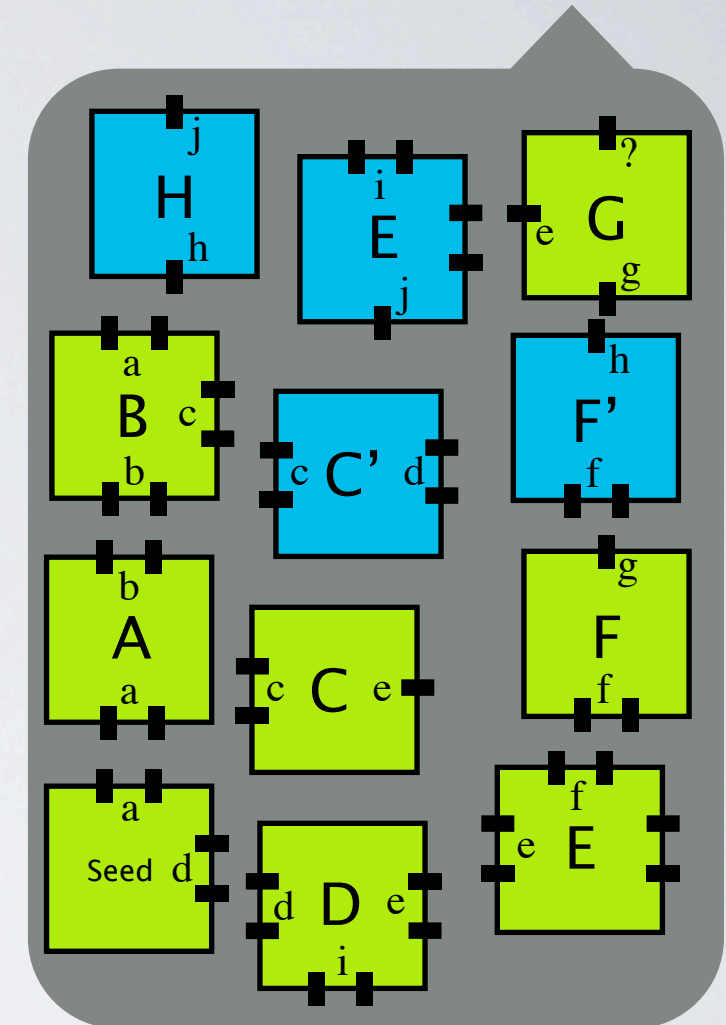
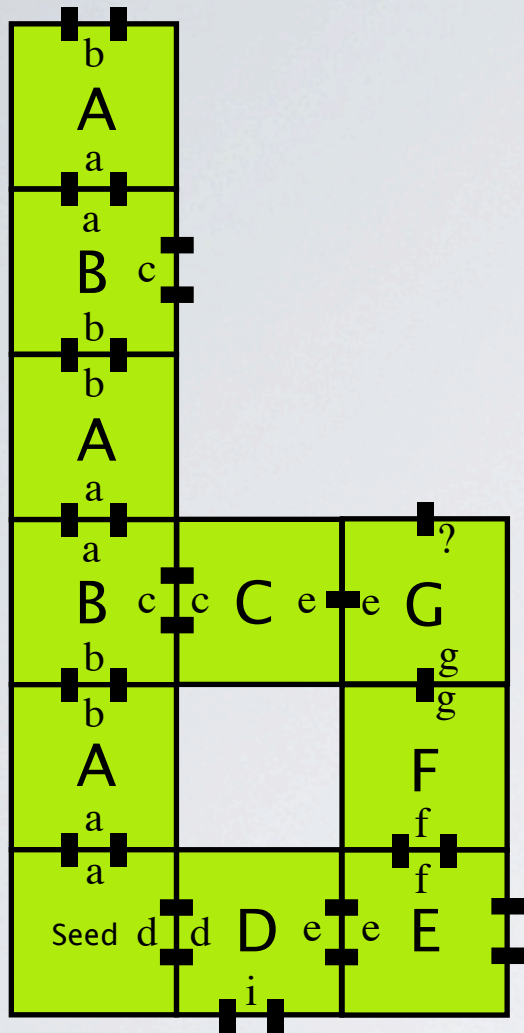


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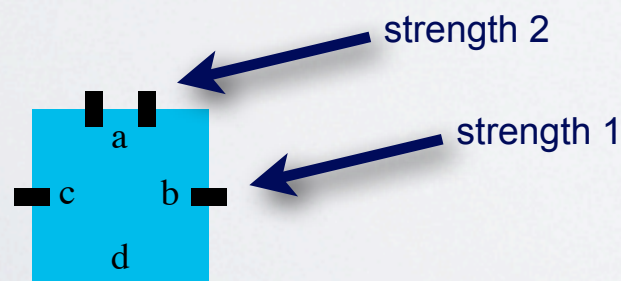
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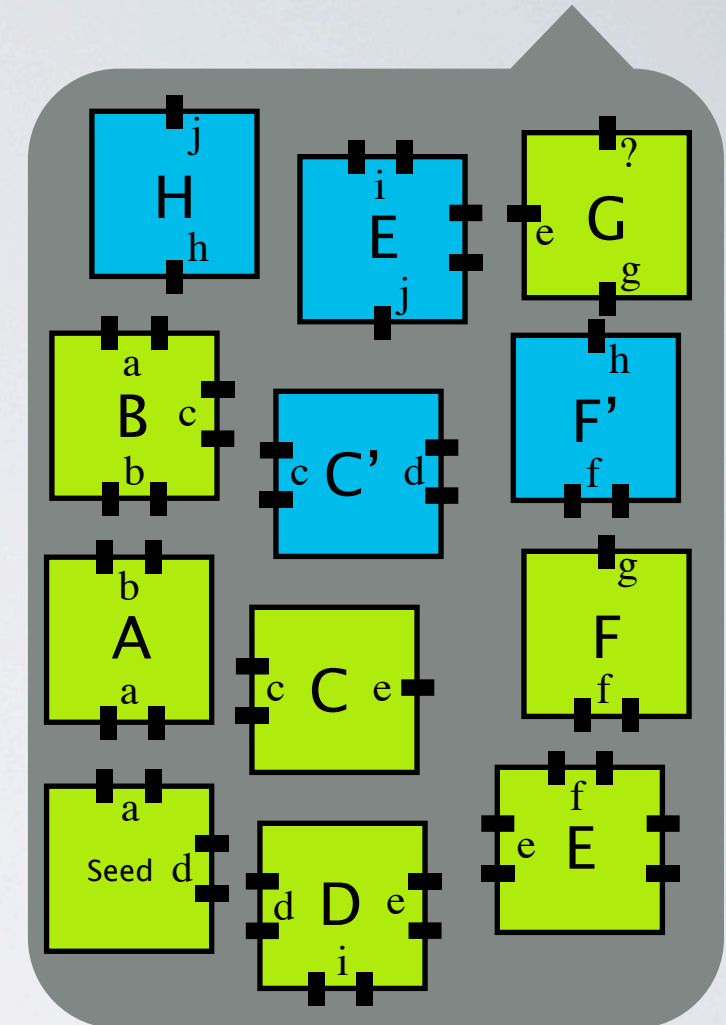
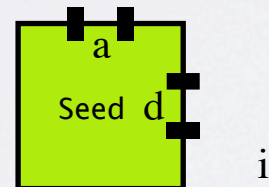
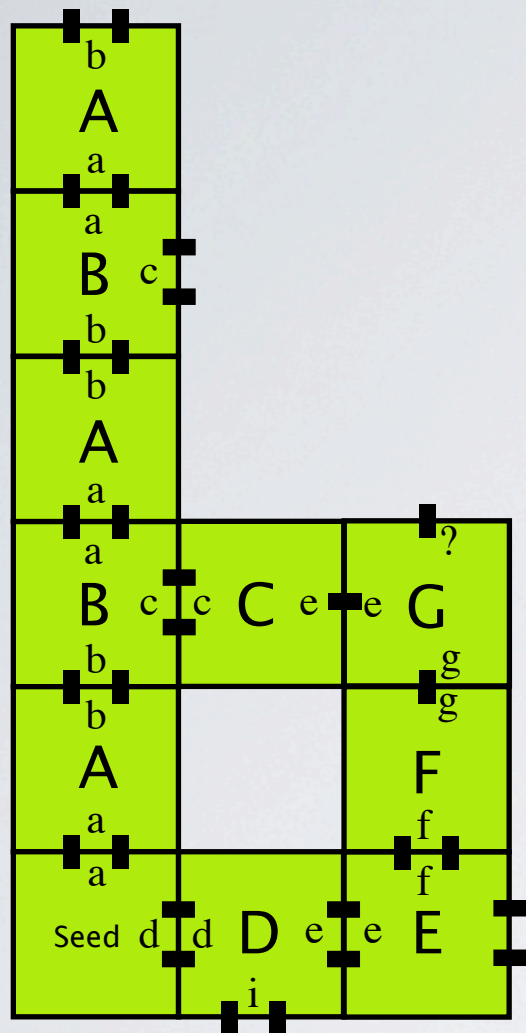


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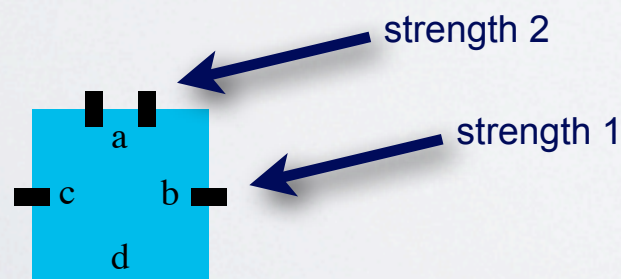
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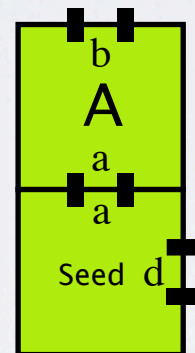
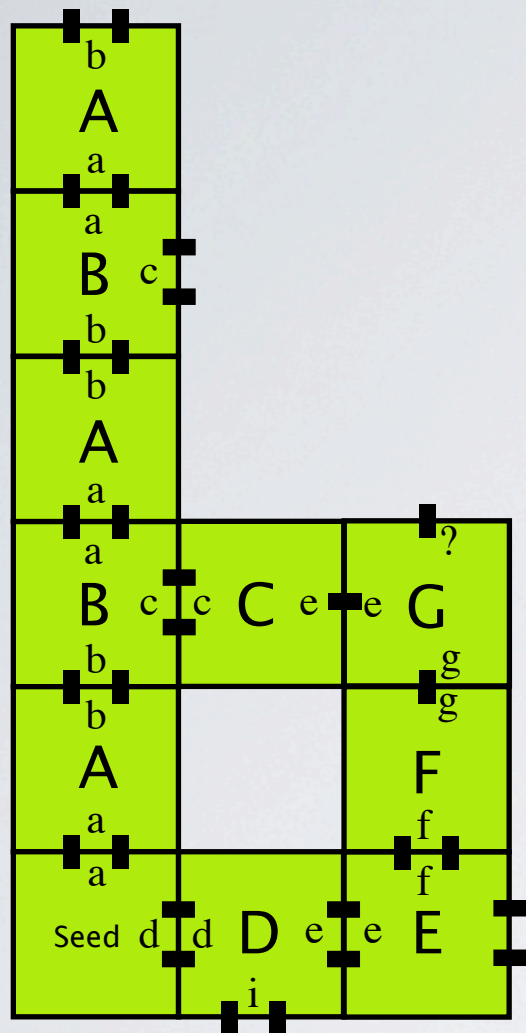


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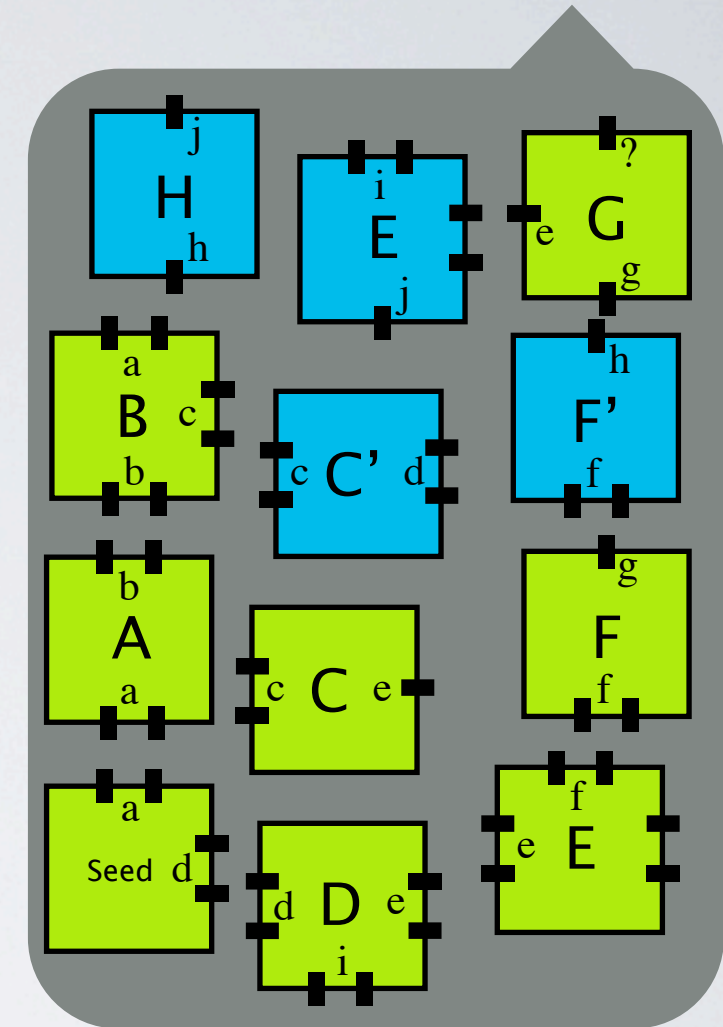
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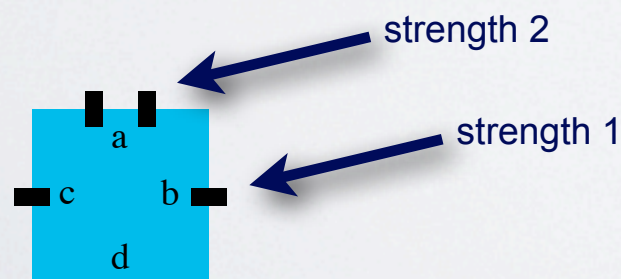
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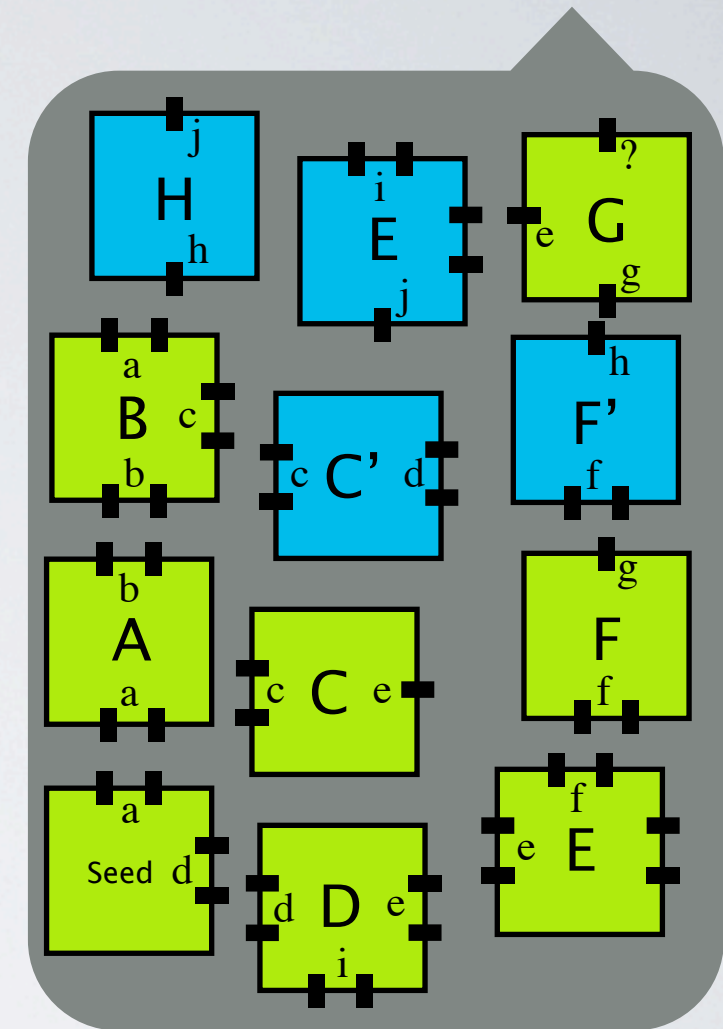
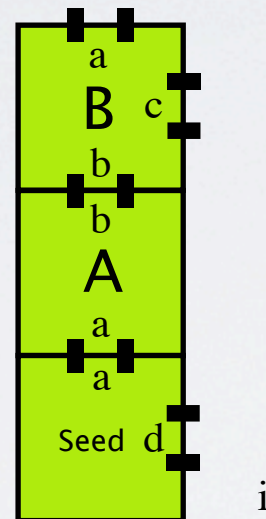
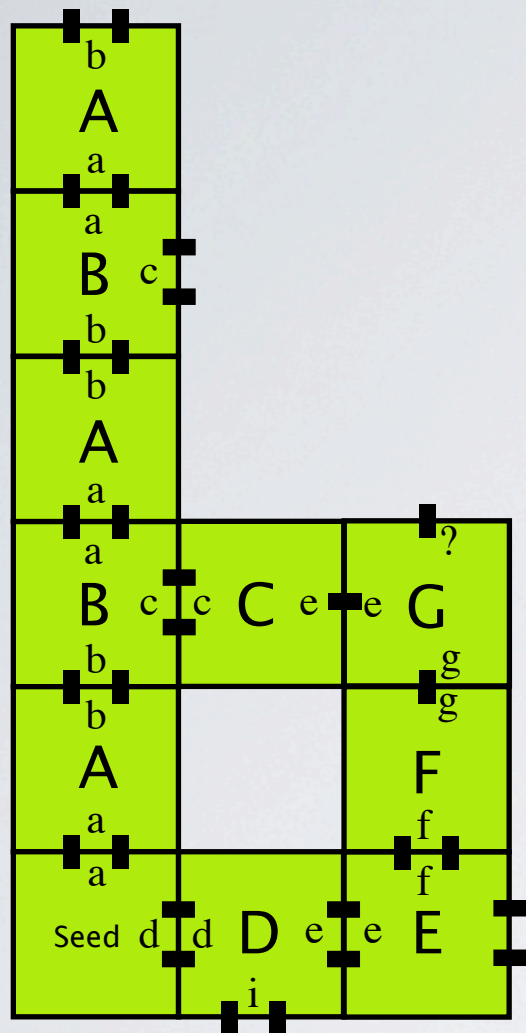


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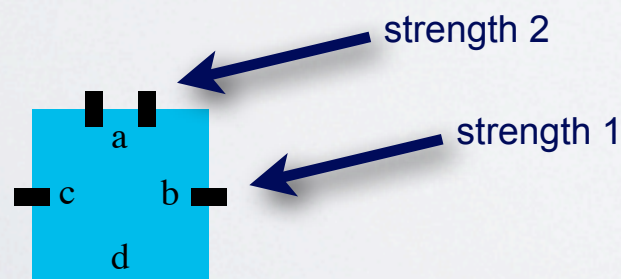
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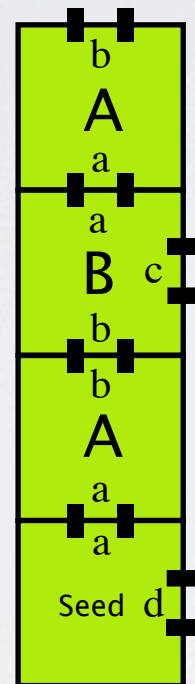
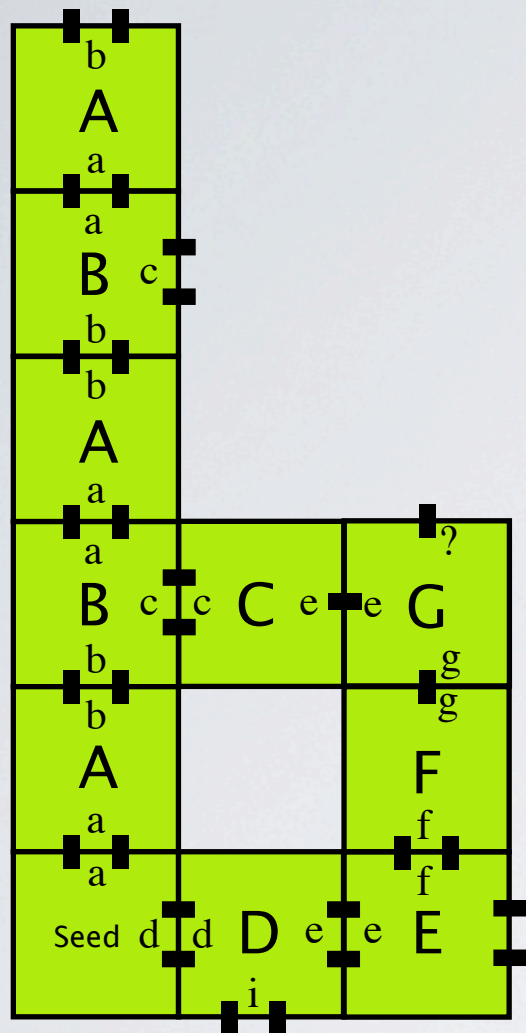


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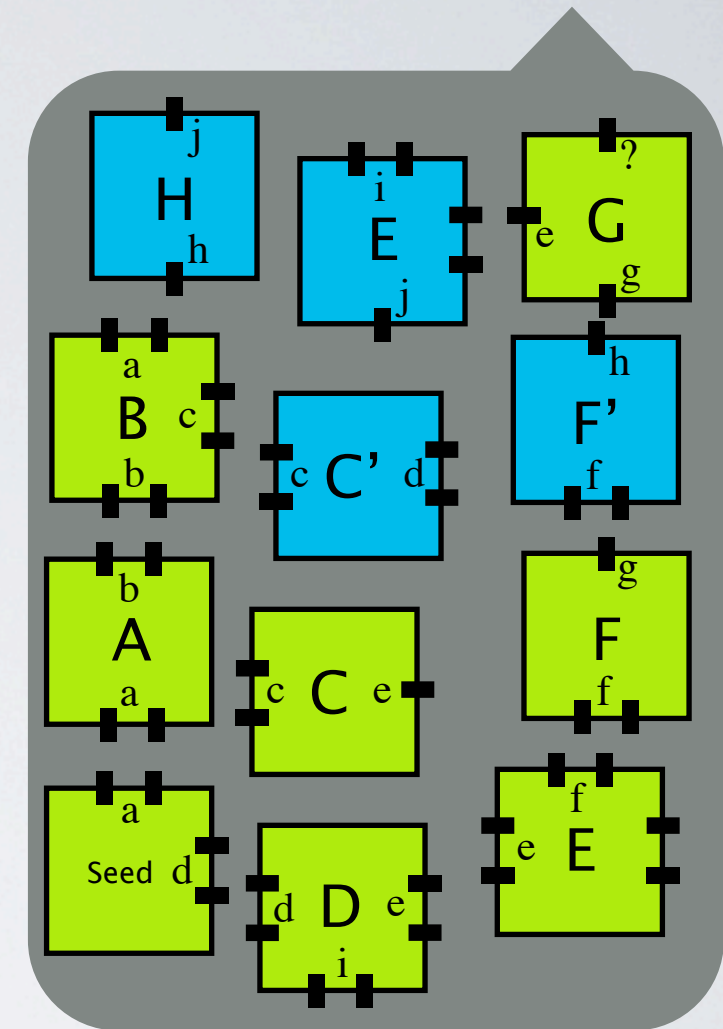
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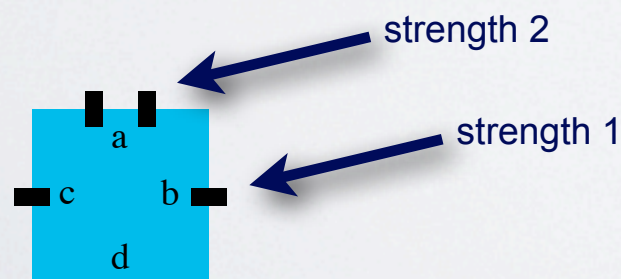
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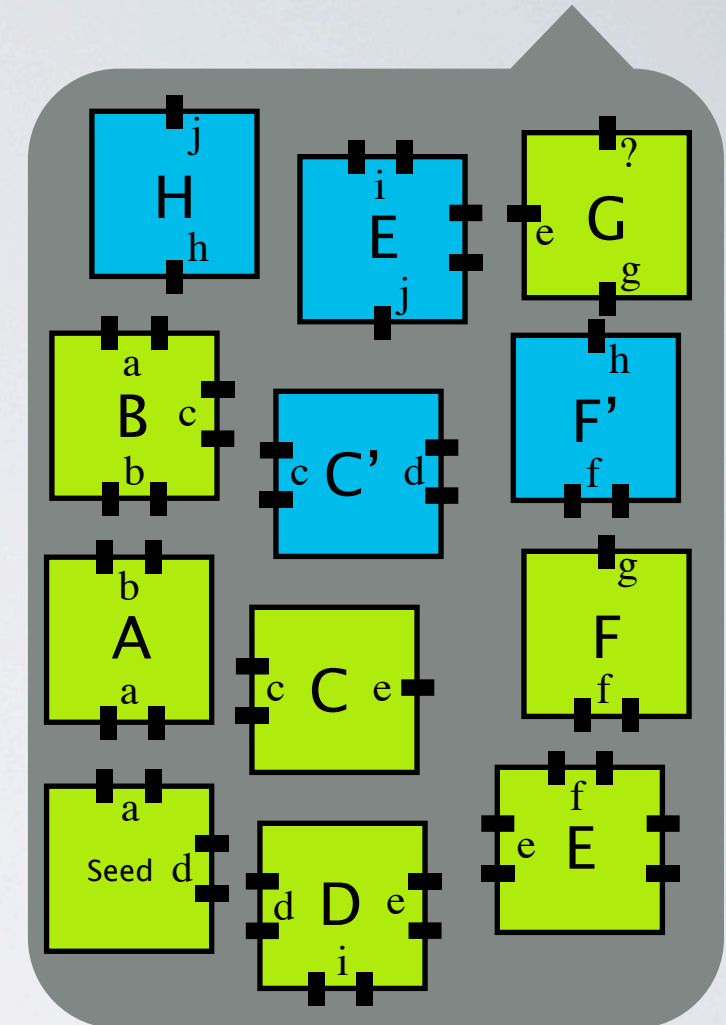
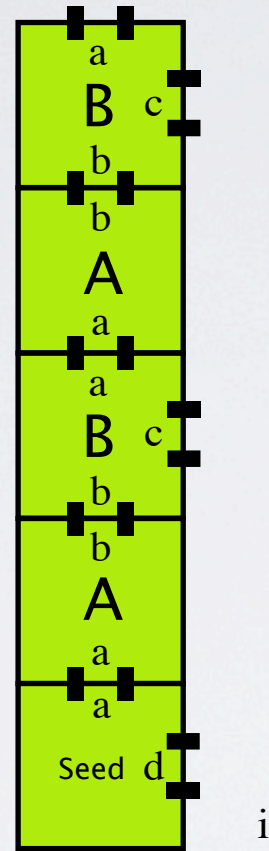
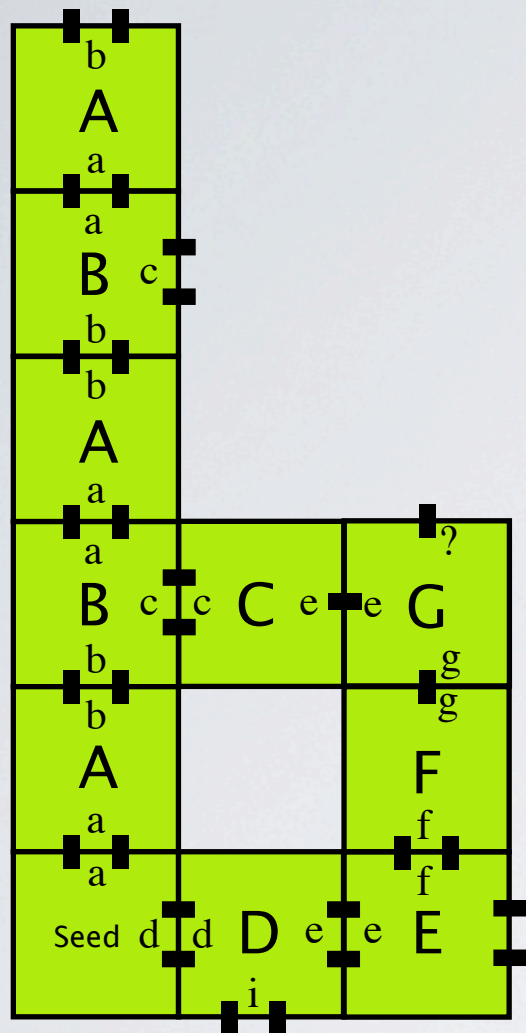


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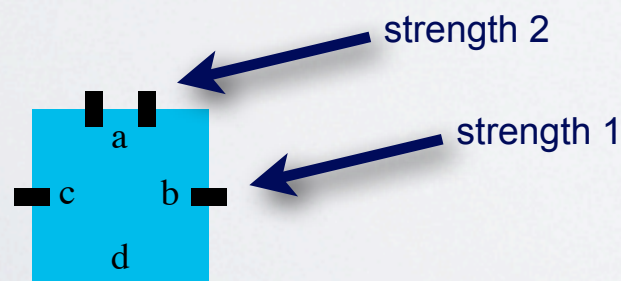
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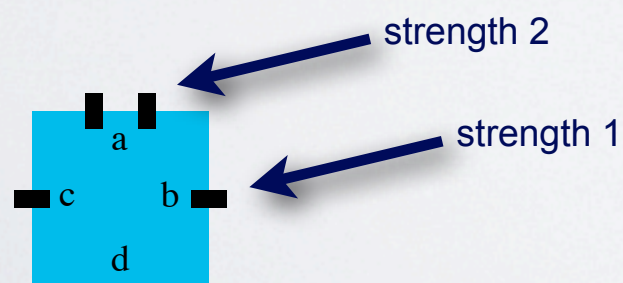
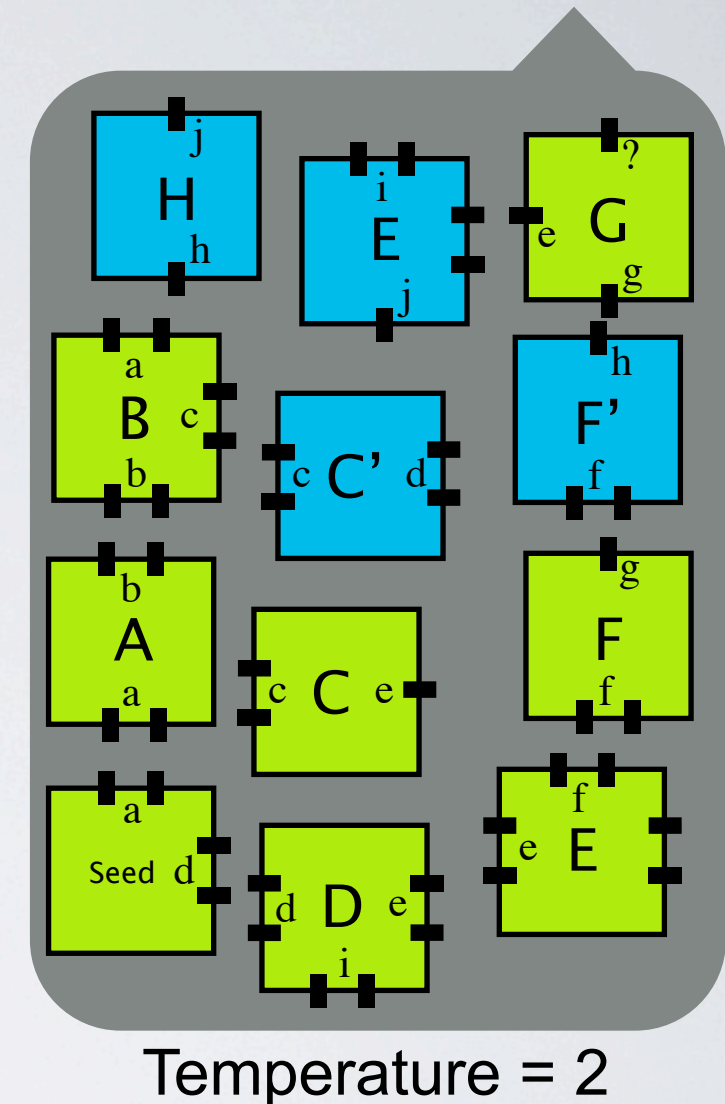
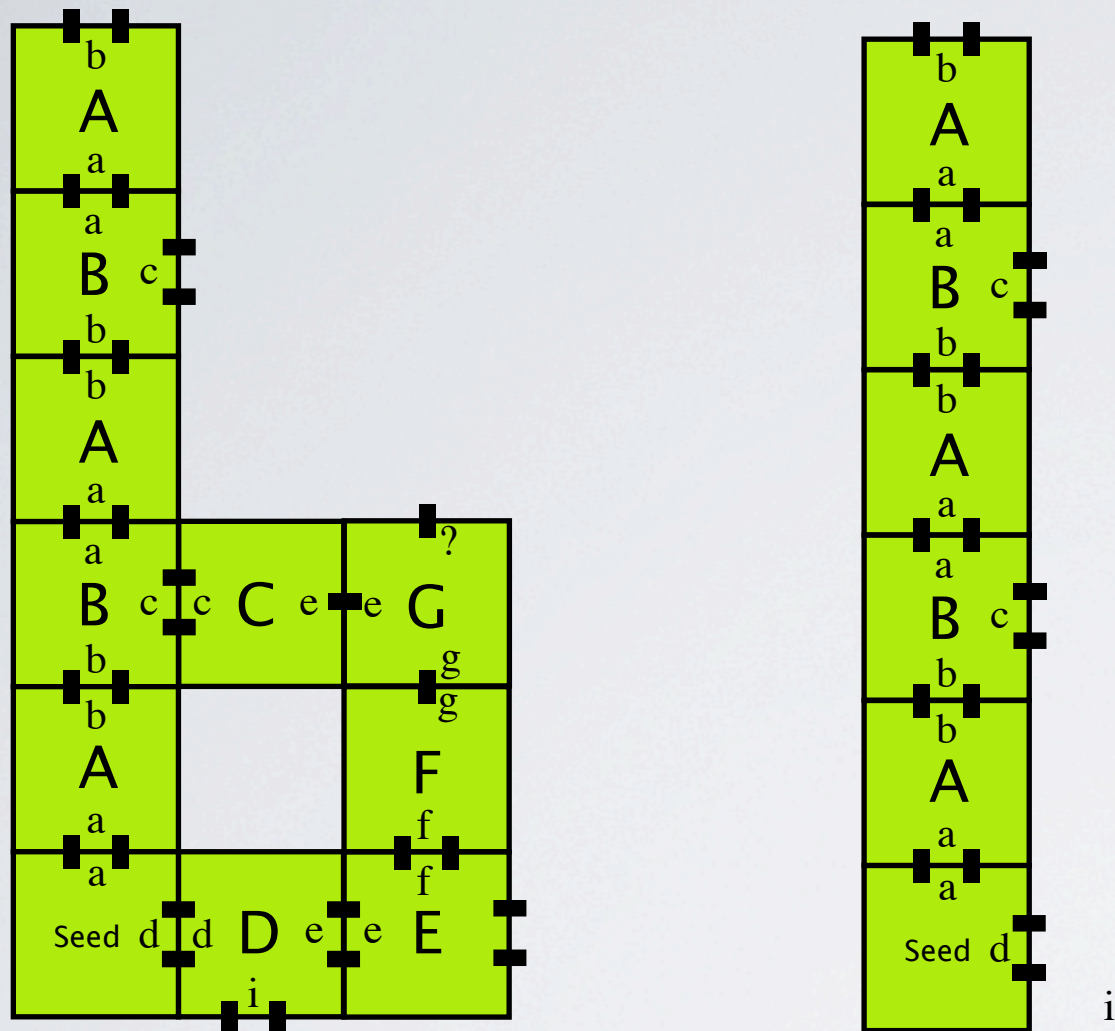


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Defn. Tile assembly system: tile set, seed tile, temperature

Winfree, 1998

Abstract tile assembly model

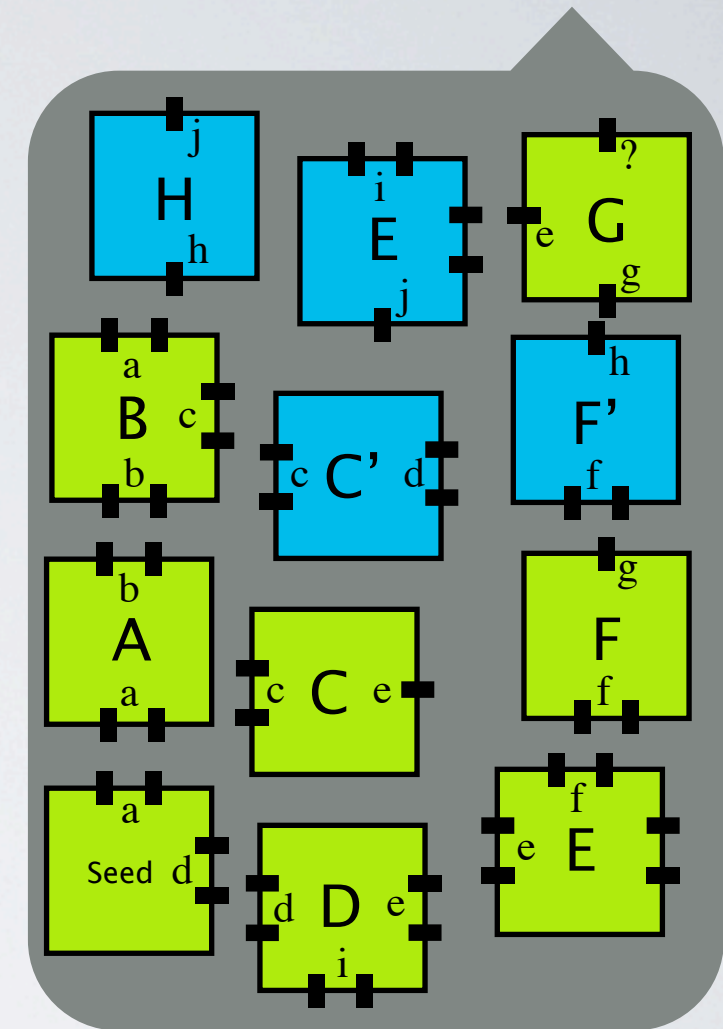
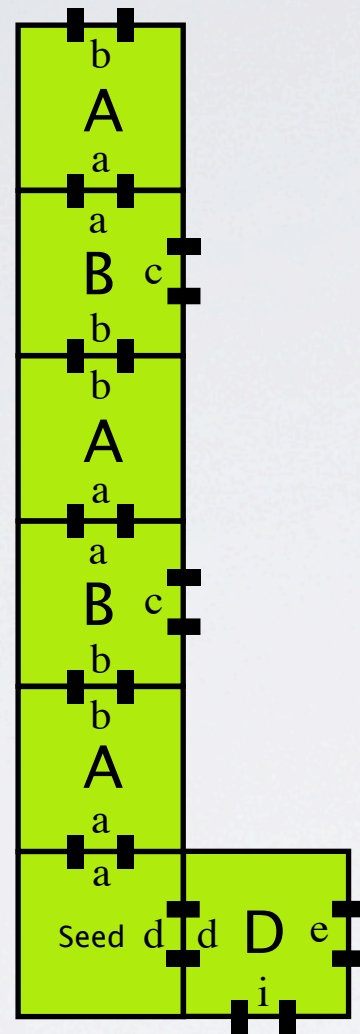
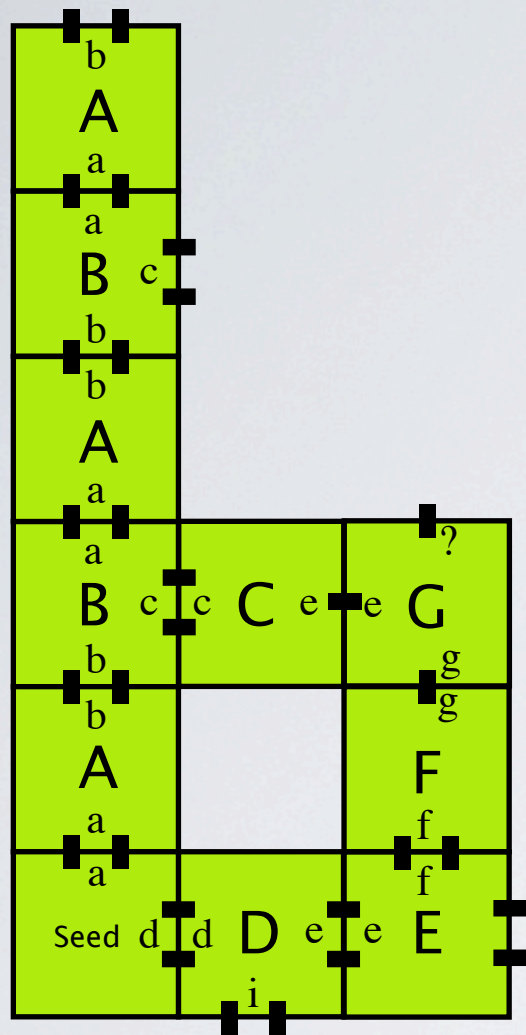


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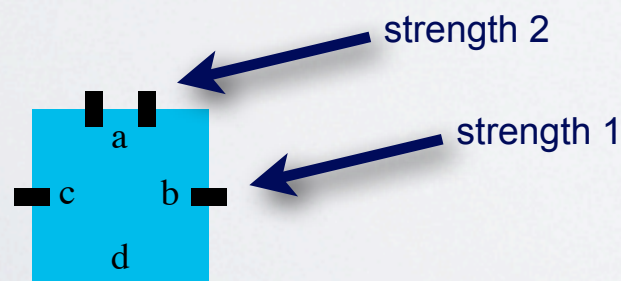
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Abstract tile assembly model



Temperature = 2

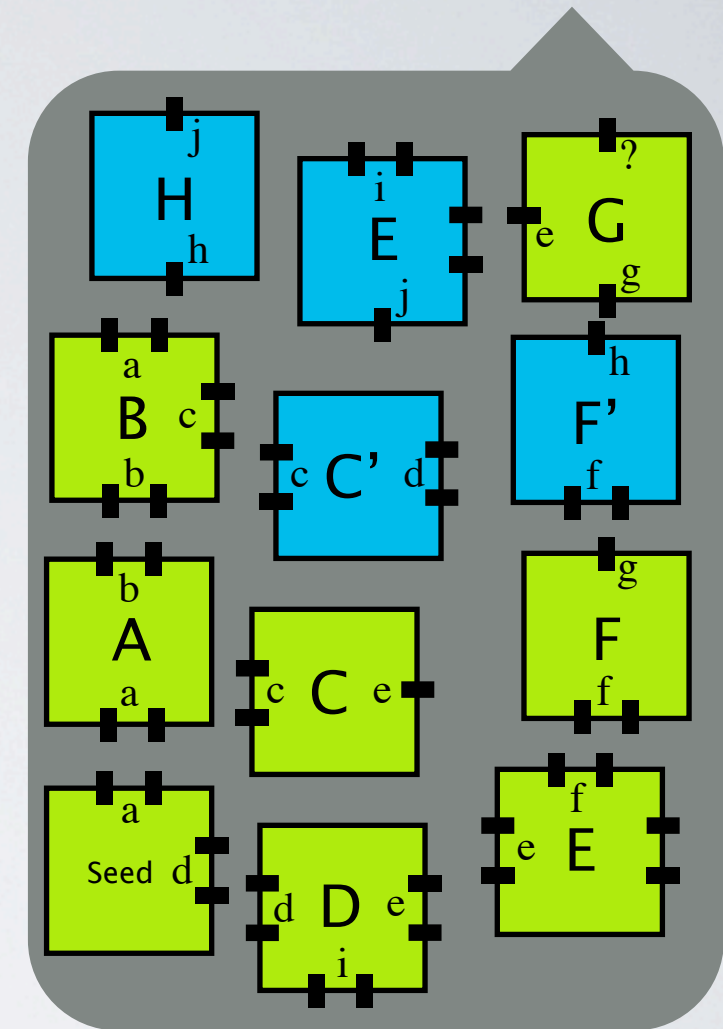
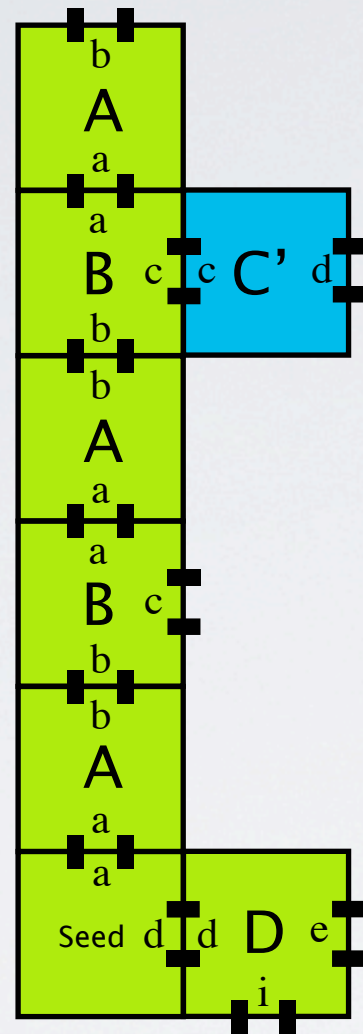
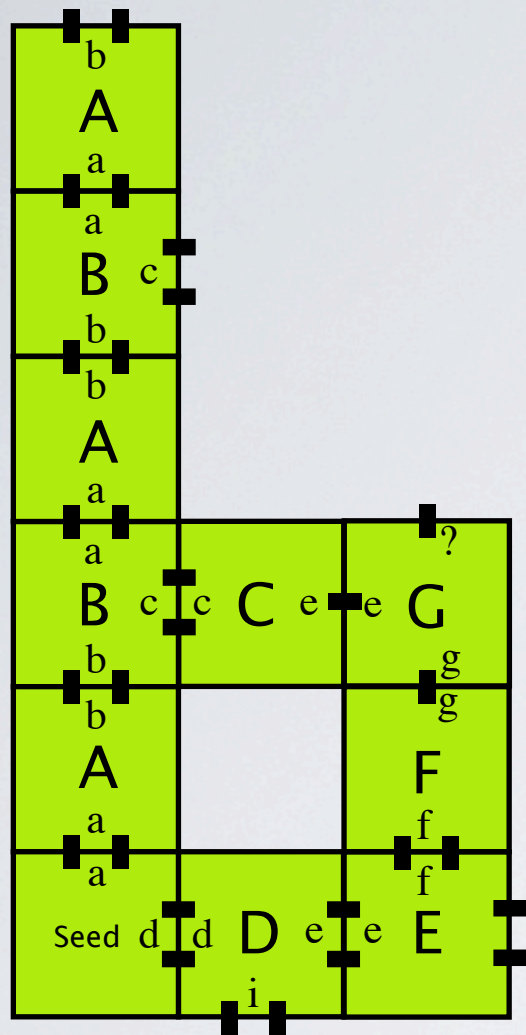


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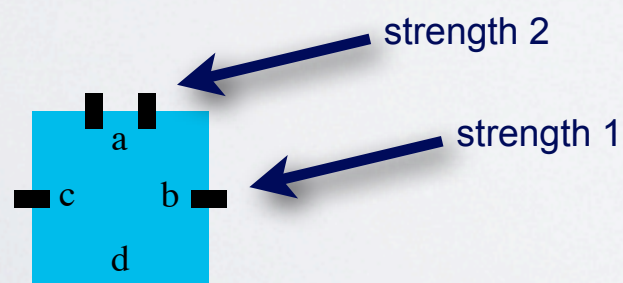
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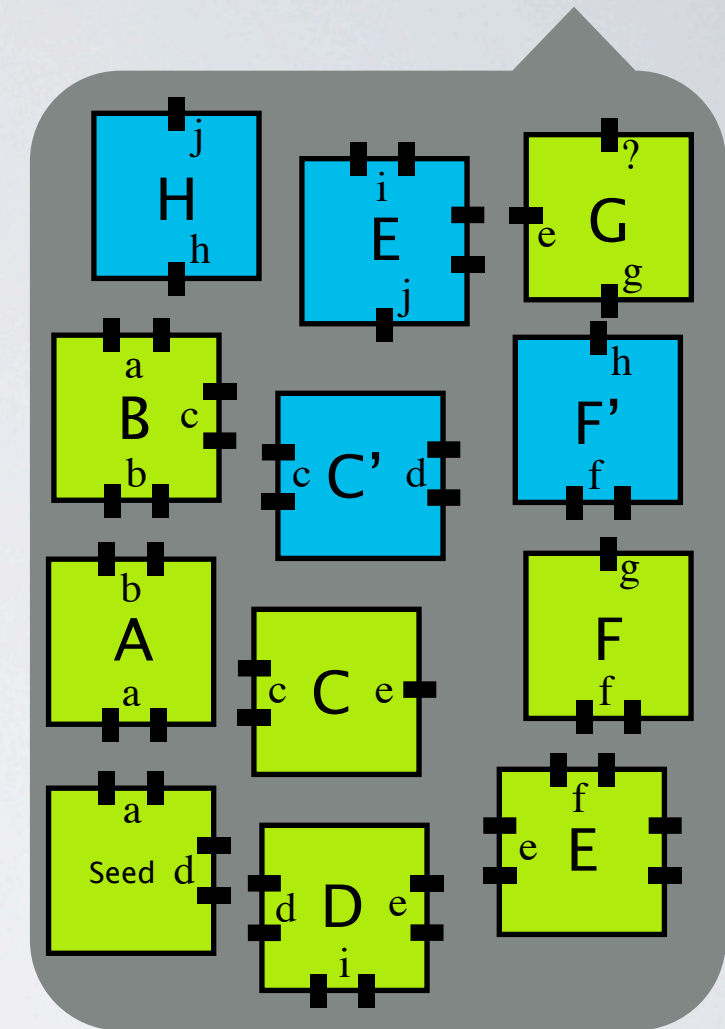
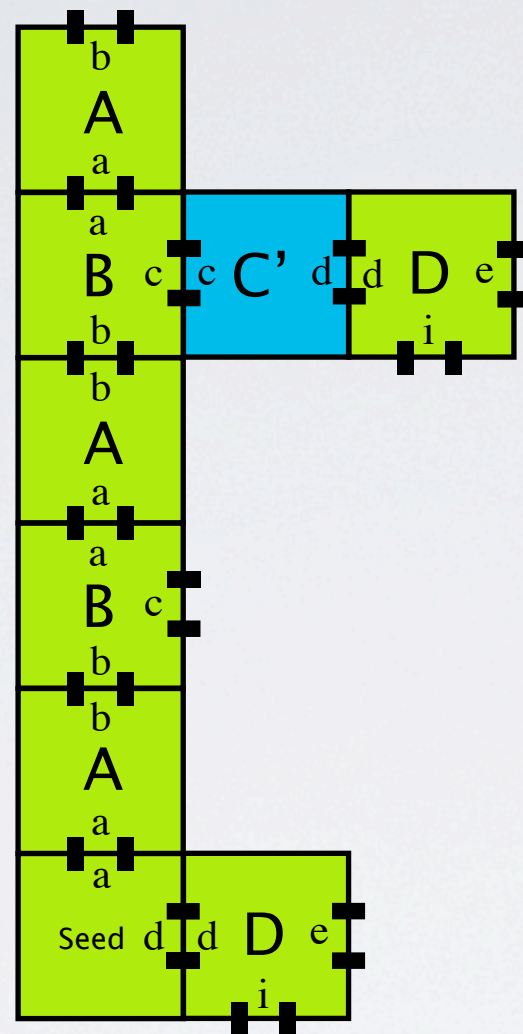
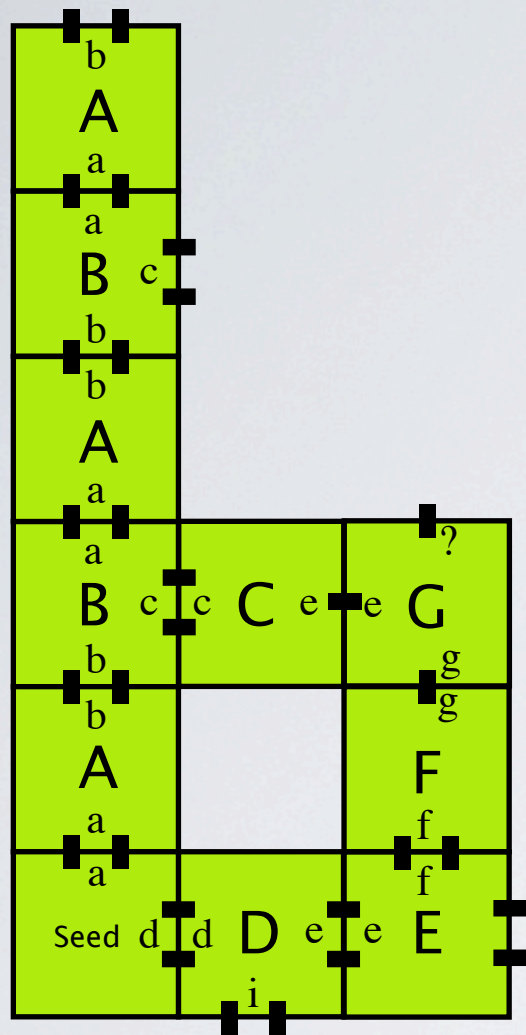


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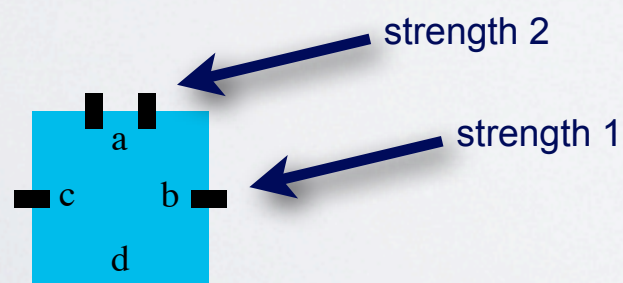
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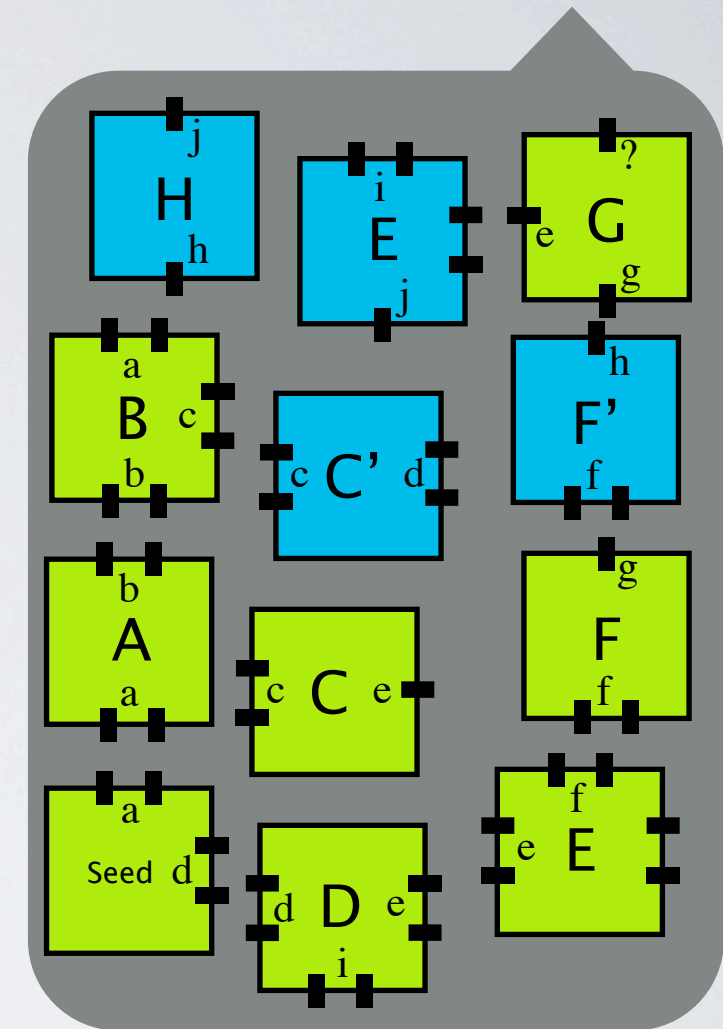
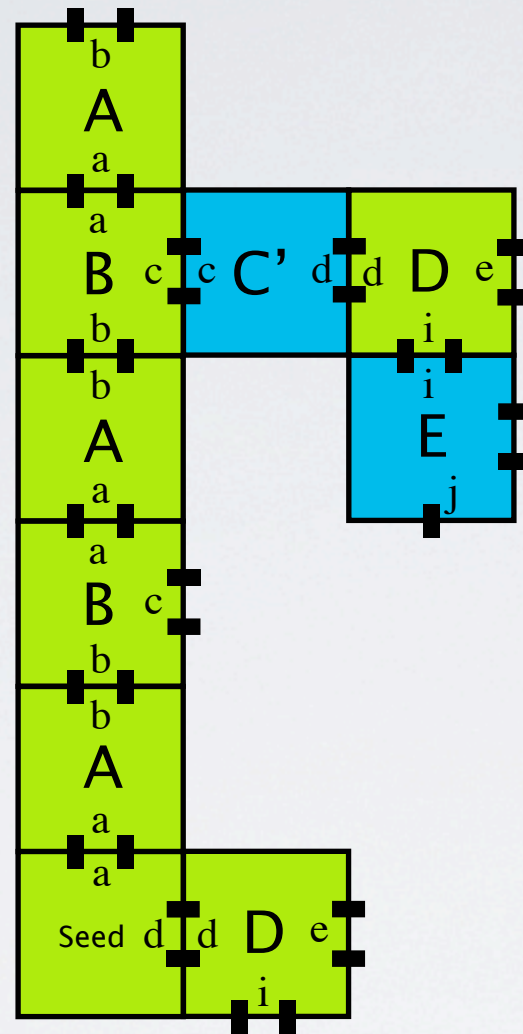
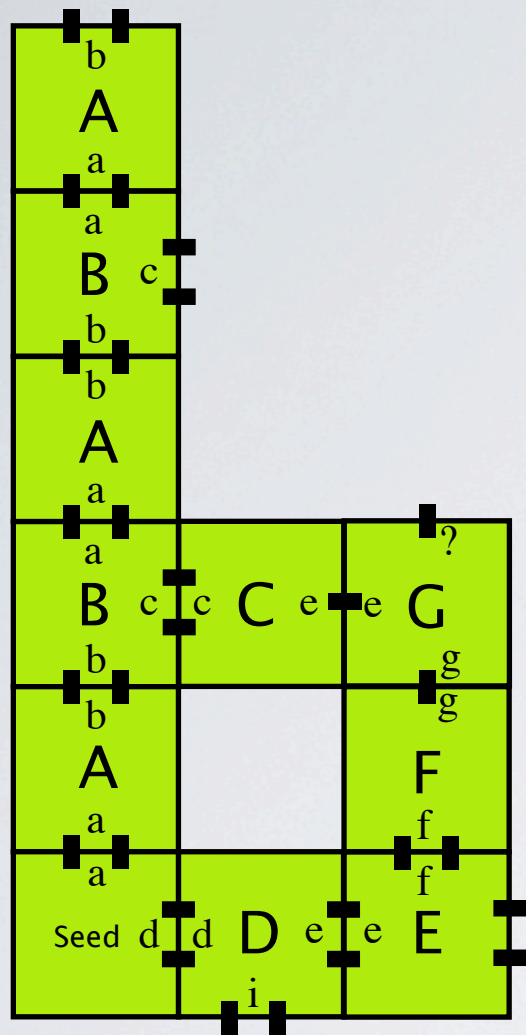


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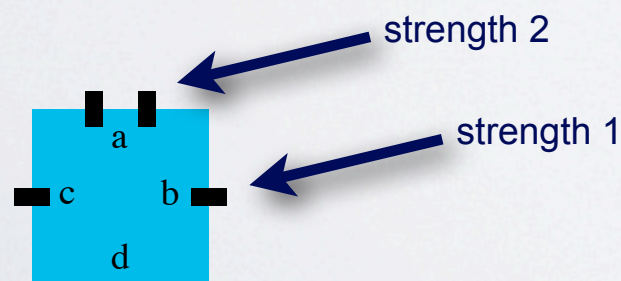
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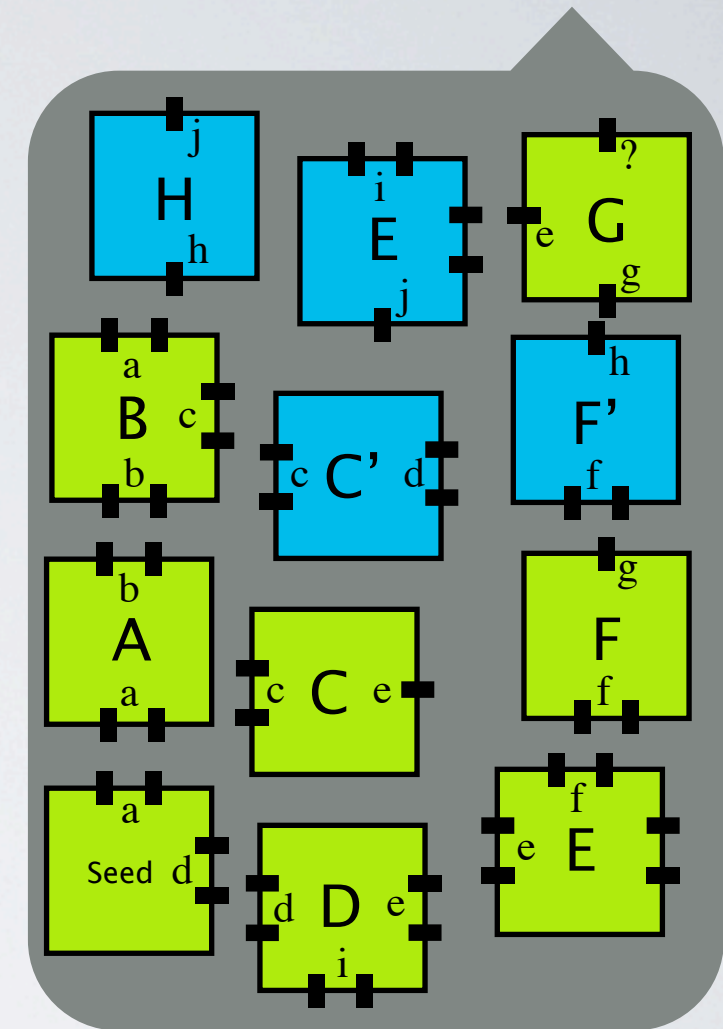
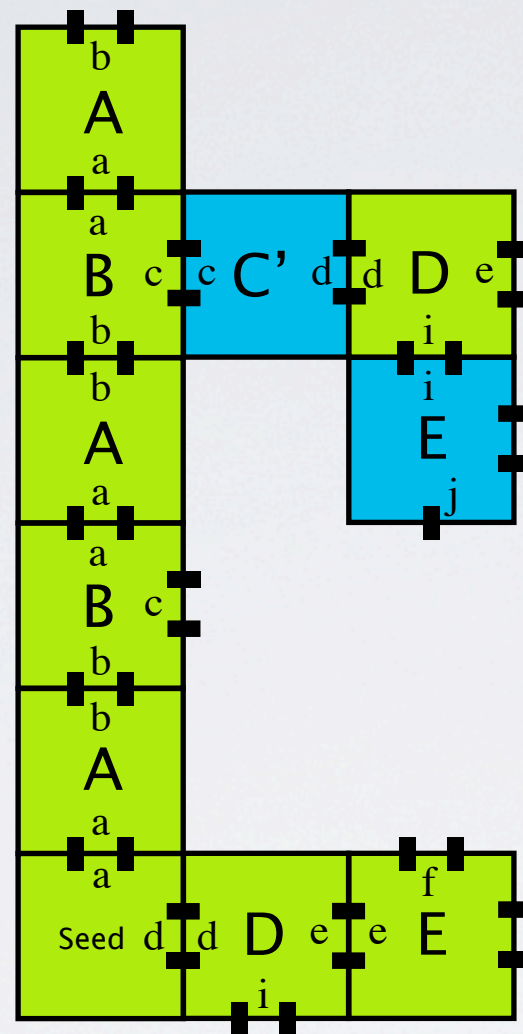
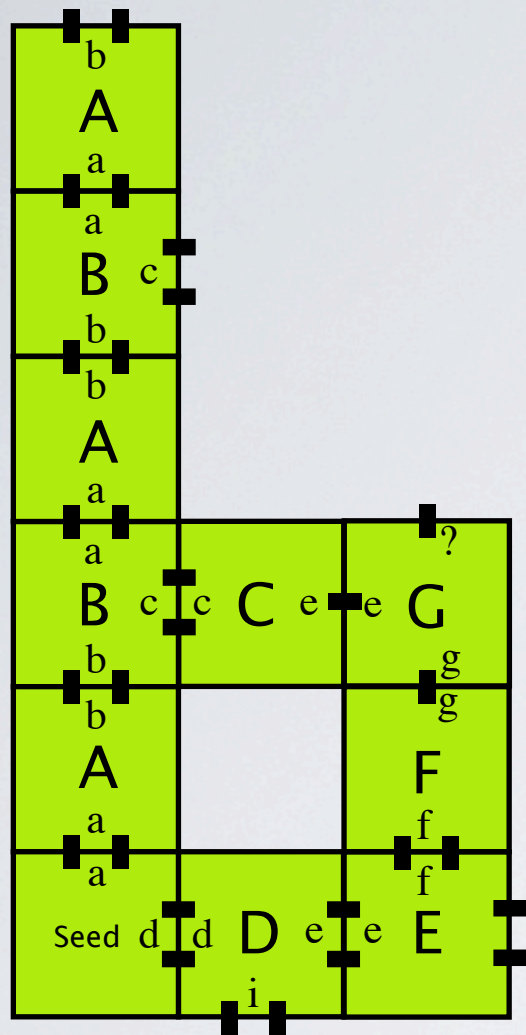


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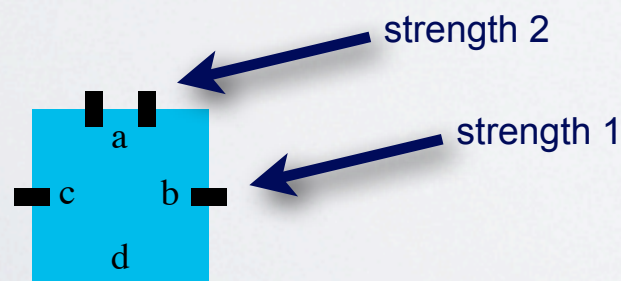
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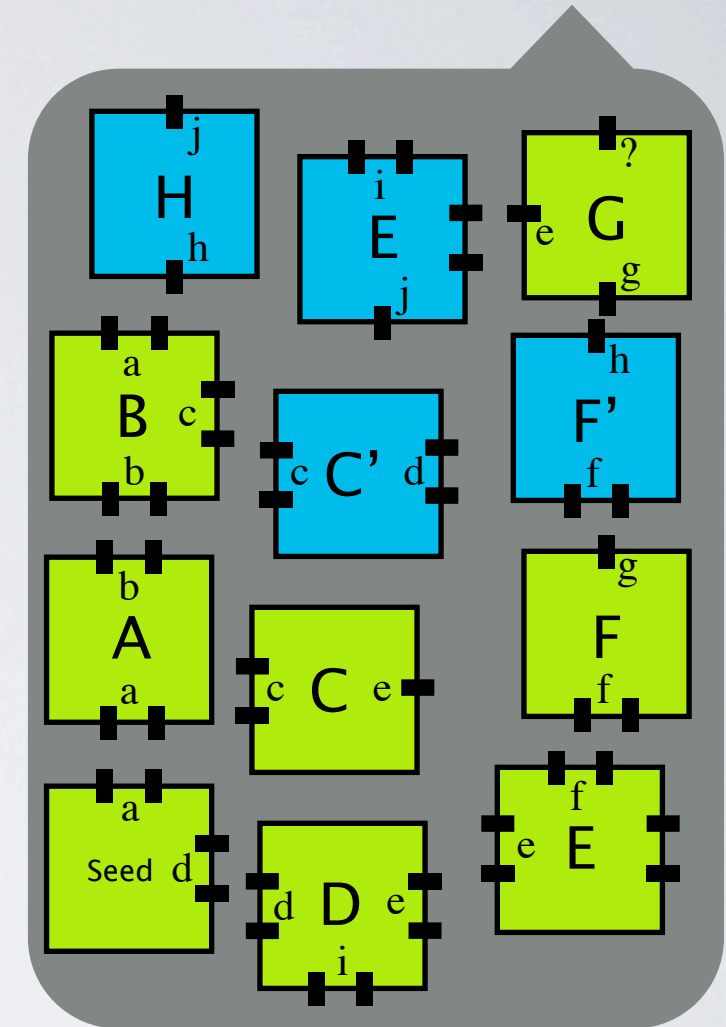
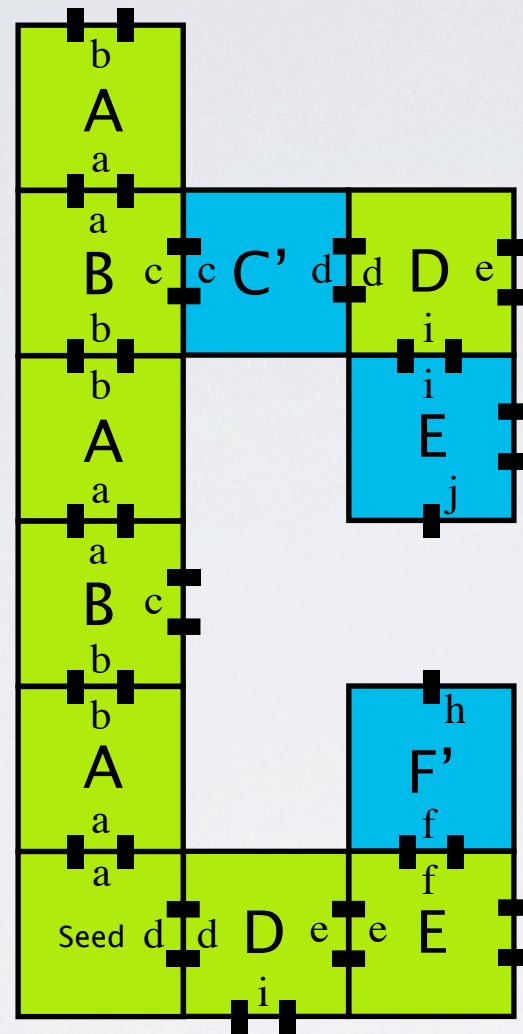
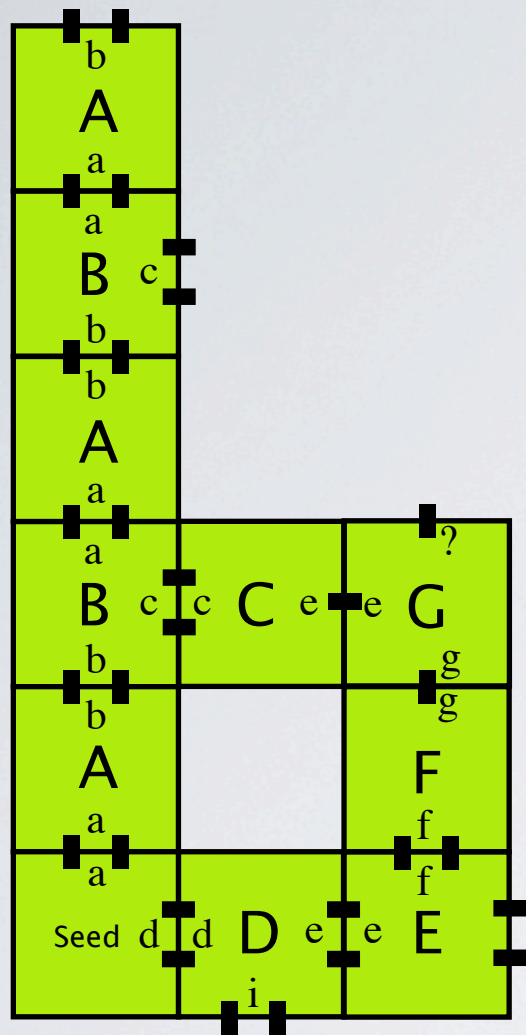


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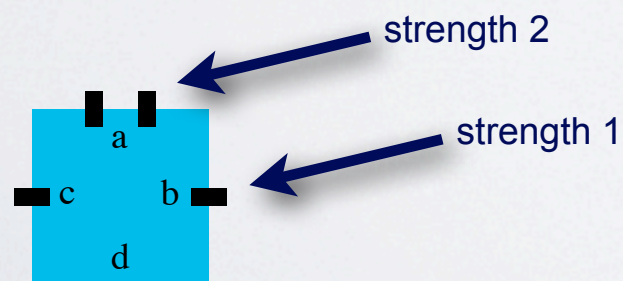
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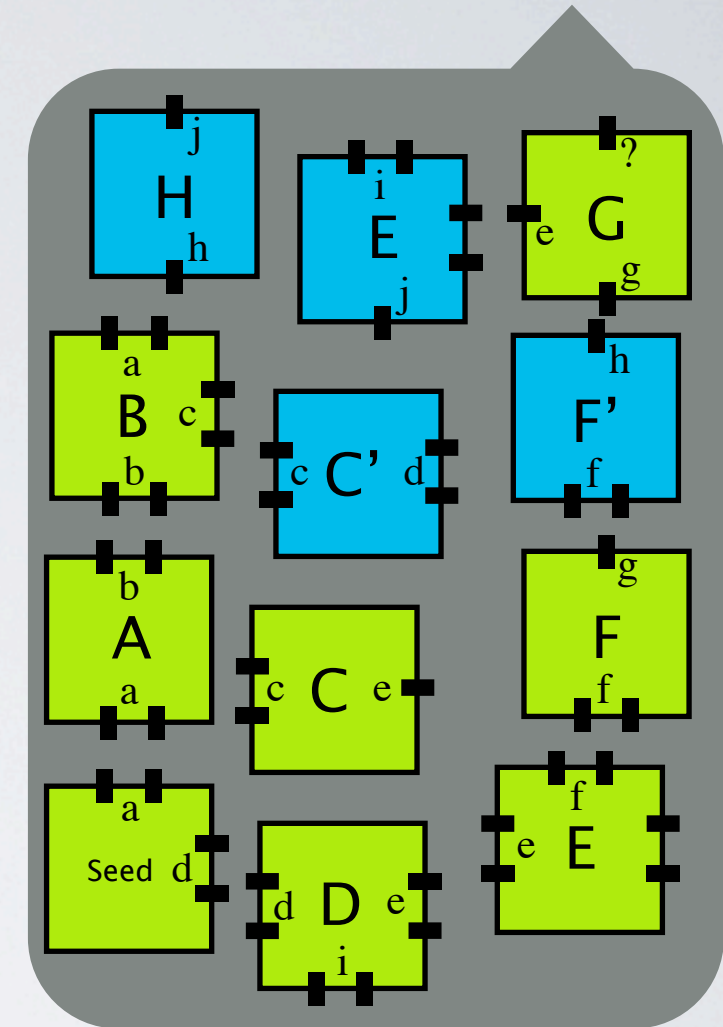
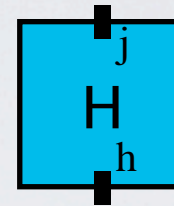
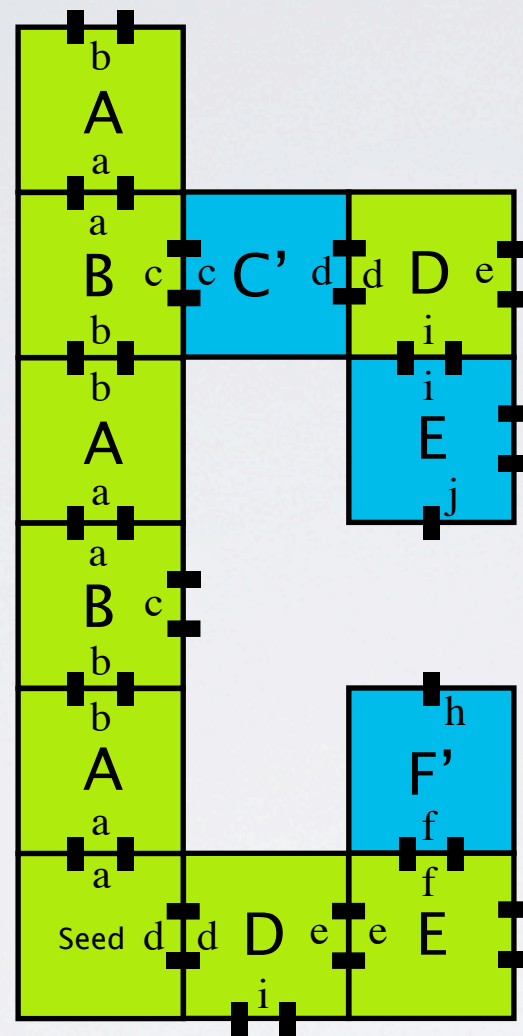
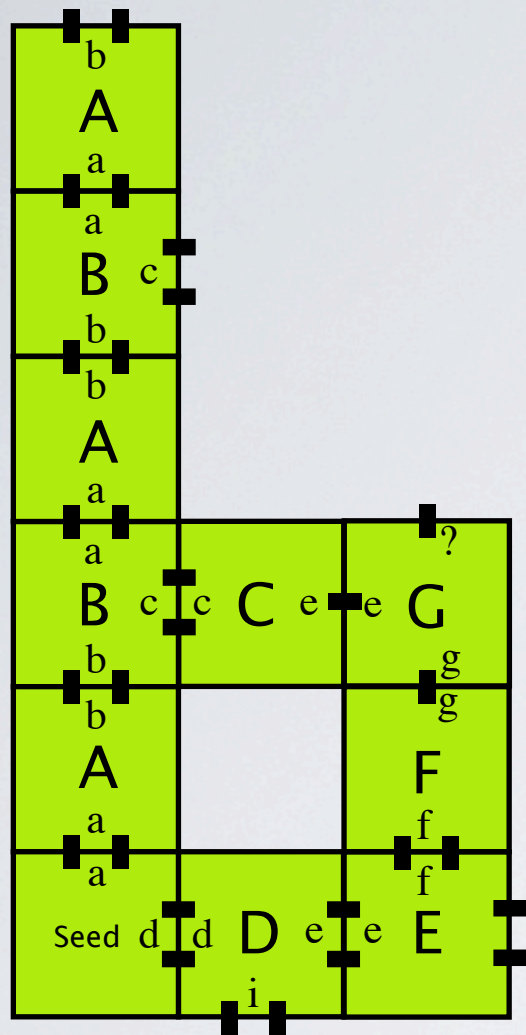


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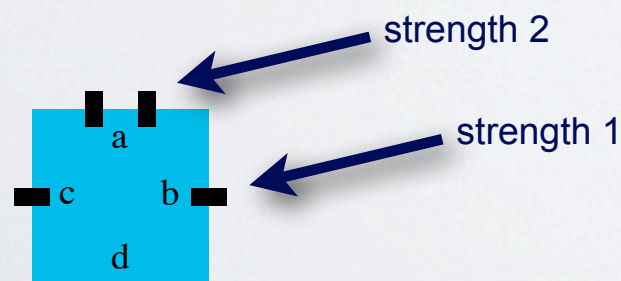
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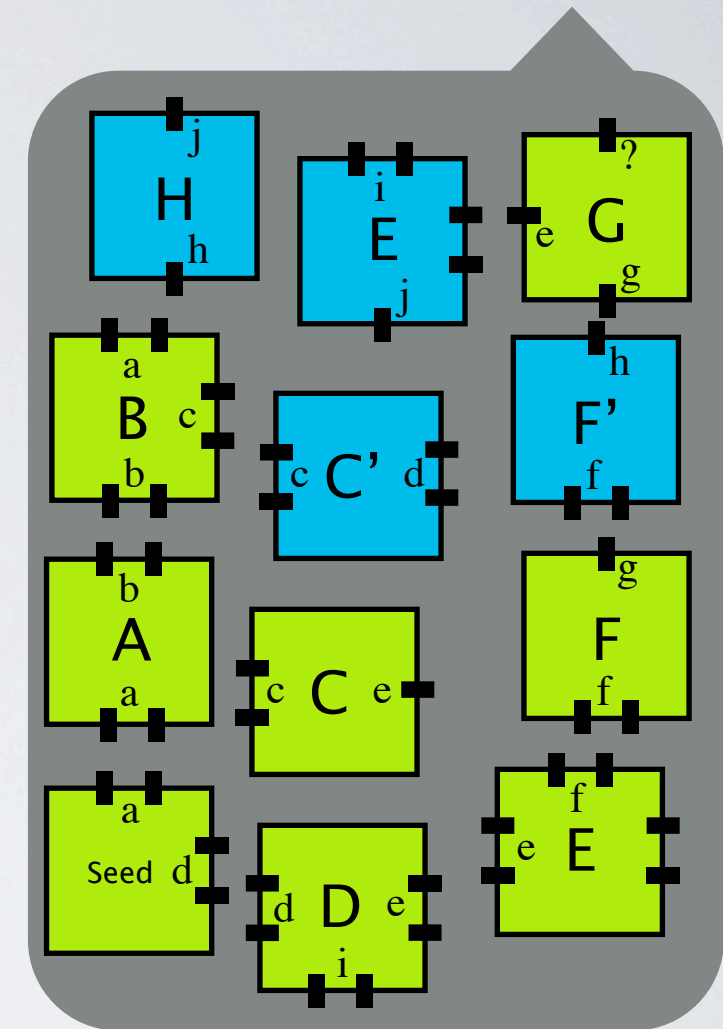
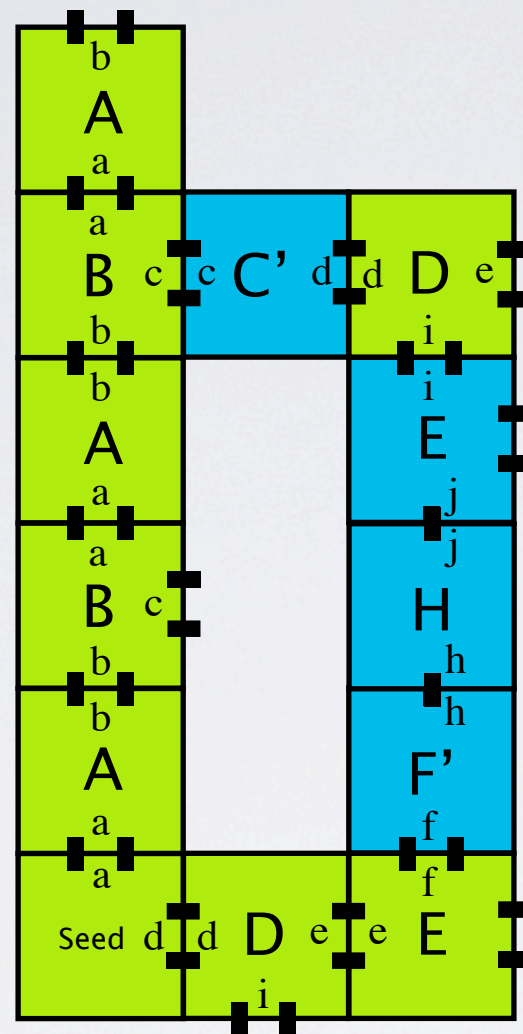
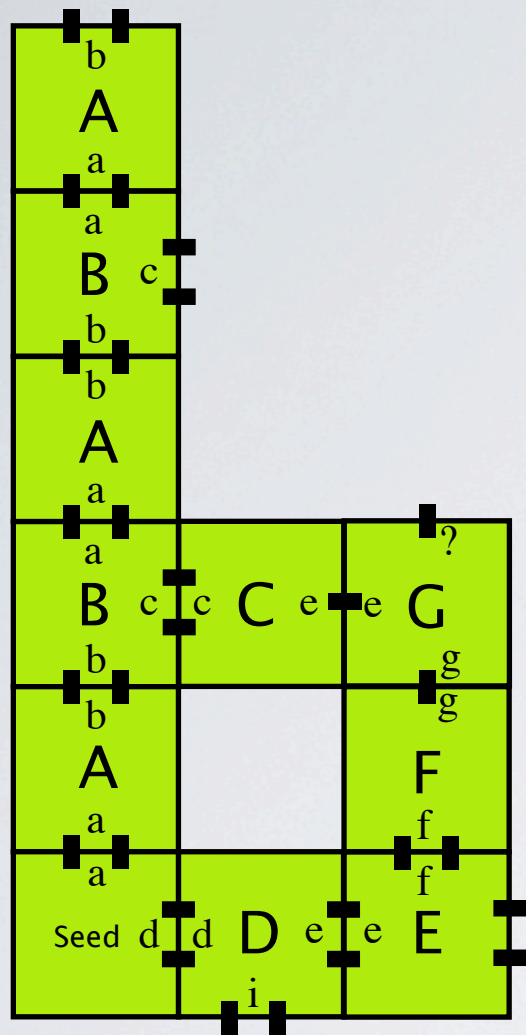


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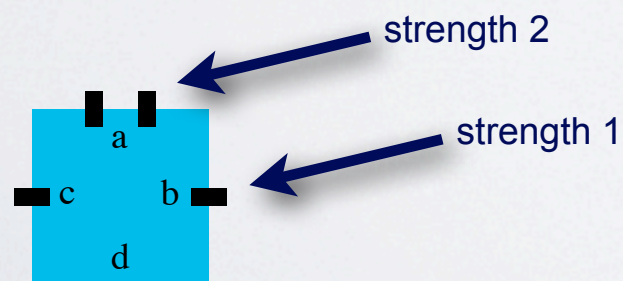
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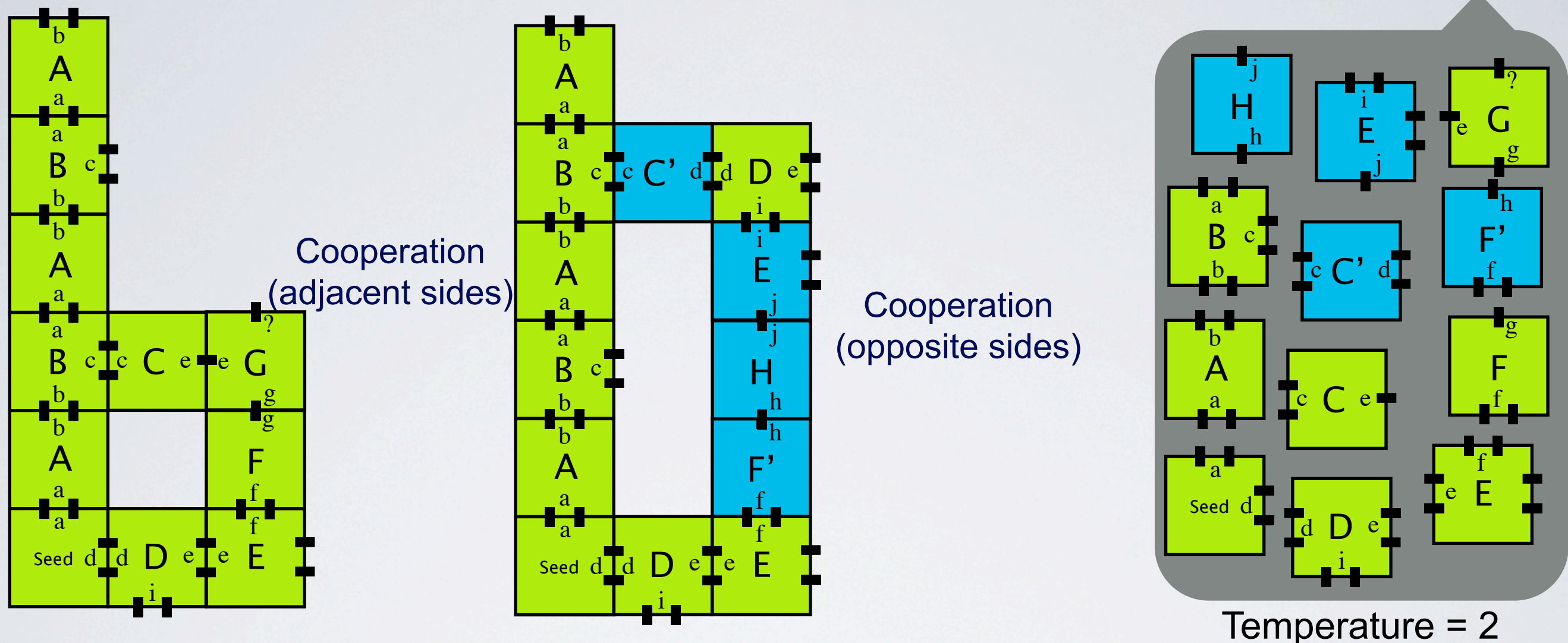


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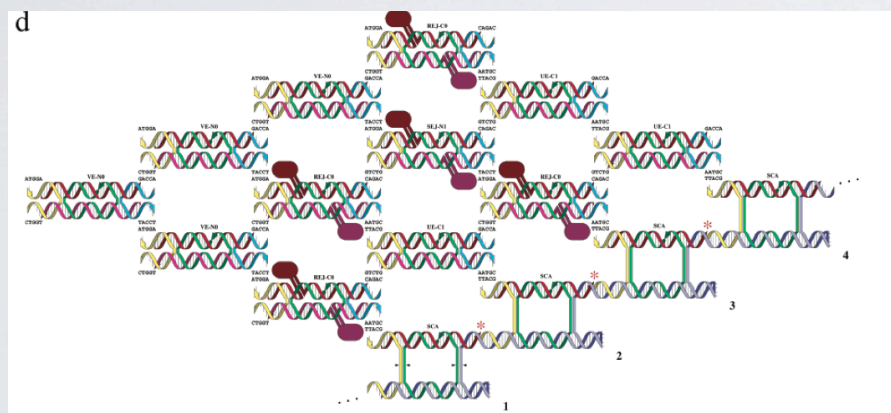
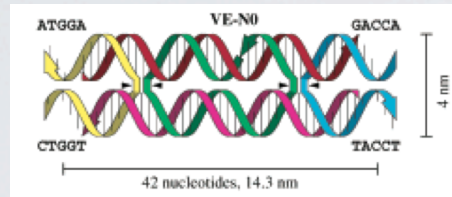
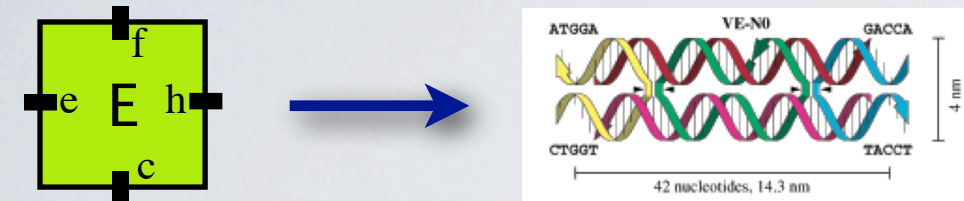


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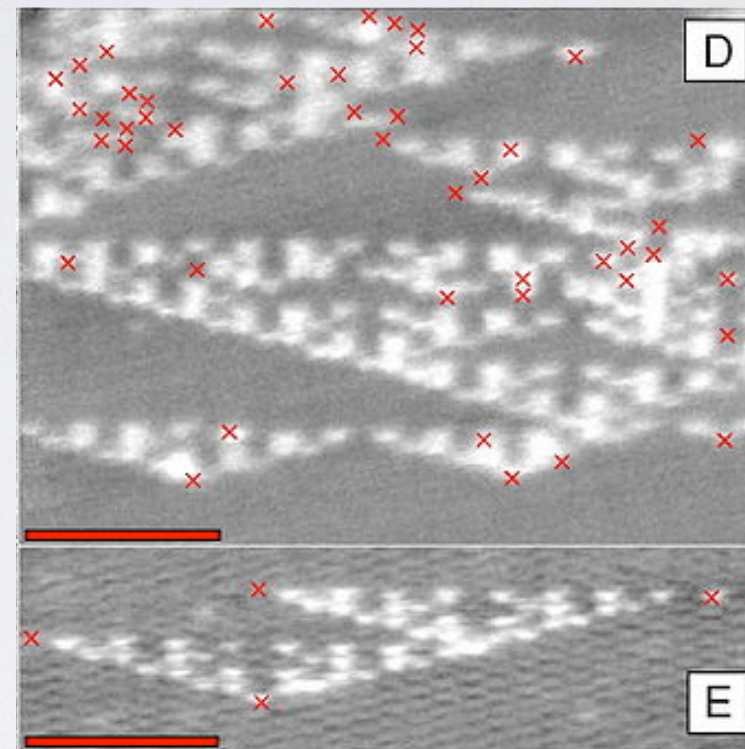
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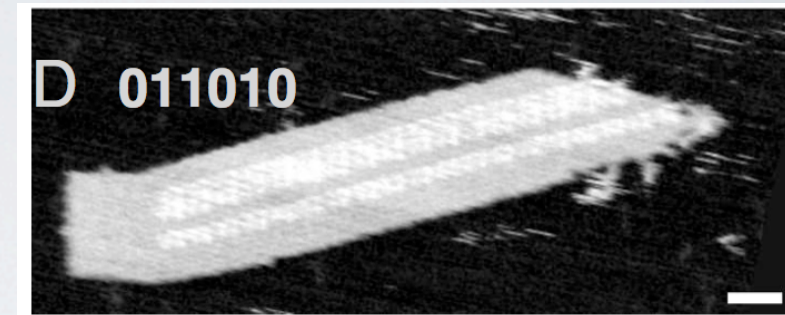
Algorithmic self-assembly with tiles: computation and geometry



Barish, Rothmund, Winfree 2005

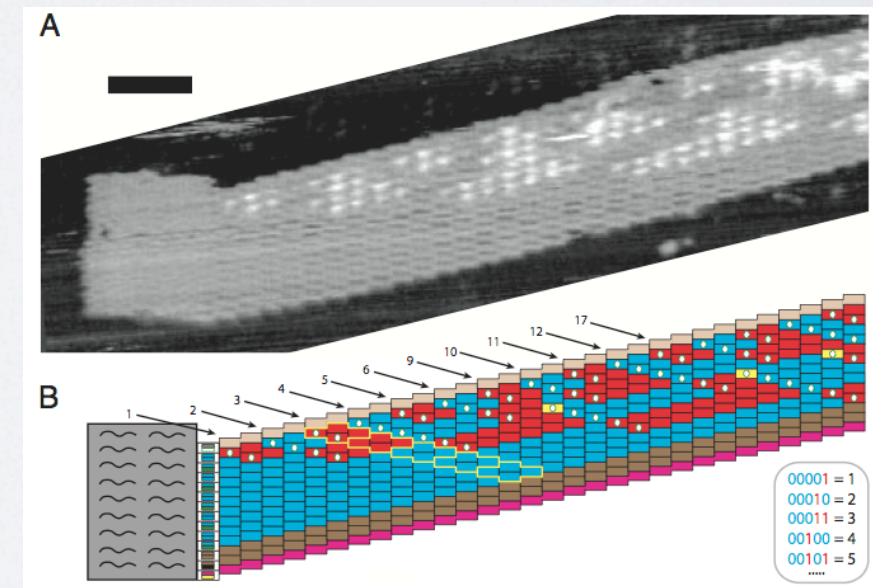


Rothmund, Papadakis, Winfree 2004



Barish, Schulman Rothmund, Winfree 2009

- Seeman built tiles out of DNA in the laboratory
- Winfree showed that DNA tiles can run algorithms (much like cellular automata, Wang tiles)
- A variety of algorithmic tile assembly systems have been built from DNA

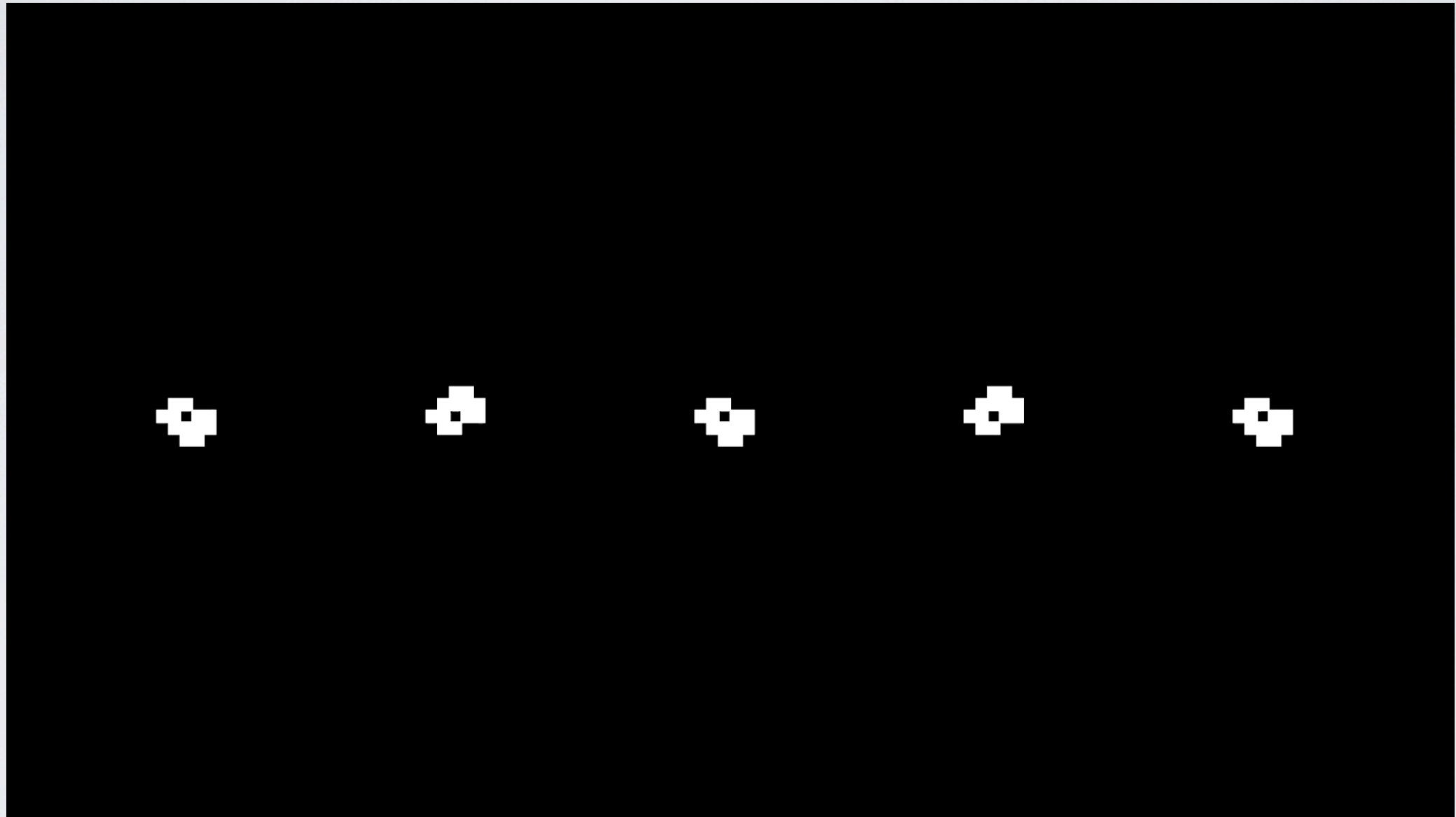


Barish, Schulman Rothmund, Winfree 2009

Intrinsic universality

<http://otcametapixel.blogspot.com/>
<http://www.youtube.com/watch?v=xP5-ileKXE8>

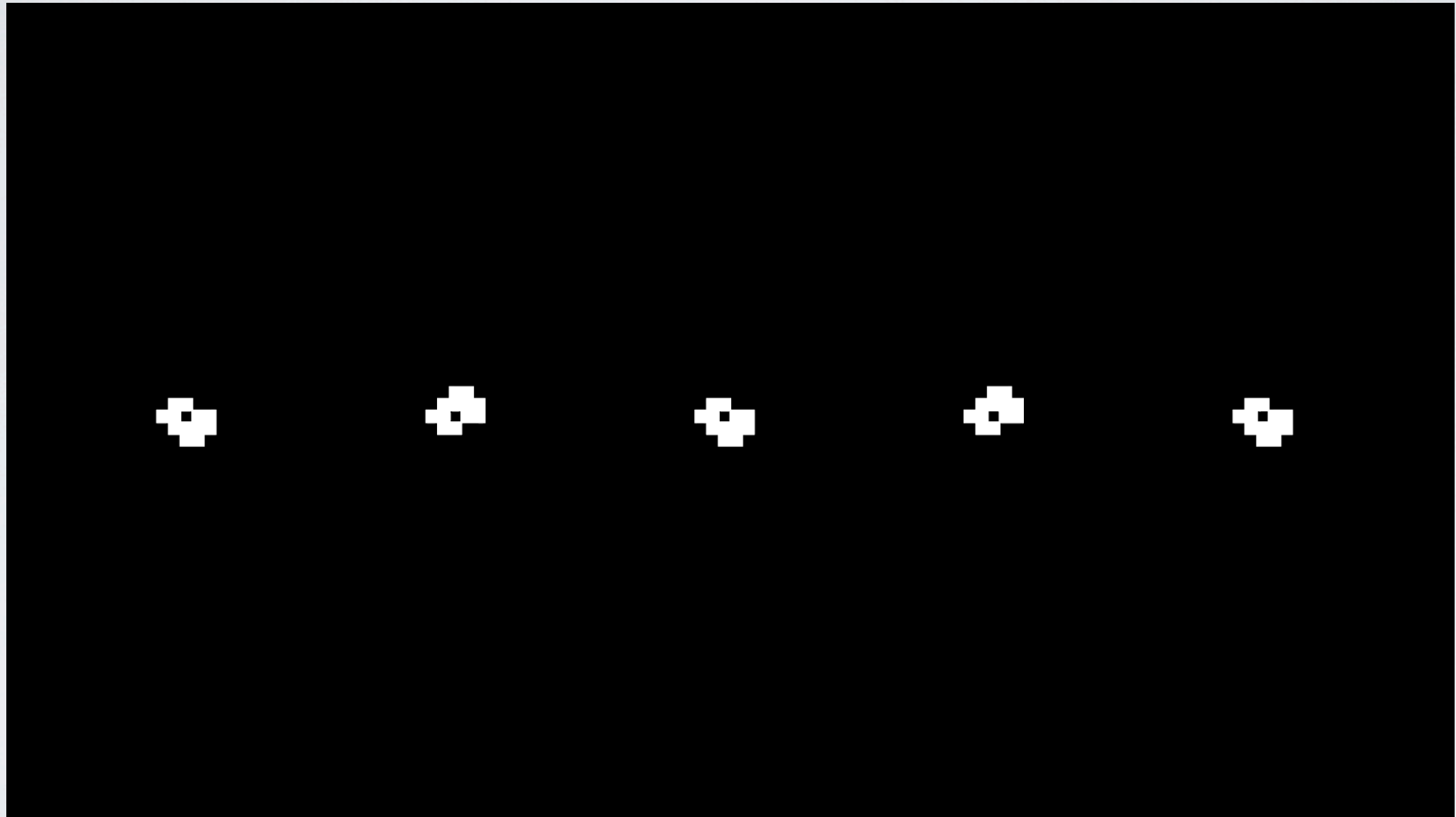
Intrinsic universality



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Intrinsic universality

- Conway's Game of Life is an intrinsically universal cellular automaton



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<http://www.youtube.com/watch?v=xP5-ileKXE8>

Intrinsic universality

- An intrinsically universal CA is one that “simulates” any other:
 - Appropriate initial conditions
 - Encodes many properties of the simulated CA
- Theory of intrinsic universality in CA
- Applications: non-intrinsic universality results to prove weakness of a model

Is the abstract tile assembly model
intrinsically universal?

Is the abstract tile assembly model intrinsically universal?

Yes!

Is the abstract tile assembly model intrinsically universal?

Yes!

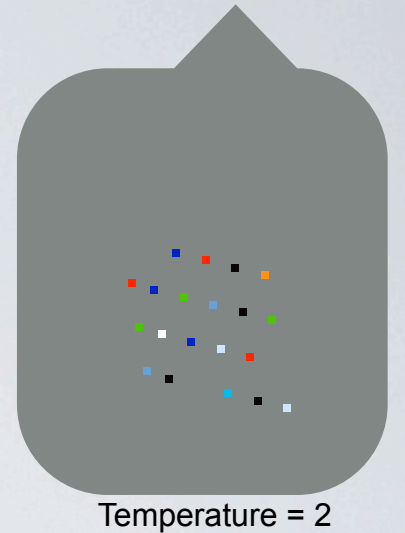
There is a single universal tile set that
simulates any tile assembly system

Simulation

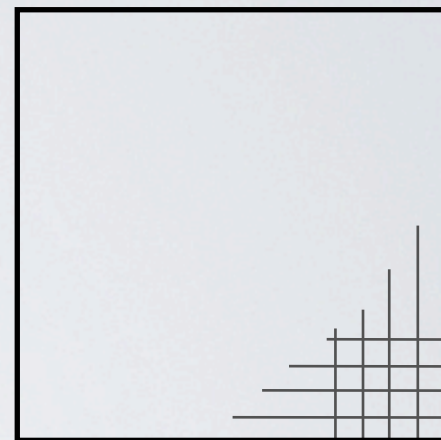
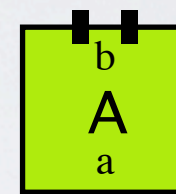
- We want a definition of simulation that captures the notion of one TAS simulating the “behavior” of another
- \mathcal{T} is a (any) simulated tile assembly system,
 - $\mathcal{T} = (\text{tileset } T, \text{ seed assembly } \sigma, \text{ temperature } \tau)$
- Tile assembly system $\mathcal{U}_\tau = (U, \sigma_\tau, 2)$ simulates \mathcal{T} if:
 - Tiles from T are simulated by $m \times m$ supertiles in \mathcal{U}_τ , i.e., a representation function R maps supertiles in assemblies of \mathcal{U}_τ to tiles in T . Oh, and R is “clean”.
 - The producible assemblies in \mathcal{U}_τ represent exactly the producible assemblies in \mathcal{T} .
 - Ignoring $m \times m$ scaling, dynamics are equivalent in \mathcal{U}_τ & \mathcal{T} .

Simulation with tiles

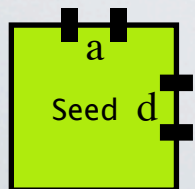
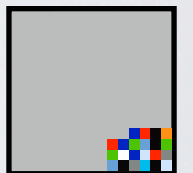
Simulator
tile set



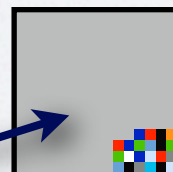
Simulated
tile



Simulator tiles



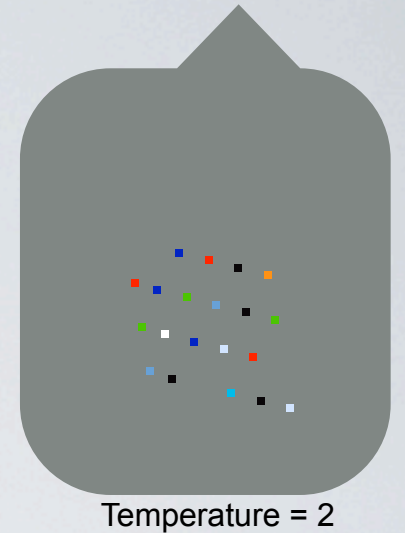
Preassembled
seed structure



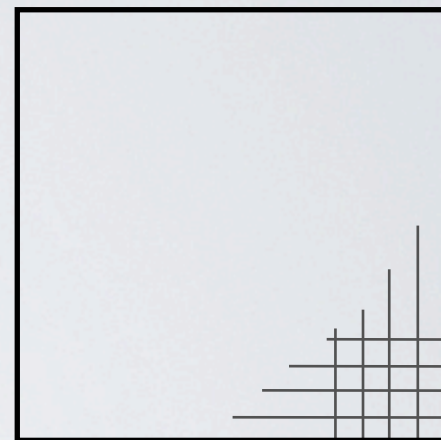
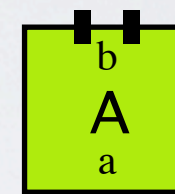
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Simulation with tiles

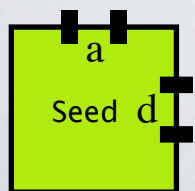
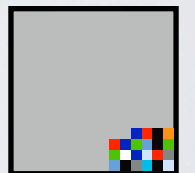
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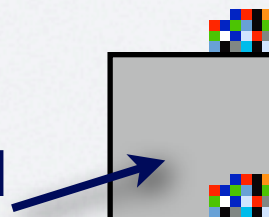
Simulated
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Simulator tiles



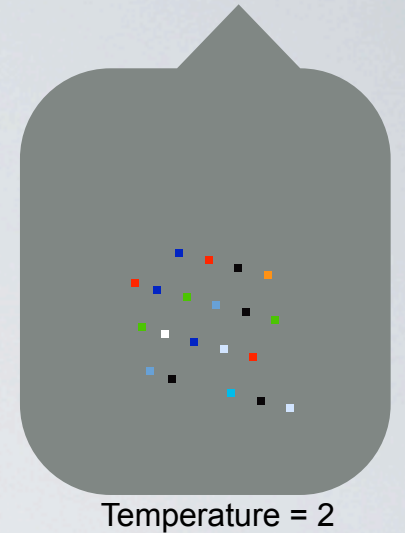
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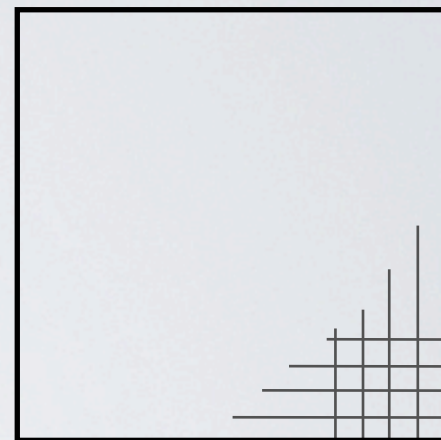
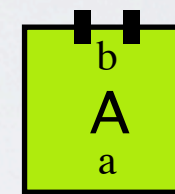
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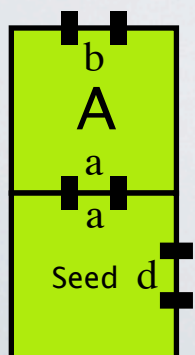
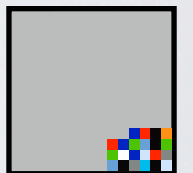
Simulator
tile set



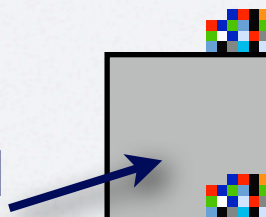
Simulated
tile



Simulator tiles



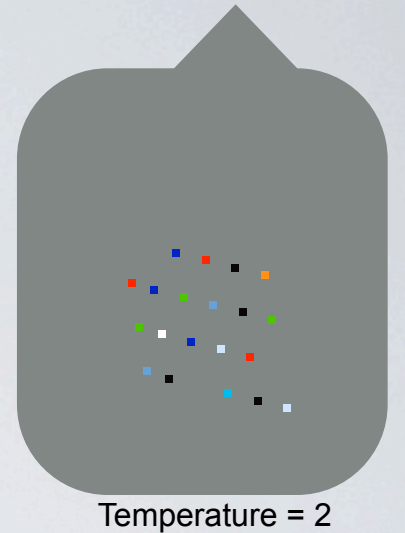
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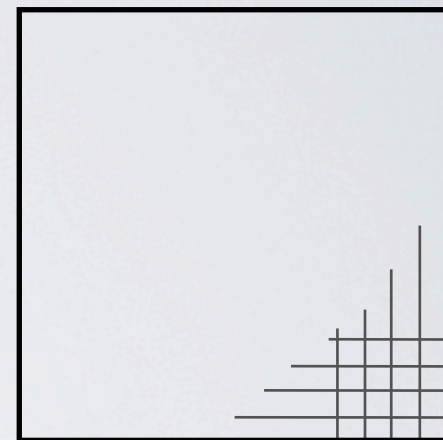
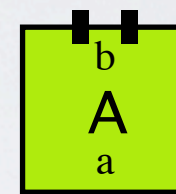
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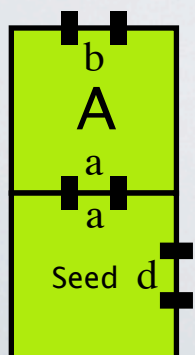
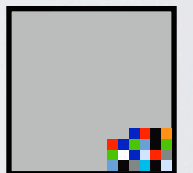
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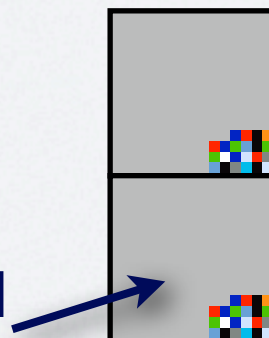
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tile



Simulator tiles



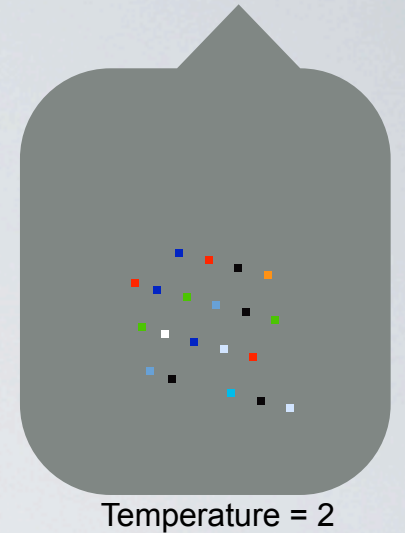
Preassembled
seed structure



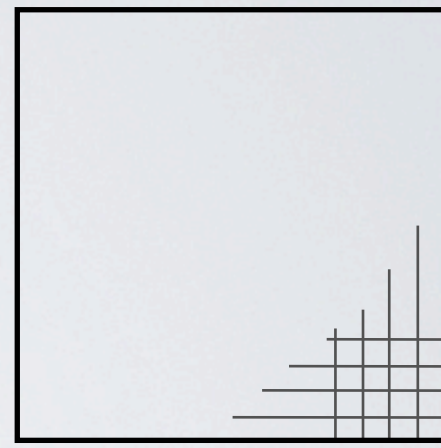
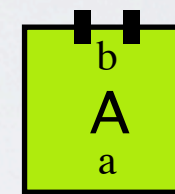
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

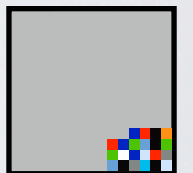
Simulator
tile set



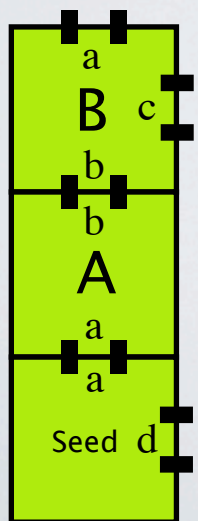
Simulated
tile



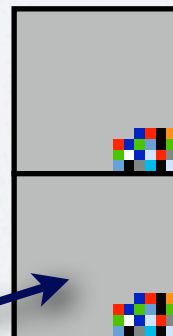
Simulator tiles



- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

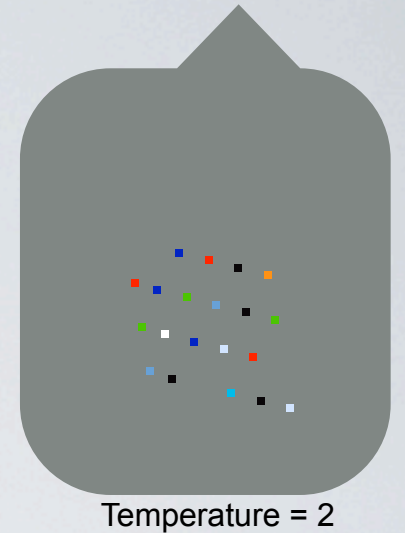


Preassembled
seed structure

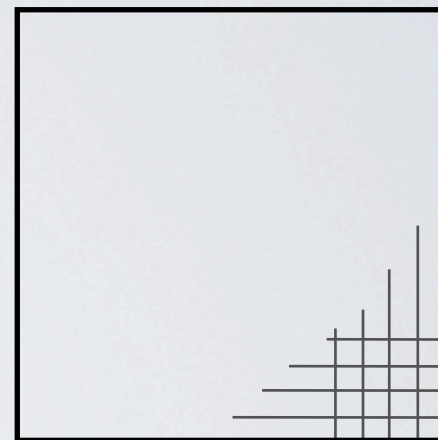
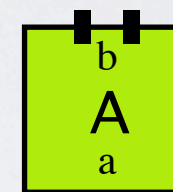


Simulation with tiles

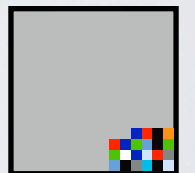
Simulator
tile set



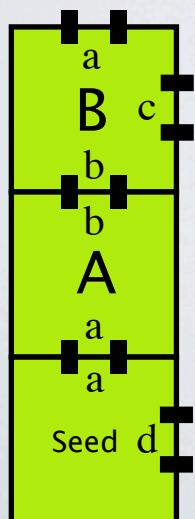
Simulated
tile



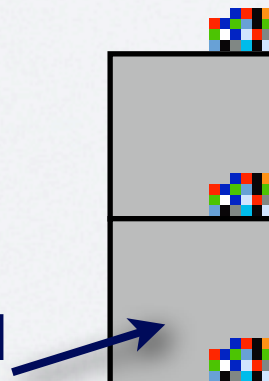
Simulator tiles



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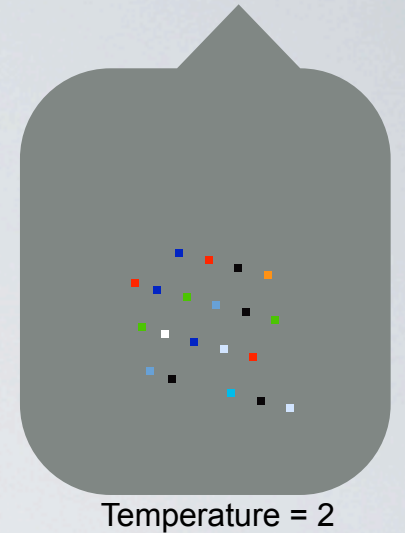


Preassembled
seed structure

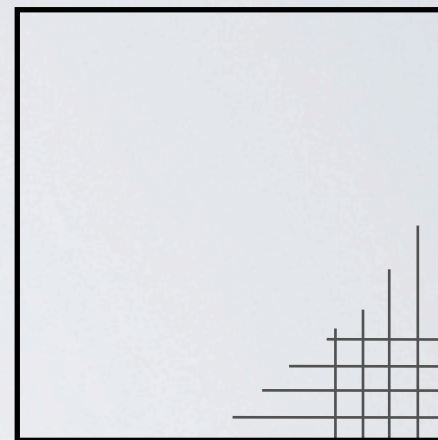
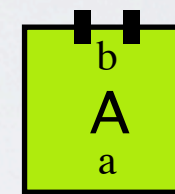


Simulation with tiles

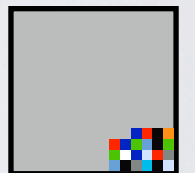
Simulator
tile set



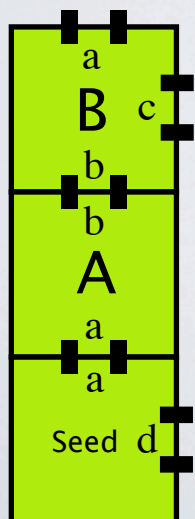
Simulated
tile



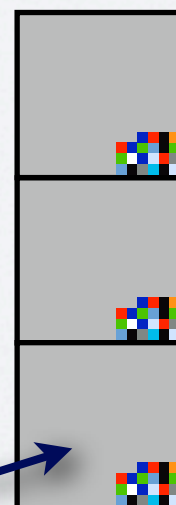
Simulator tiles



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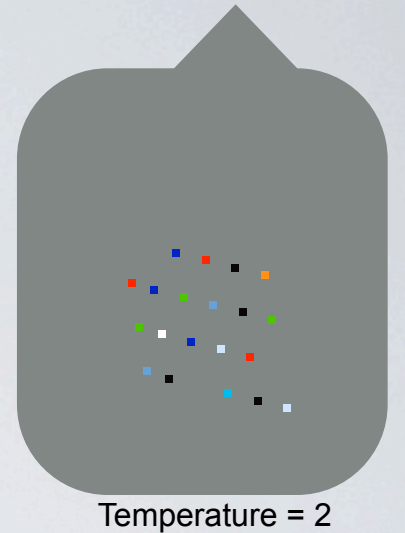


Preassembled
seed structure

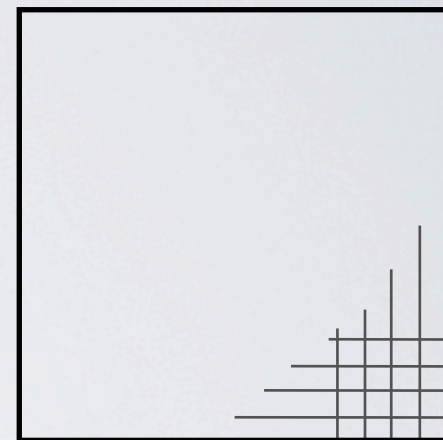
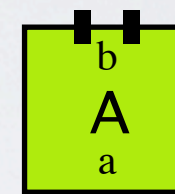


Simulation with tiles

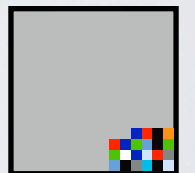
Simulator
tile set



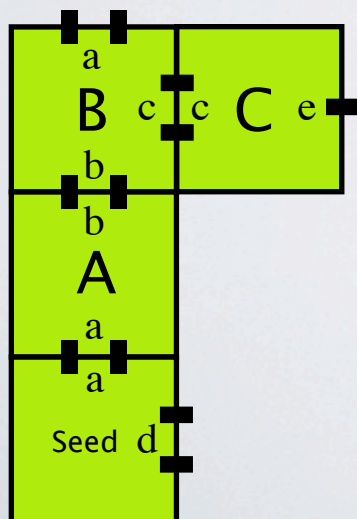
Simulated
tile



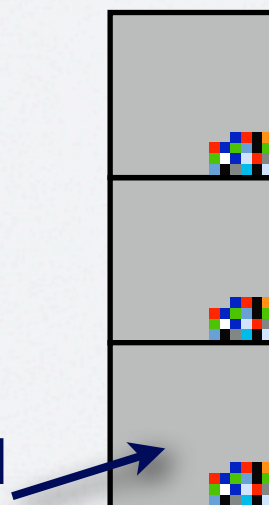
Simulator tiles



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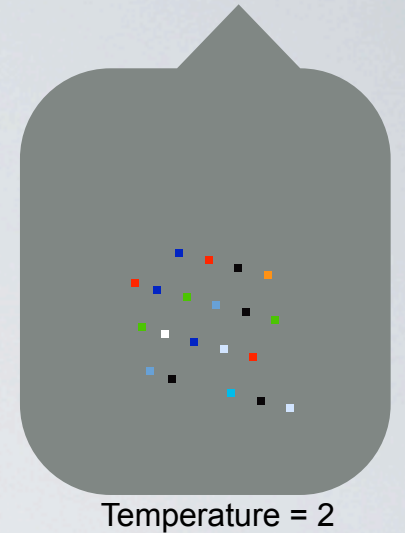


Preassembled
seed structure

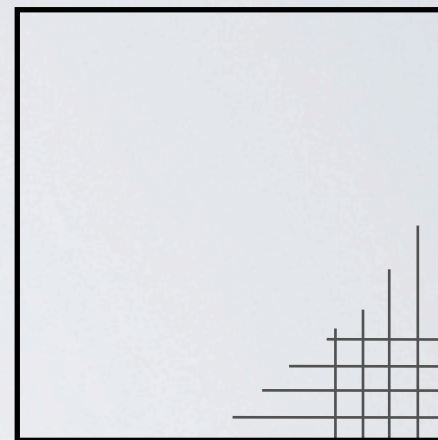
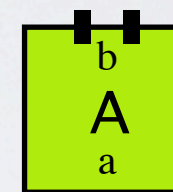


Simulation with tiles

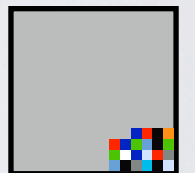
Simulator
tile set



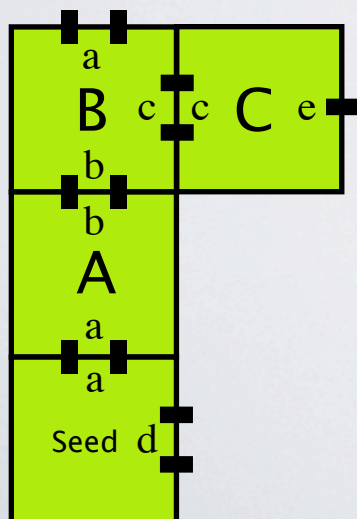
Simulated
tile



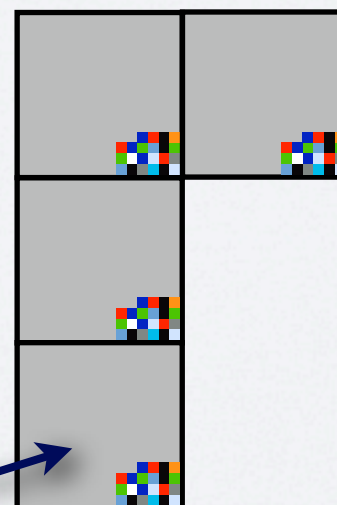
Simulator tiles



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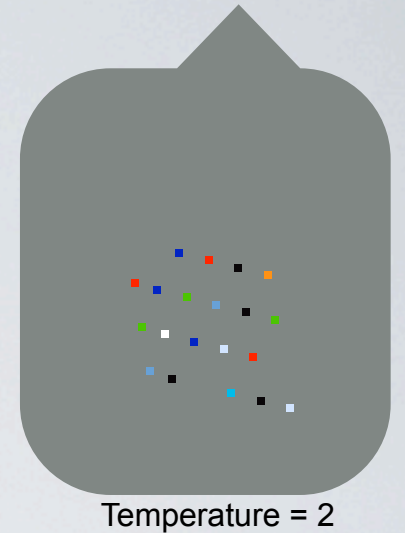


Preassembled
seed structure

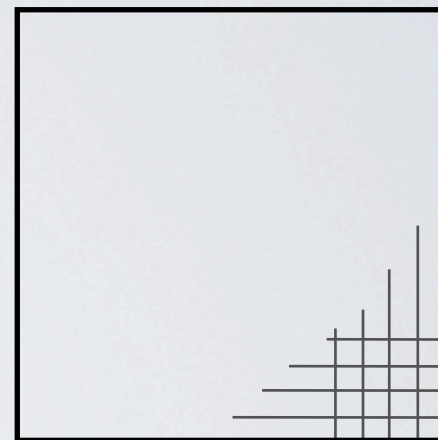
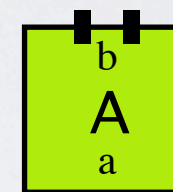


Simulation with tiles

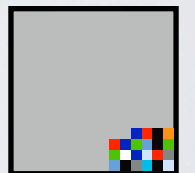
Simulator
tile set



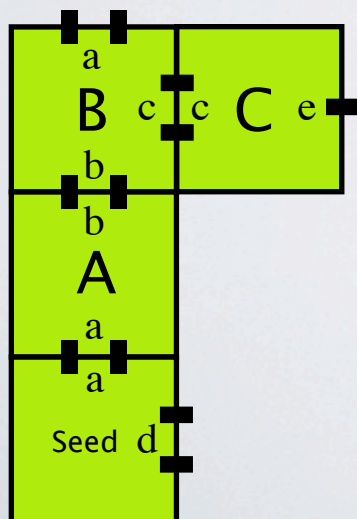
Simulated
tile



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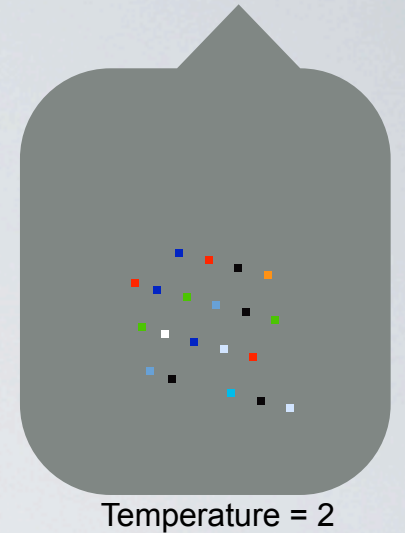


Preassembled
seed structure

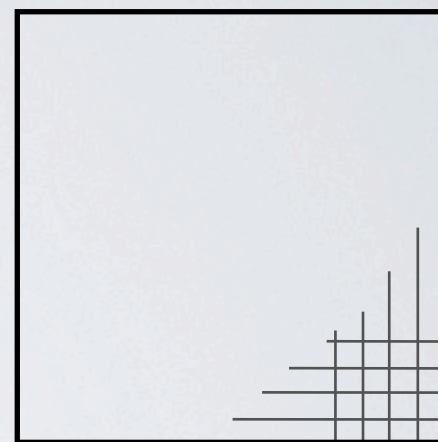
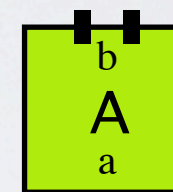


Simulation with tiles

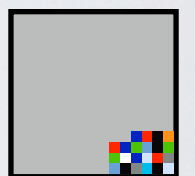
Simulator
tile set



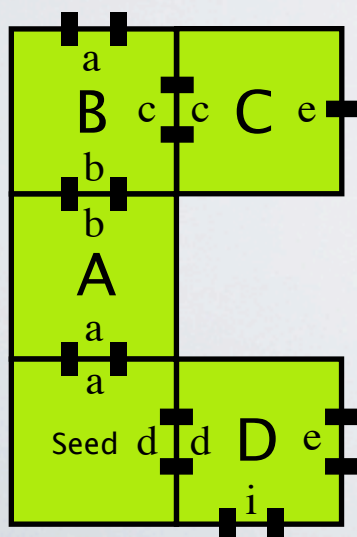
Simulated
tile



Simulator tiles



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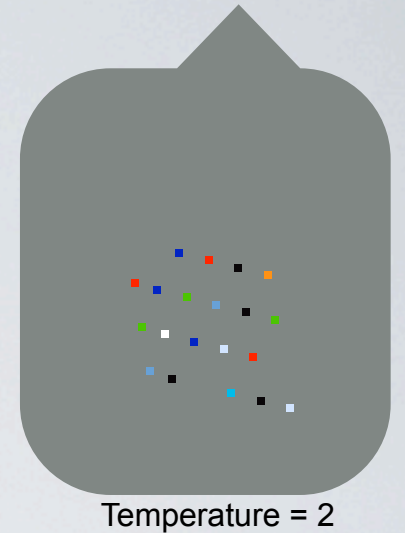


Preassembled
seed structure

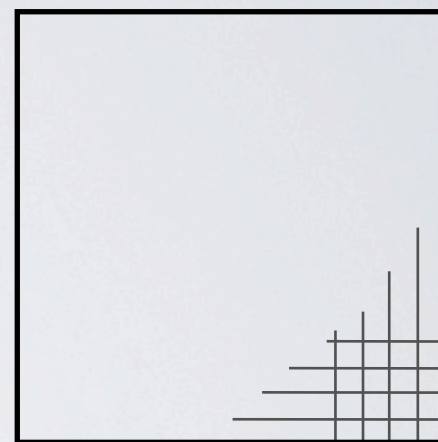
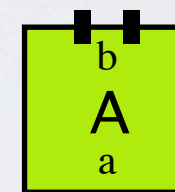


Simulation with tiles

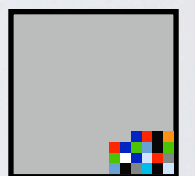
Simulator
tile set



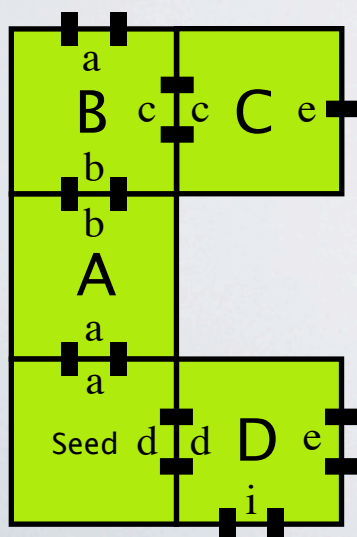
Simulated
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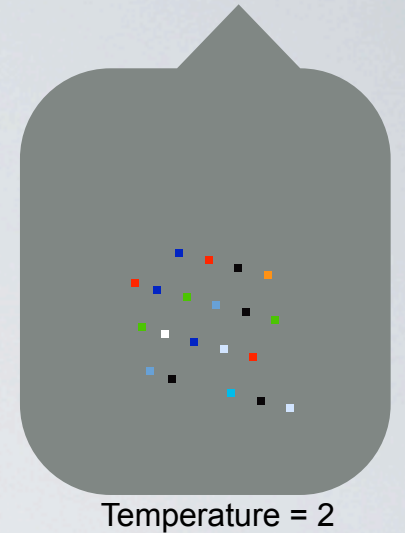


Preassembled
seed structure

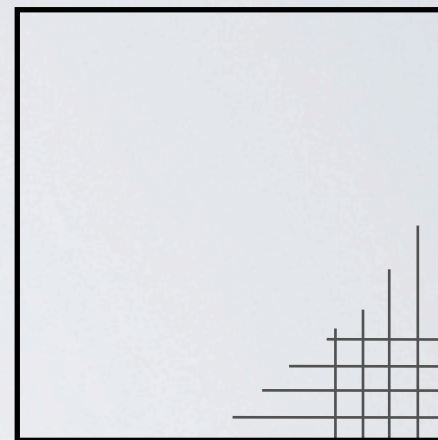
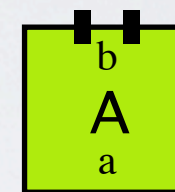


Simulation with tiles

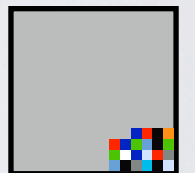
Simulator
tile set



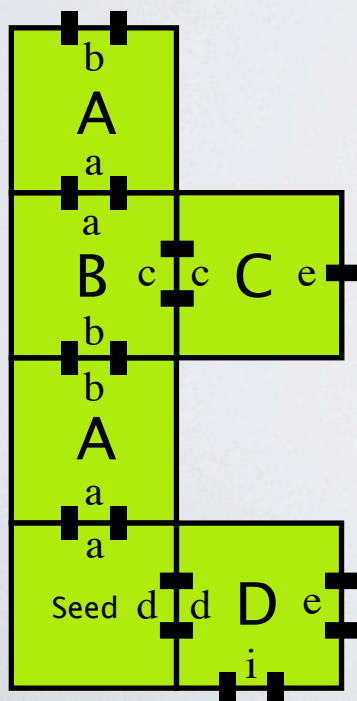
Simulated
tile



Simulator tiles



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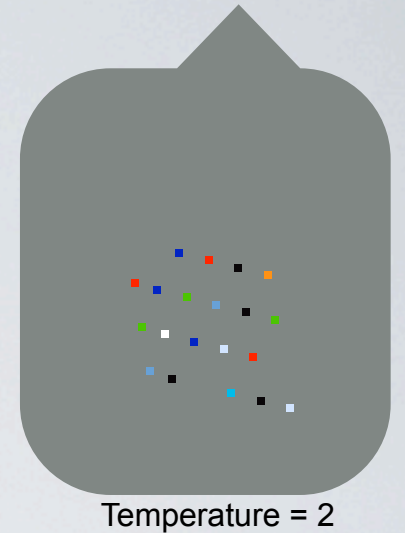


Preassembled
seed structure

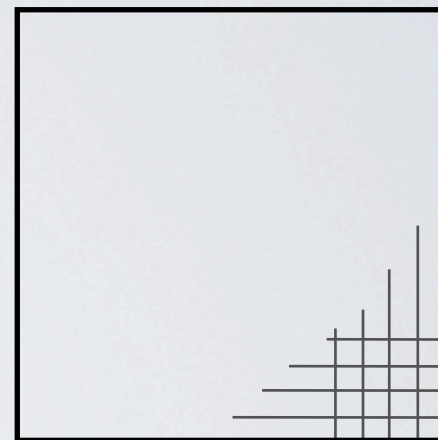
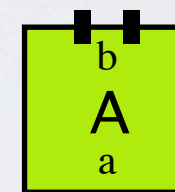


Simulation with tiles

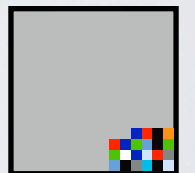
Simulator
tile set



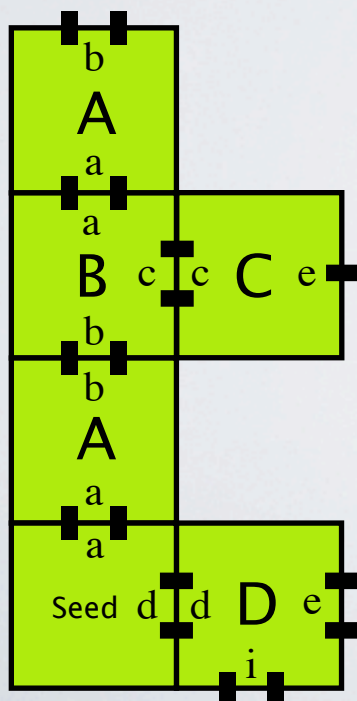
Simulated
tile



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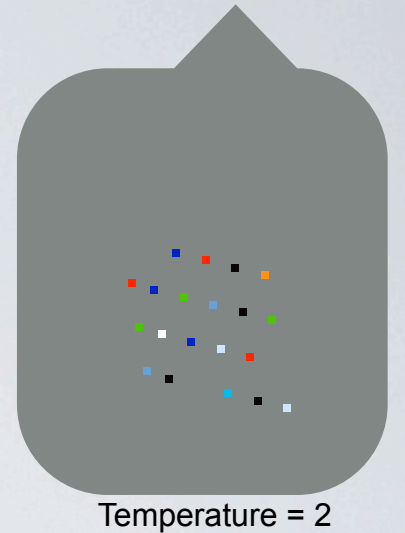


Preassembled
seed structure

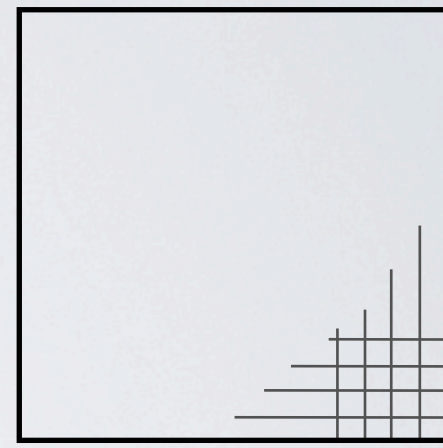
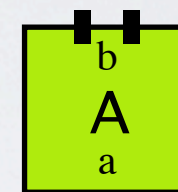


Simulation with tiles

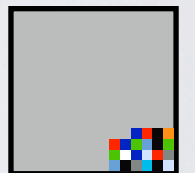
Simulator
tile set



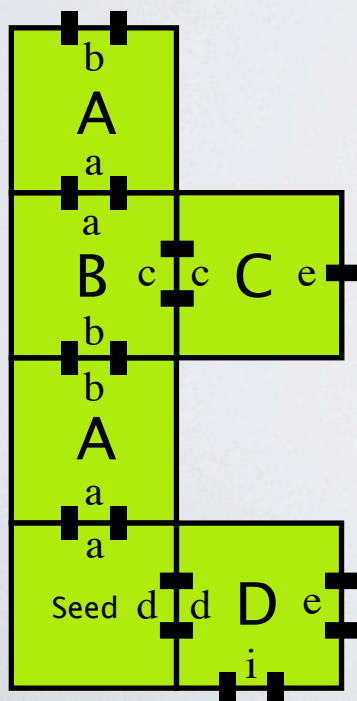
Simulated
tile



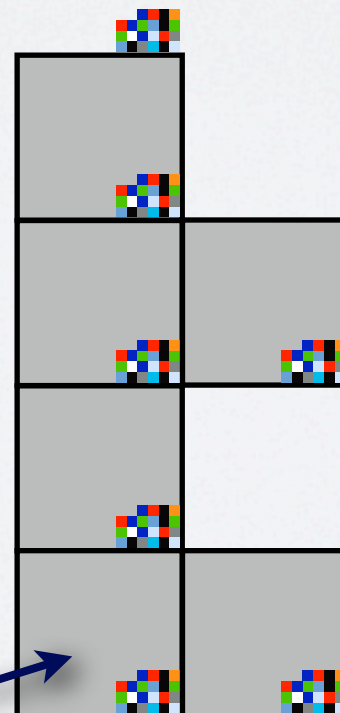
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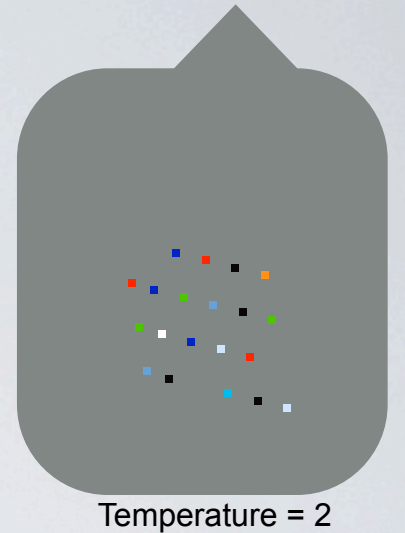


Preassembled
seed structure

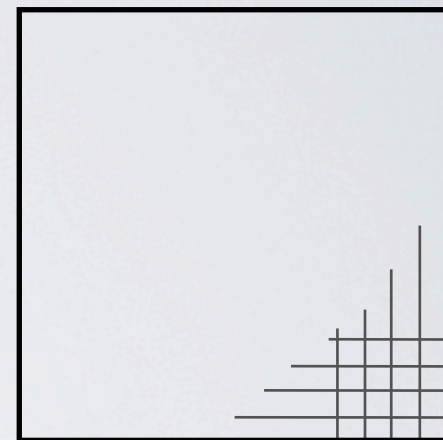
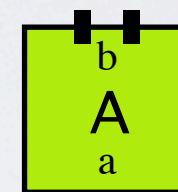


Simulation with tiles

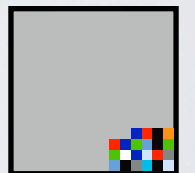
Simulator
tile set



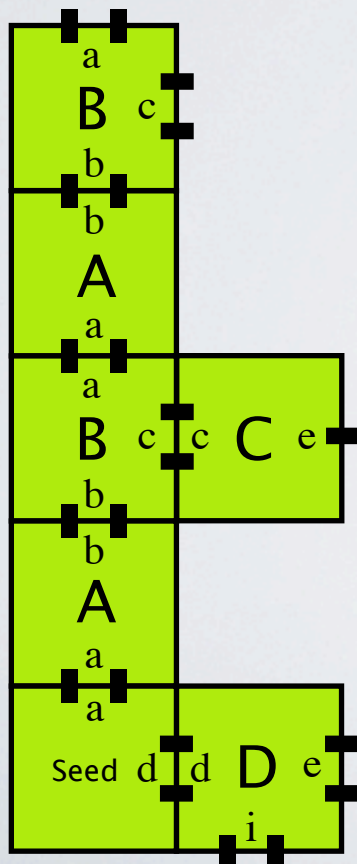
Simulated
tile



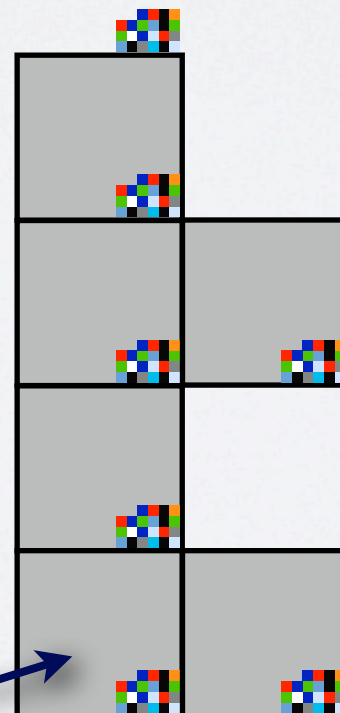
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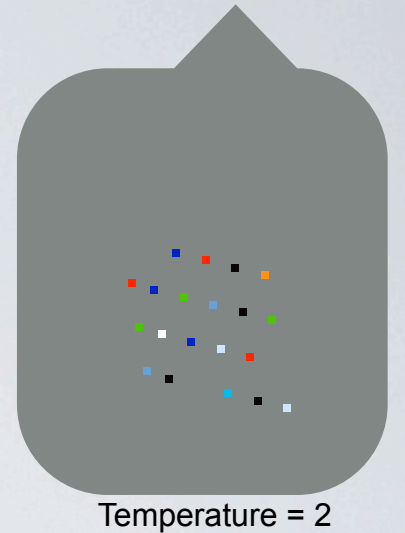


Preassembled
seed structure

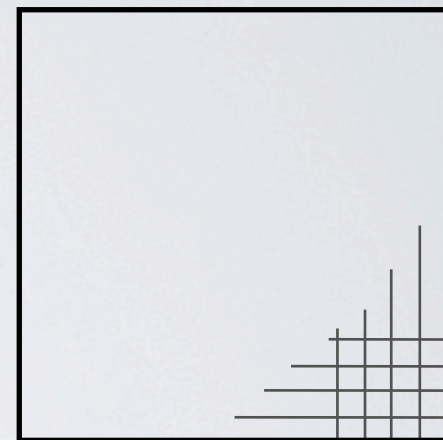
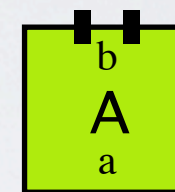


Simulation with tiles

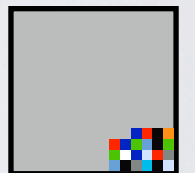
Simulator
tile set



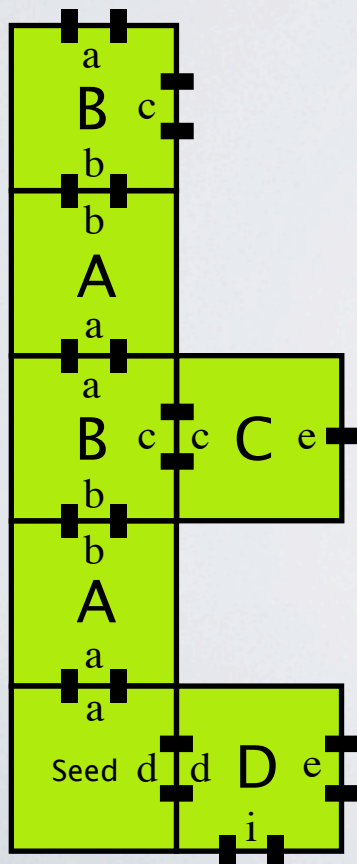
Simulated
tile



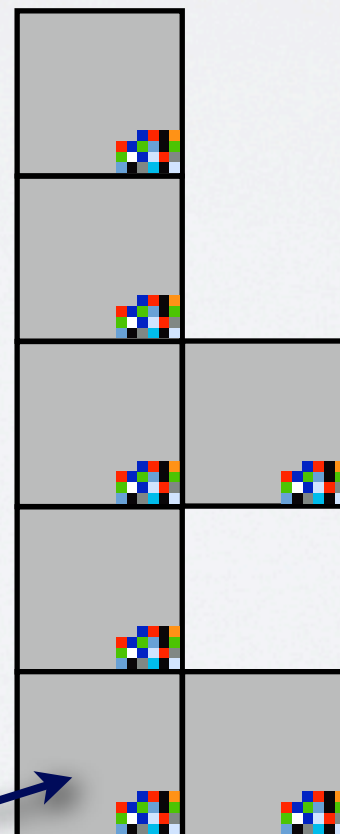
Simulator tiles



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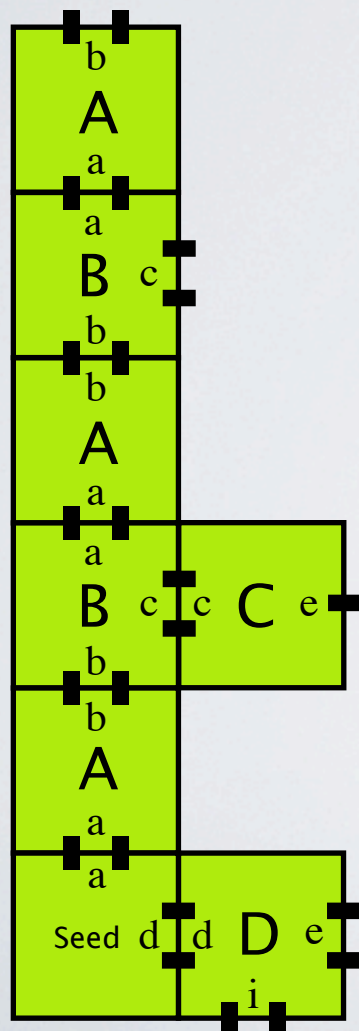
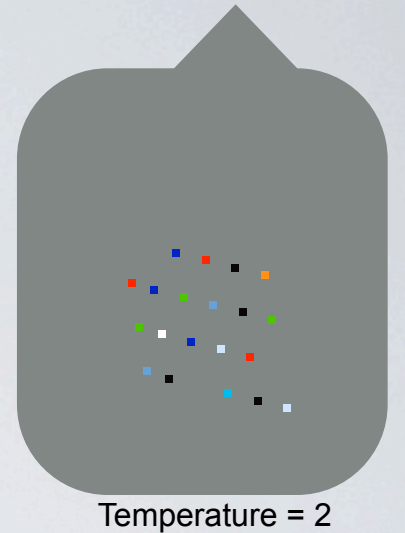


Preassembled
seed structure

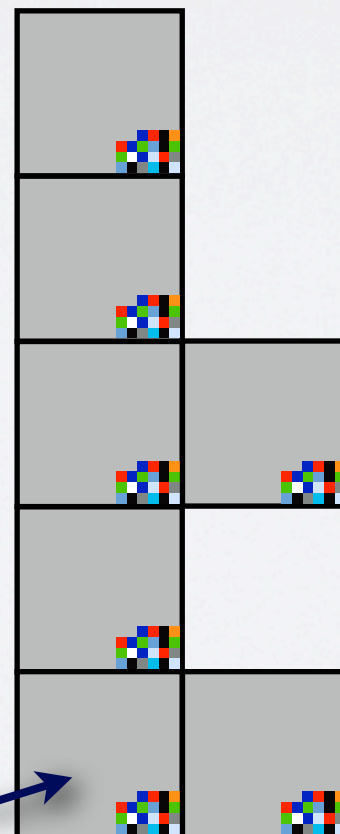


Simulation with tiles

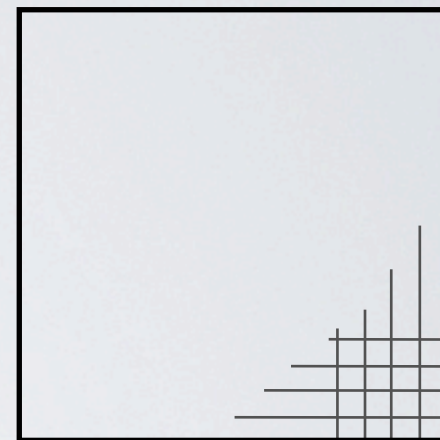
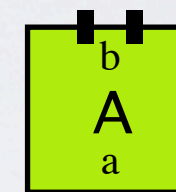
Simulator
tile set



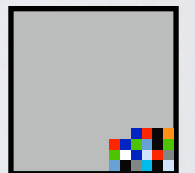
Preassembled
seed structure



Simulated
tile



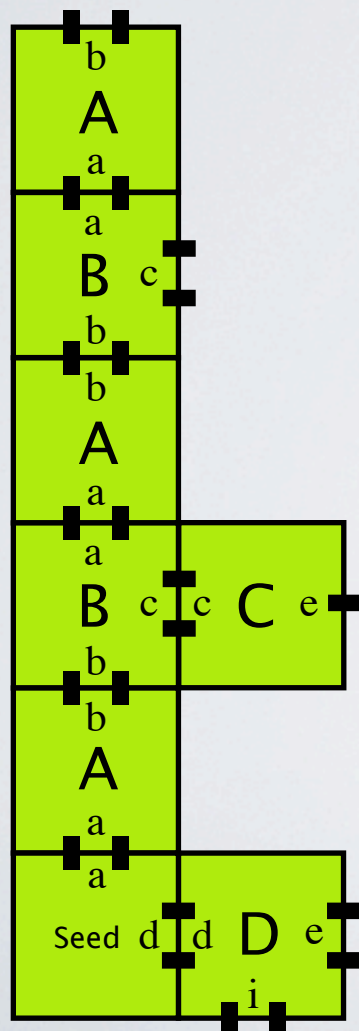
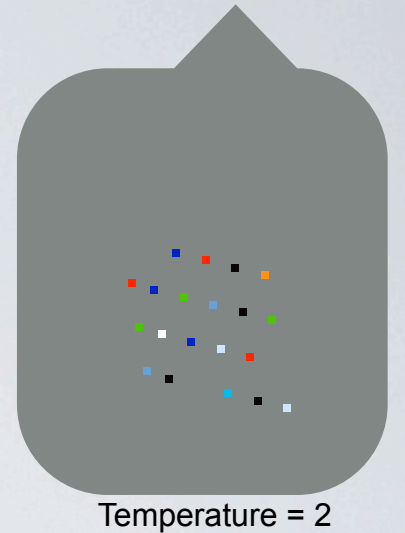
Simulator tiles



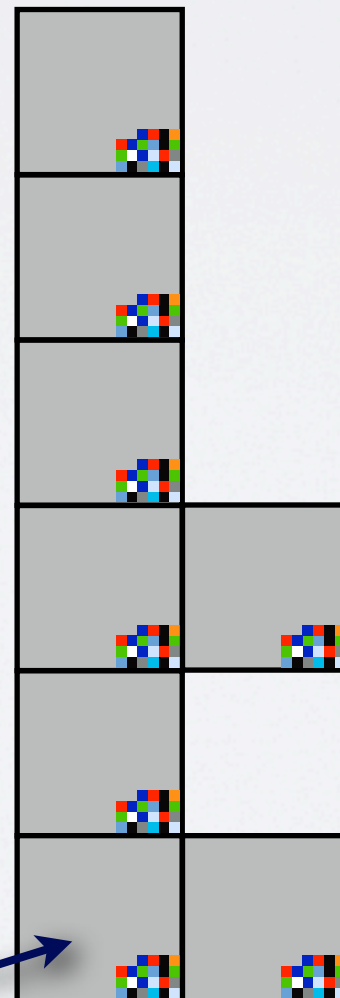
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Simulation with tiles

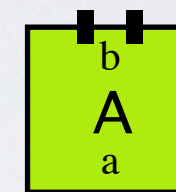
Simulator
tile set



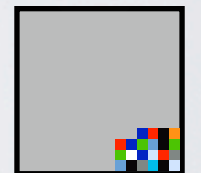
Preassembled
seed structure



Simulated
tile



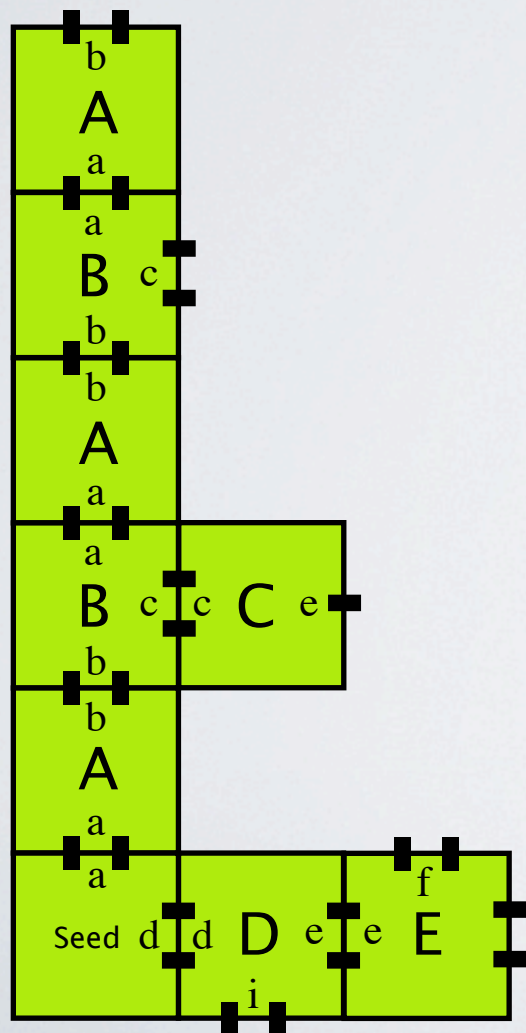
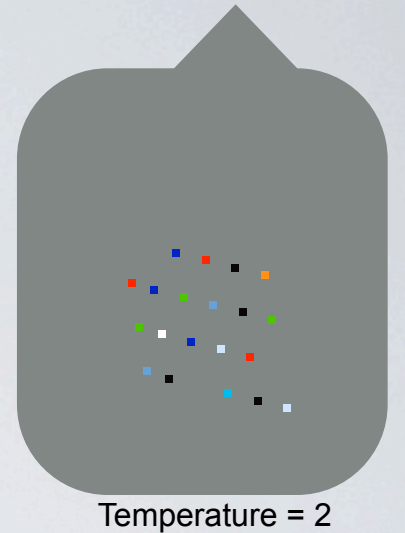
Simulator tiles



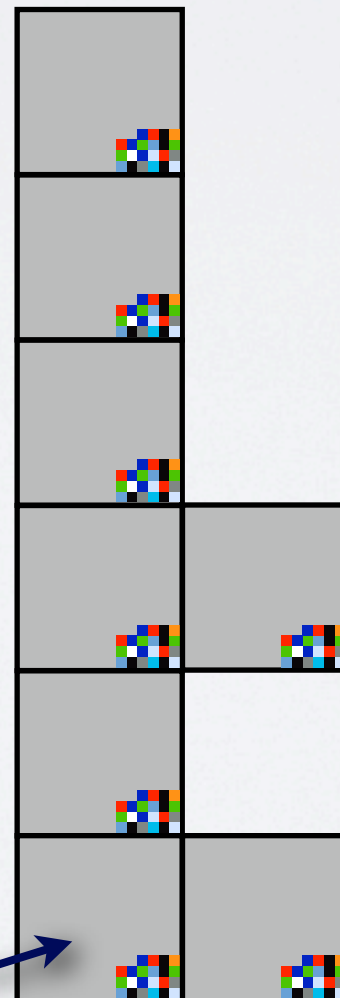
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

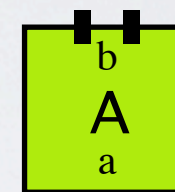
Simulator
tile set



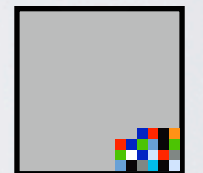
Preassembled
seed structure



Simulated
tile



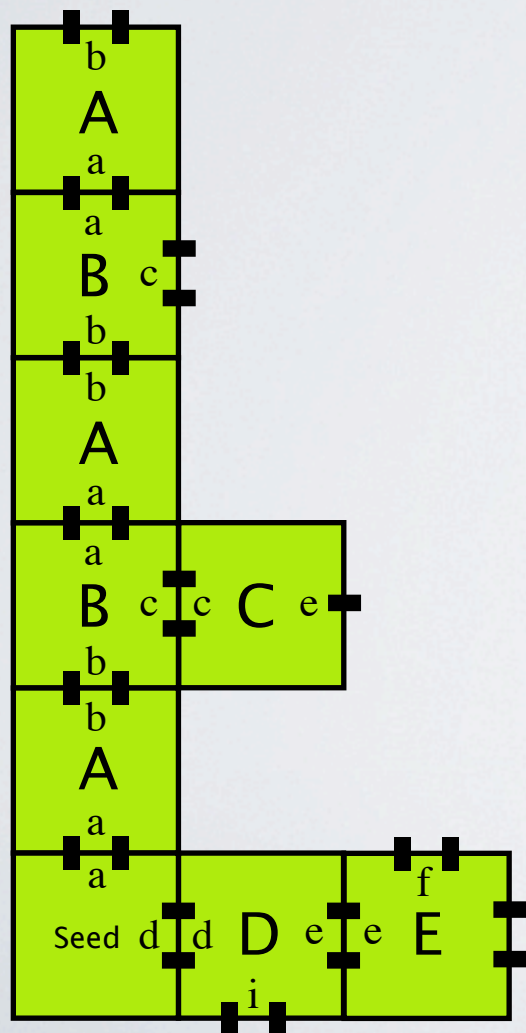
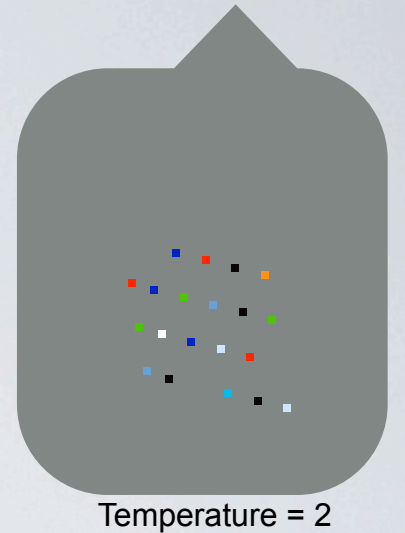
Simulator tiles



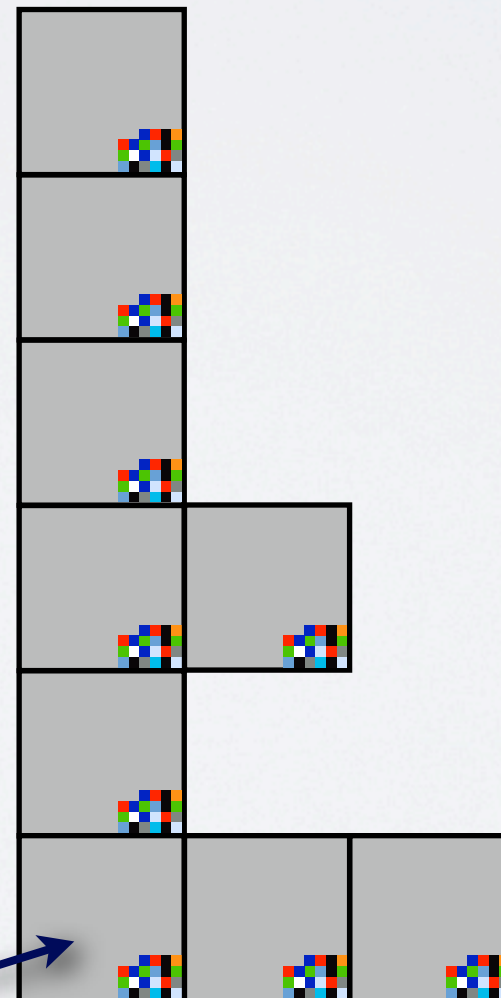
- Tiles are simulated by supertiles
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Simulation with tiles

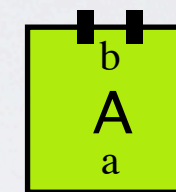
Simulator
tile set



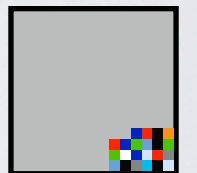
Preassembled
seed structure



Simulated
tile



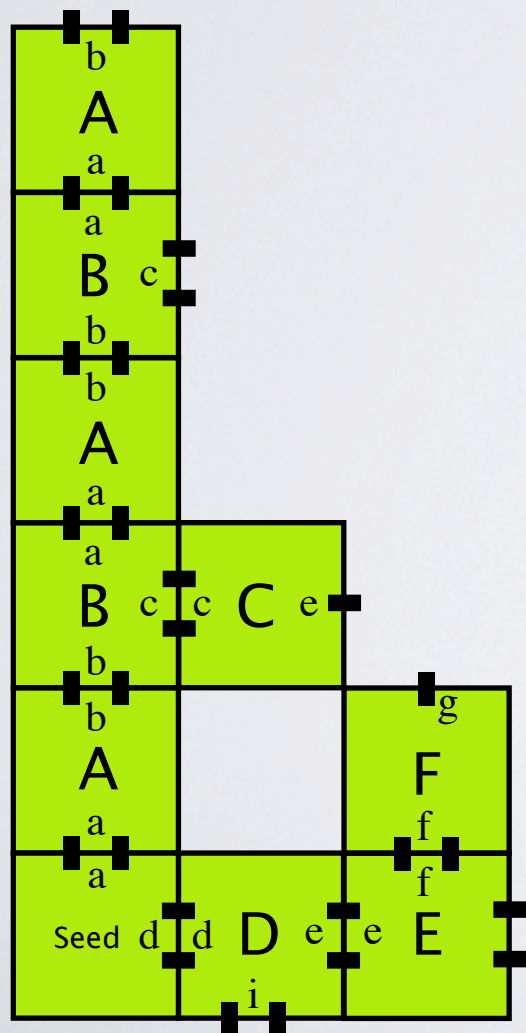
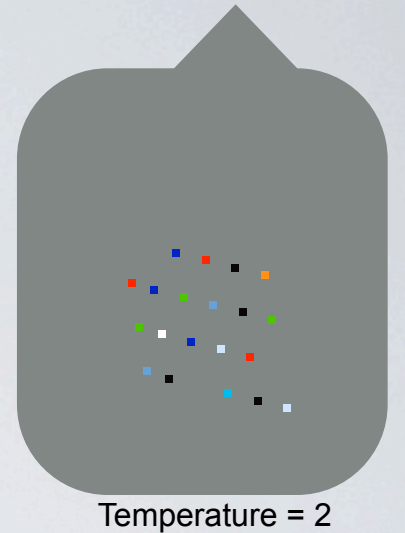
Simulator tiles



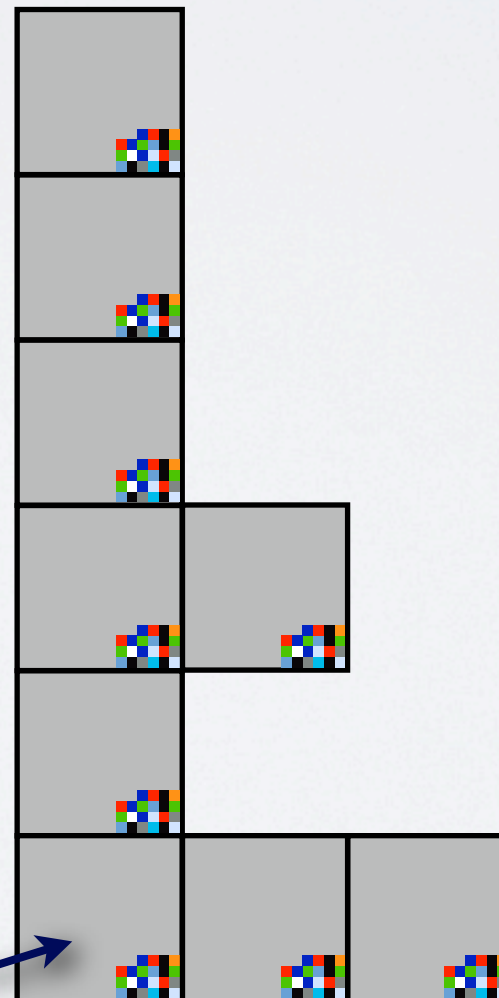
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

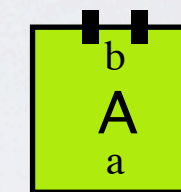
Simulator
tile set



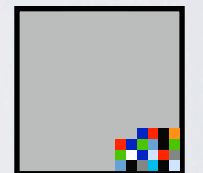
Preassembled
seed structure



Simulated
tile



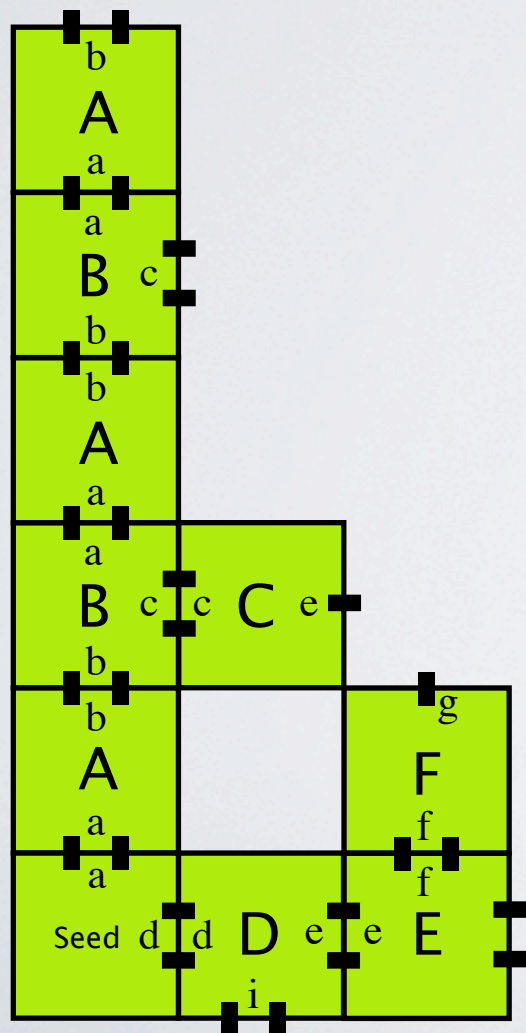
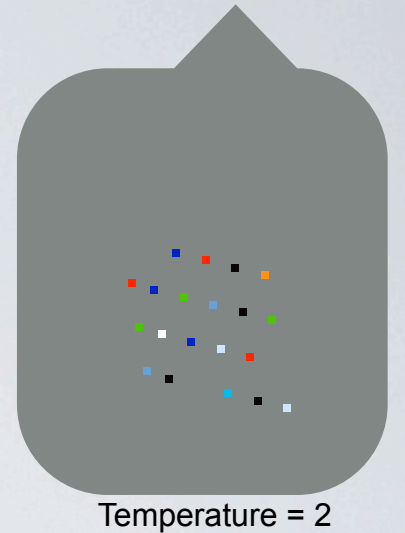
Simulator tiles



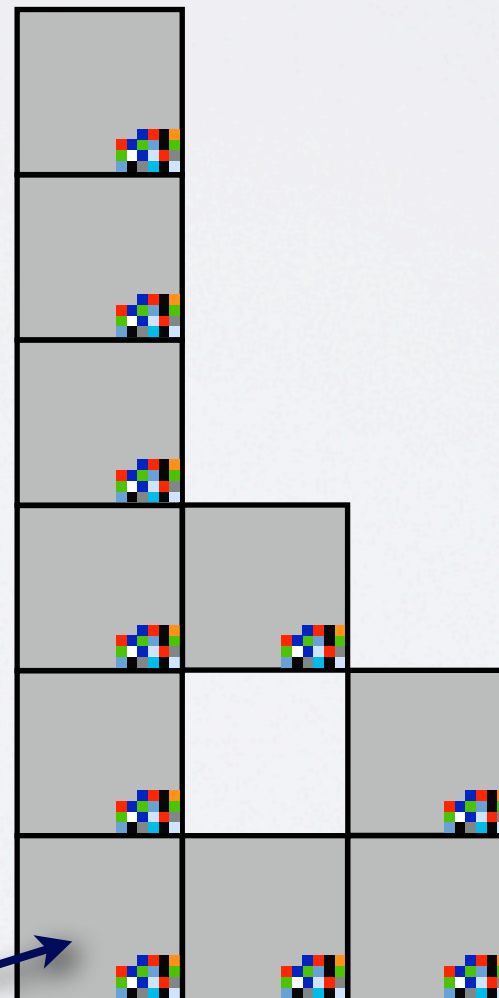
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

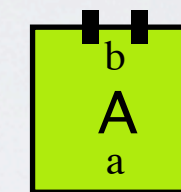
Simulator
tile set



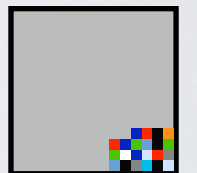
Preassembled
seed structure



Simulated
tile



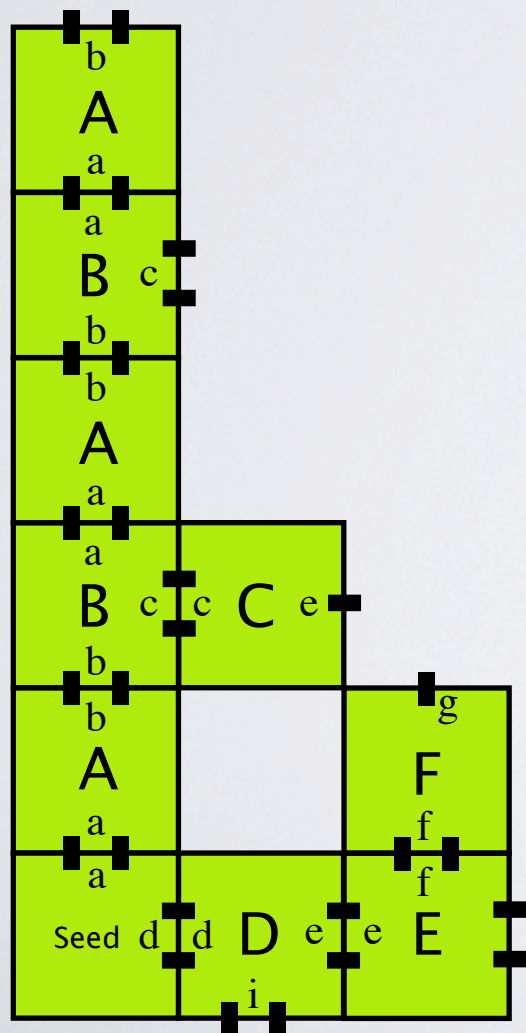
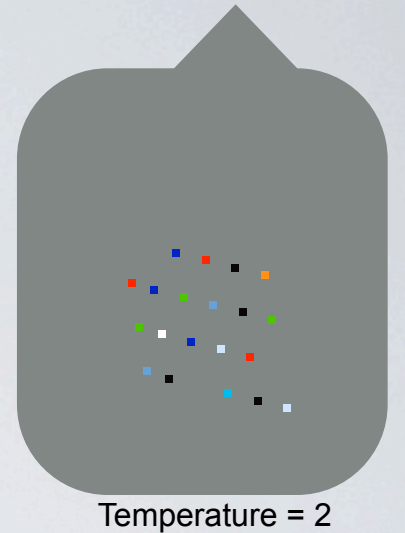
Simulator tiles



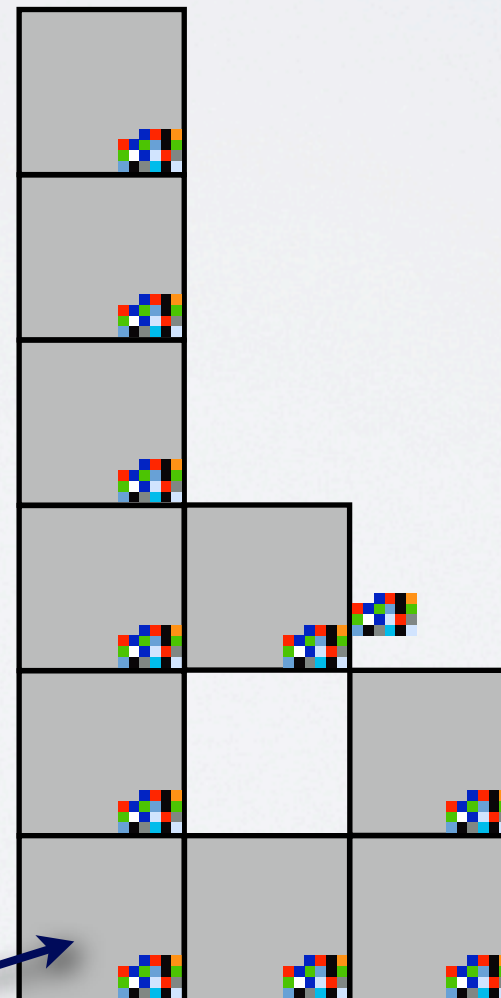
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

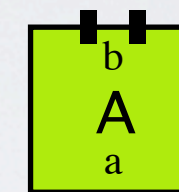
Simulator
tile set



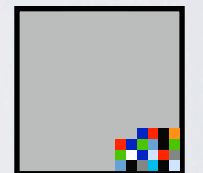
Preassembled
seed structure



Simulated
tile



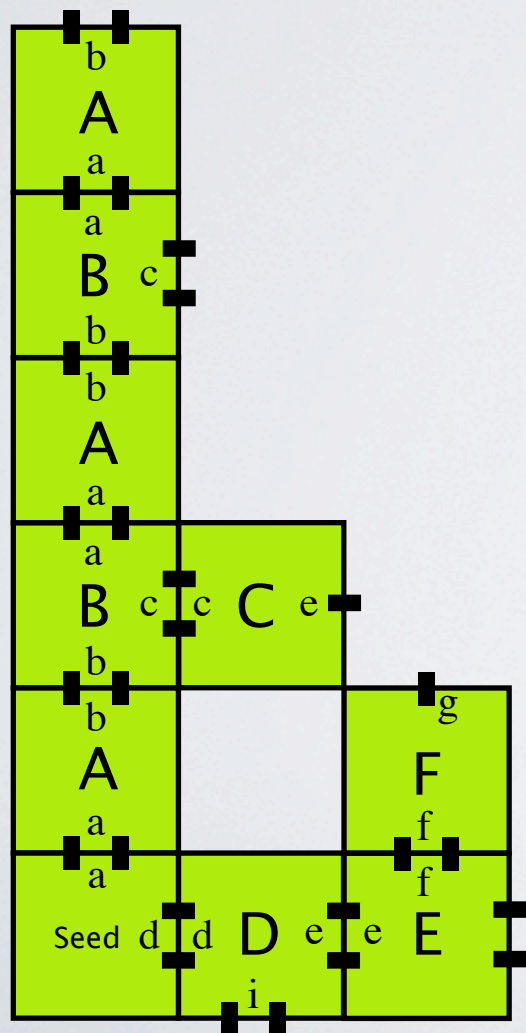
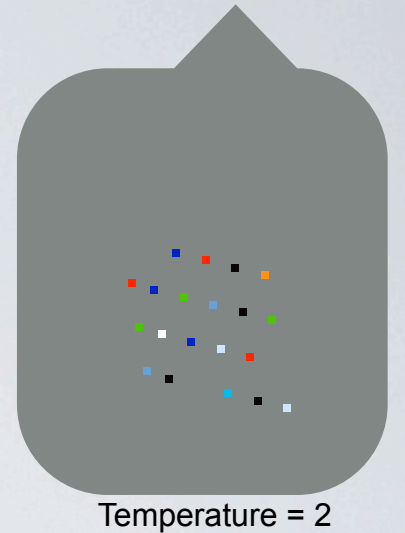
Simulator tiles



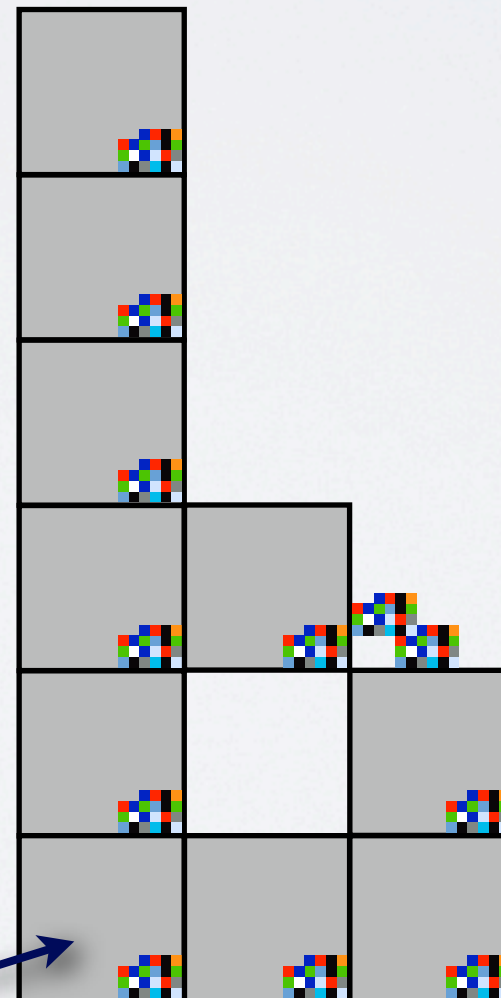
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

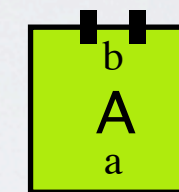
Simulator
tile set



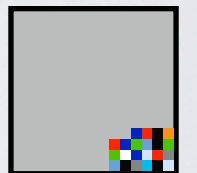
Preassembled
seed structure



Simulated
tile



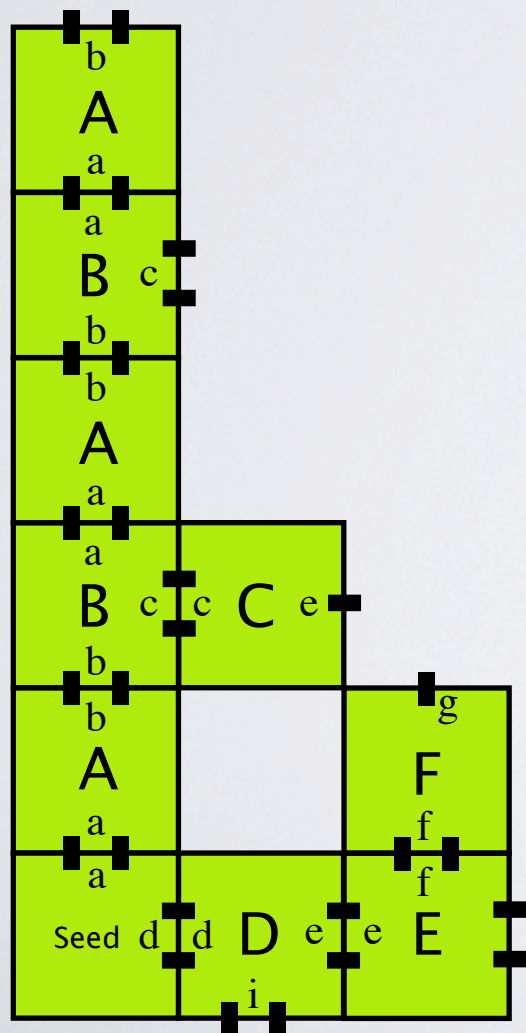
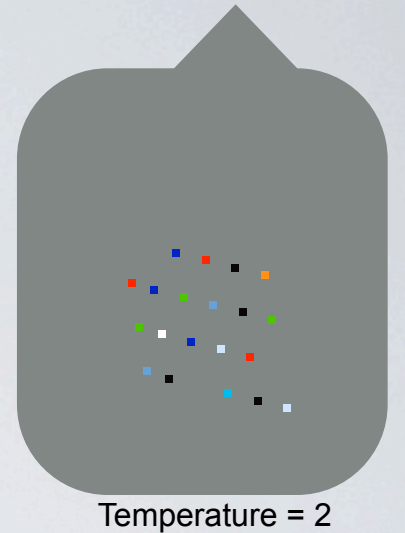
Simulator tiles



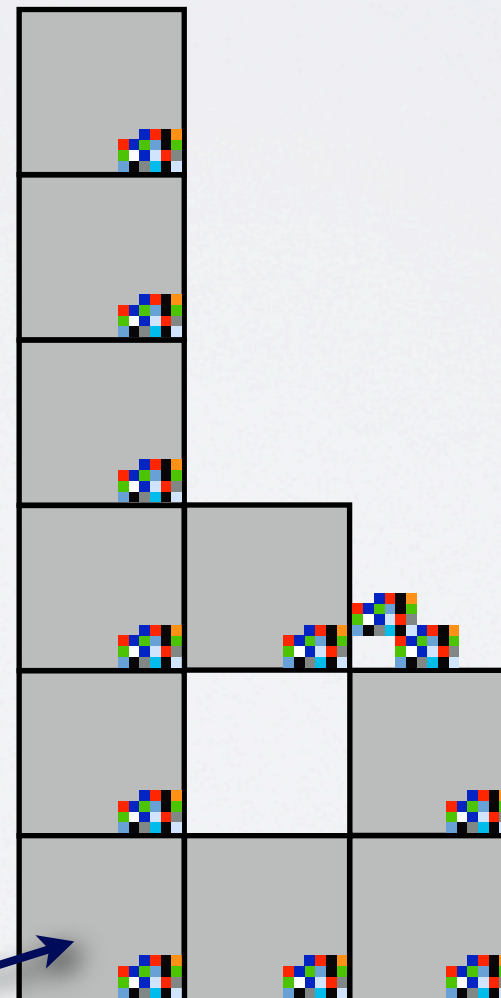
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation with tiles

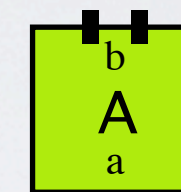
Simulator
tile set



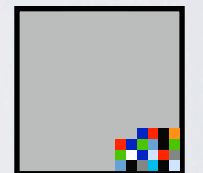
Preassembled
seed structure



Simulated
tile



Simulator tiles



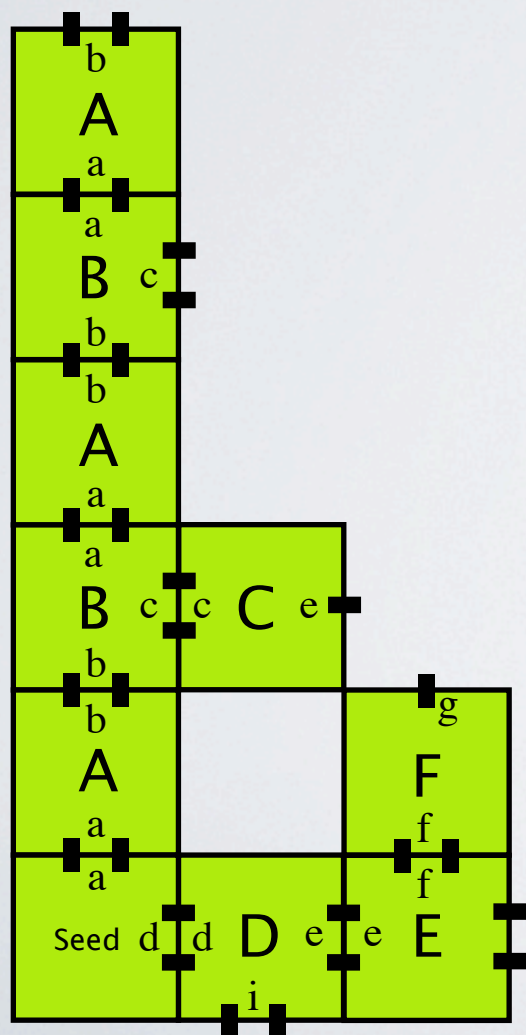
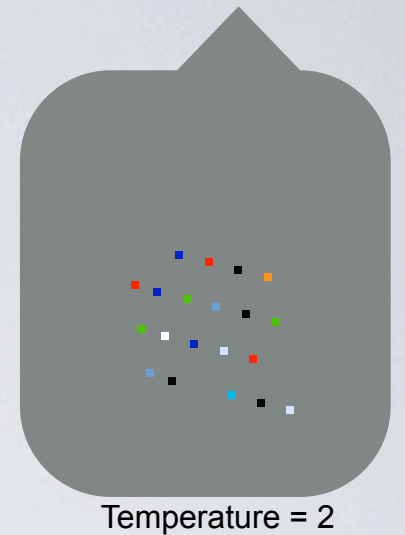
- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

etc.

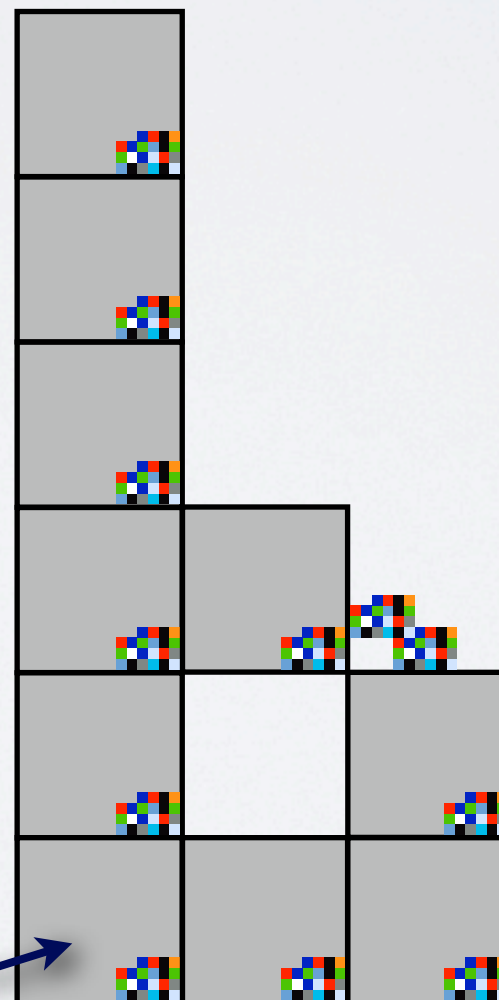
Simulation with tiles

Ignoring $m \times m$ scaling, dynamics are equivalent in the simulated system and simulator

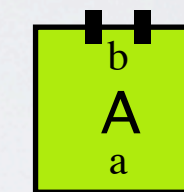
Simulator
tile set



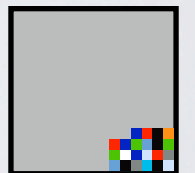
Preassembled
seed structure



Simulated
tile



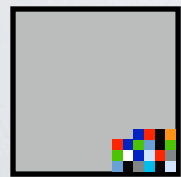
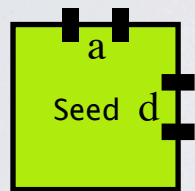
Simulator tiles



- Tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

etc.

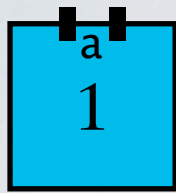
seed supertile



Preassembled
seed structure

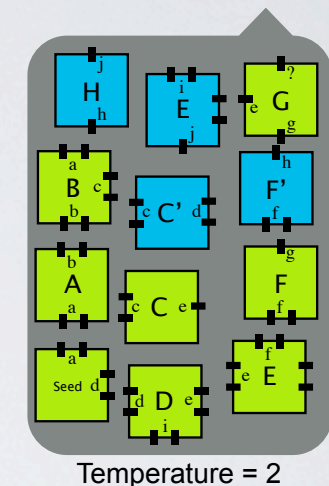


Superside



Encoded **glue** of
this superside
(e.g. “a”)

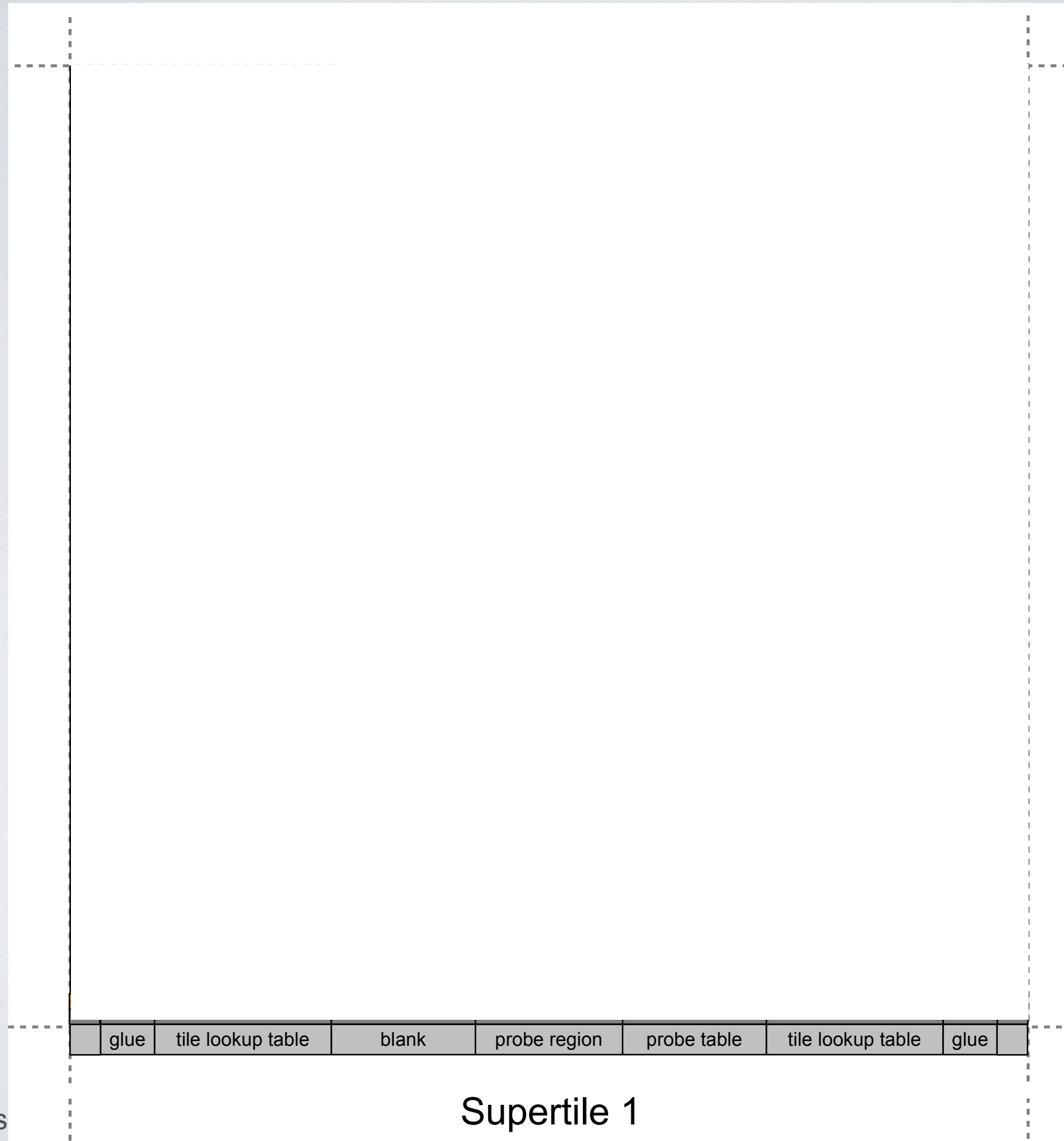
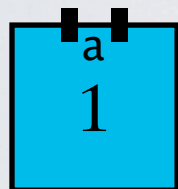
Encoding of the
entire simulated **tile**
assembly system



frame	glue	tile lookup table	blank	probe region	probe table	tile lookup table	glue	frame
4	$O(\log T)$	$O(T ^4 \log T)$	$O(T ^2)$	$O(T ^2)$	$O(T ^2)$	$O(T ^4 \log T)$	$O(\log T)$	4

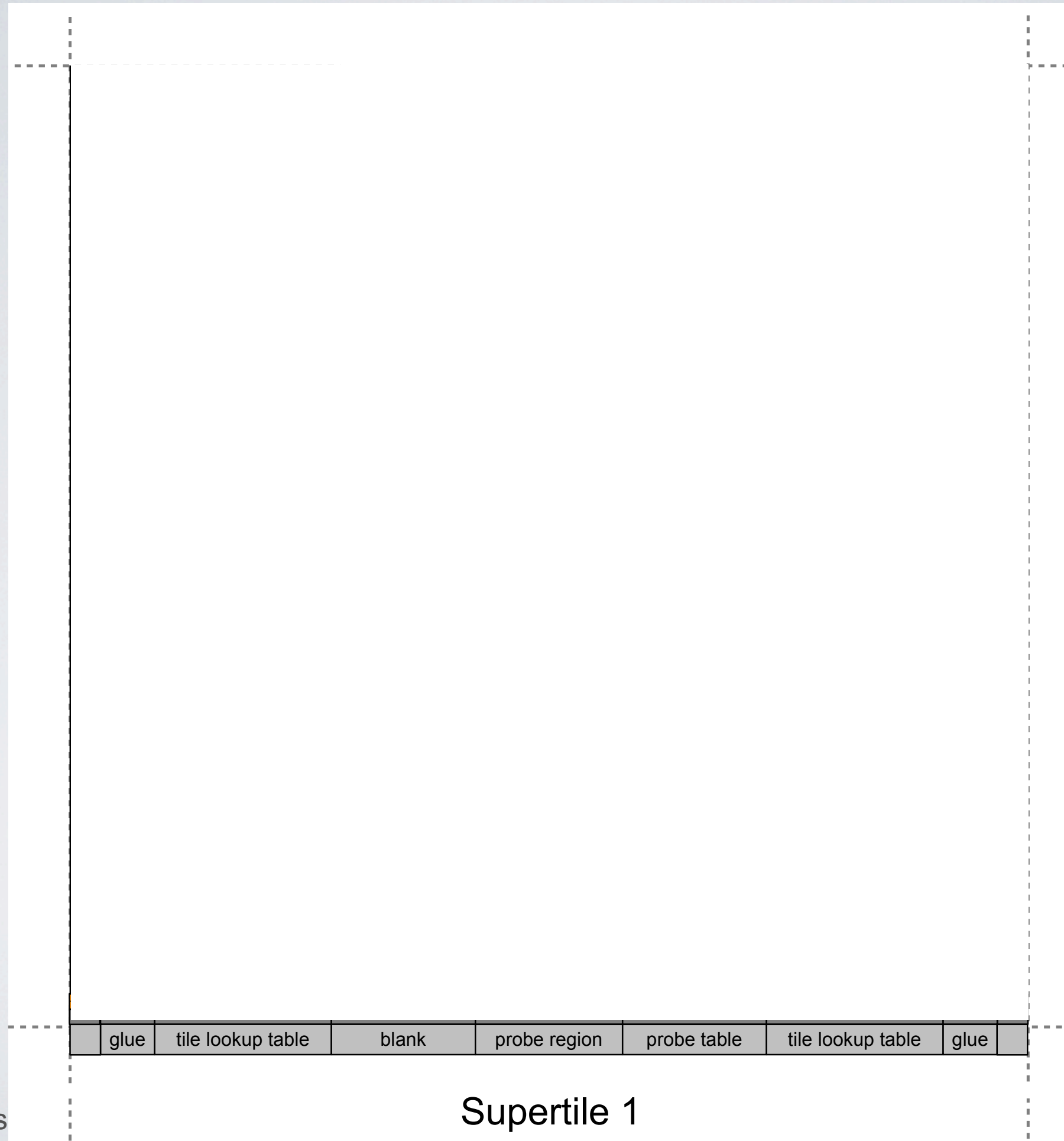
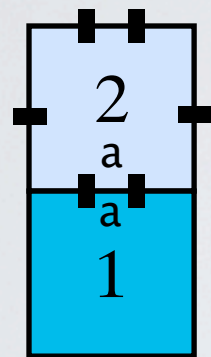
$|T|$ is number of tiles in the simulated tileset T .

One-sided binding with a single strength- τ south supertile



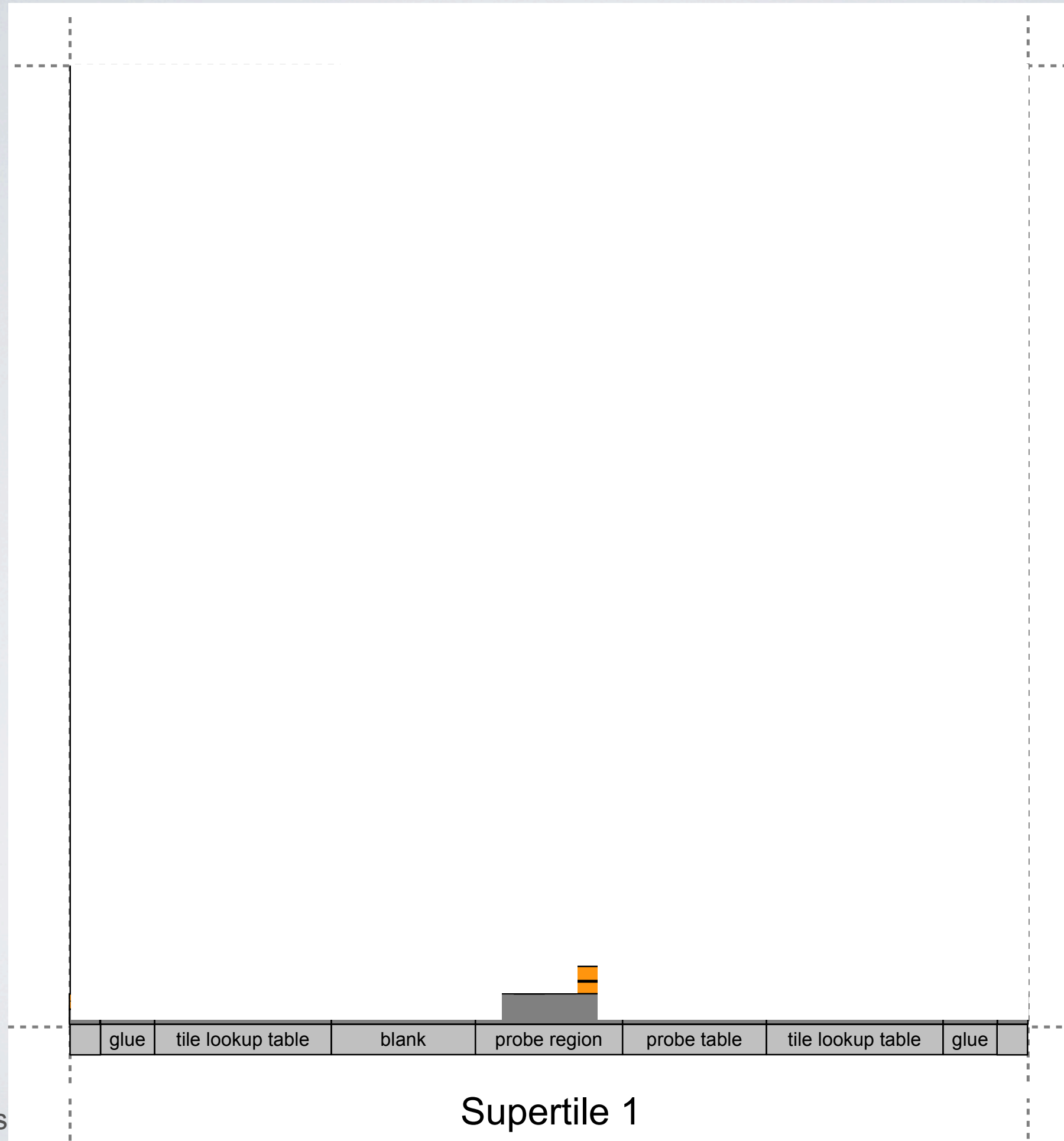
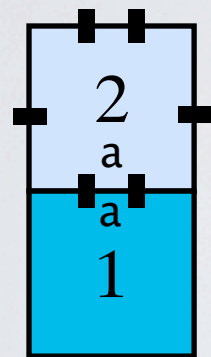
Supertile 1

One-sided binding with a single strength- τ south superside



Supertile 1

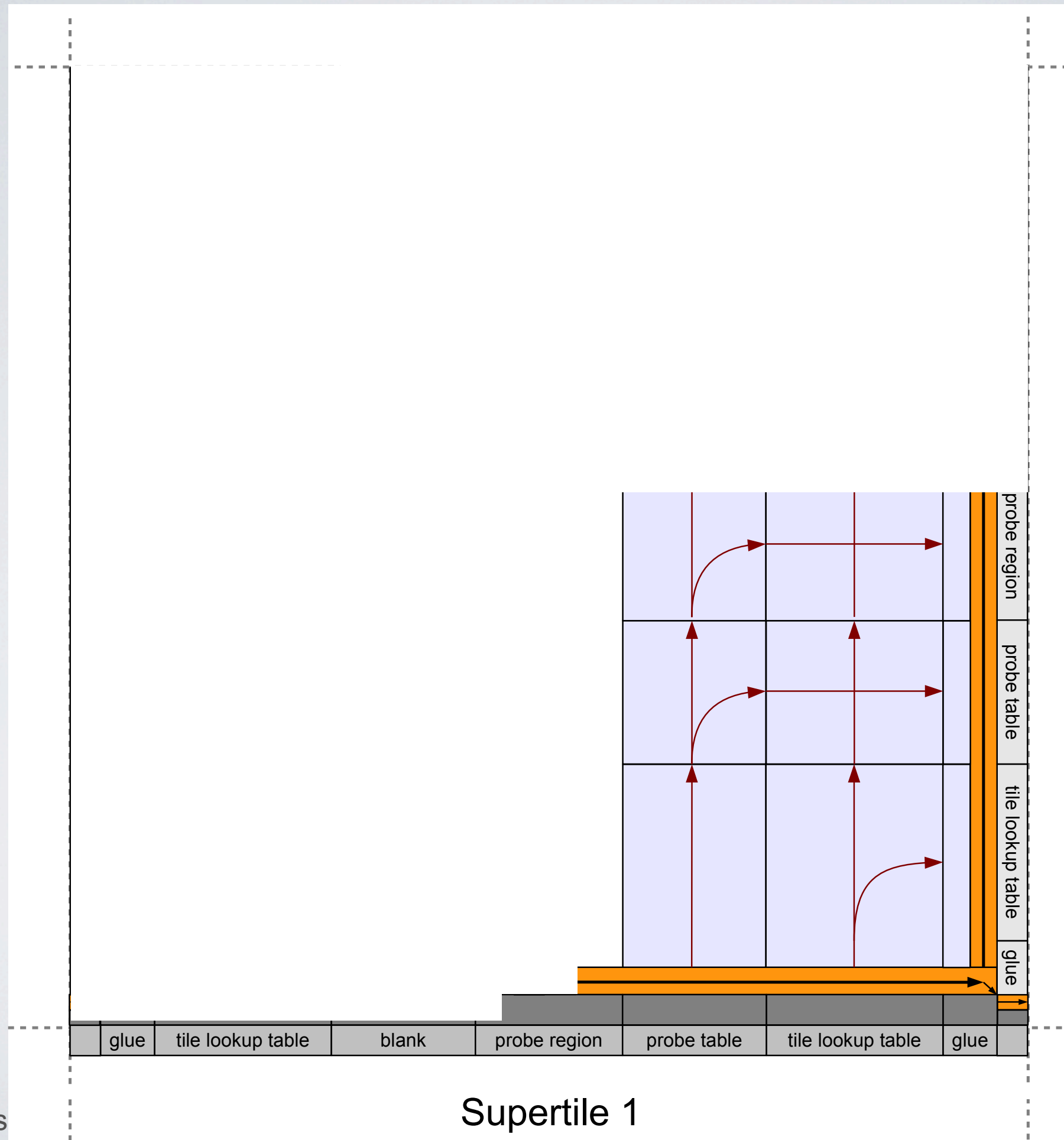
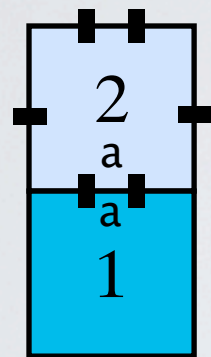
One-sided binding with a single strength- τ south superside



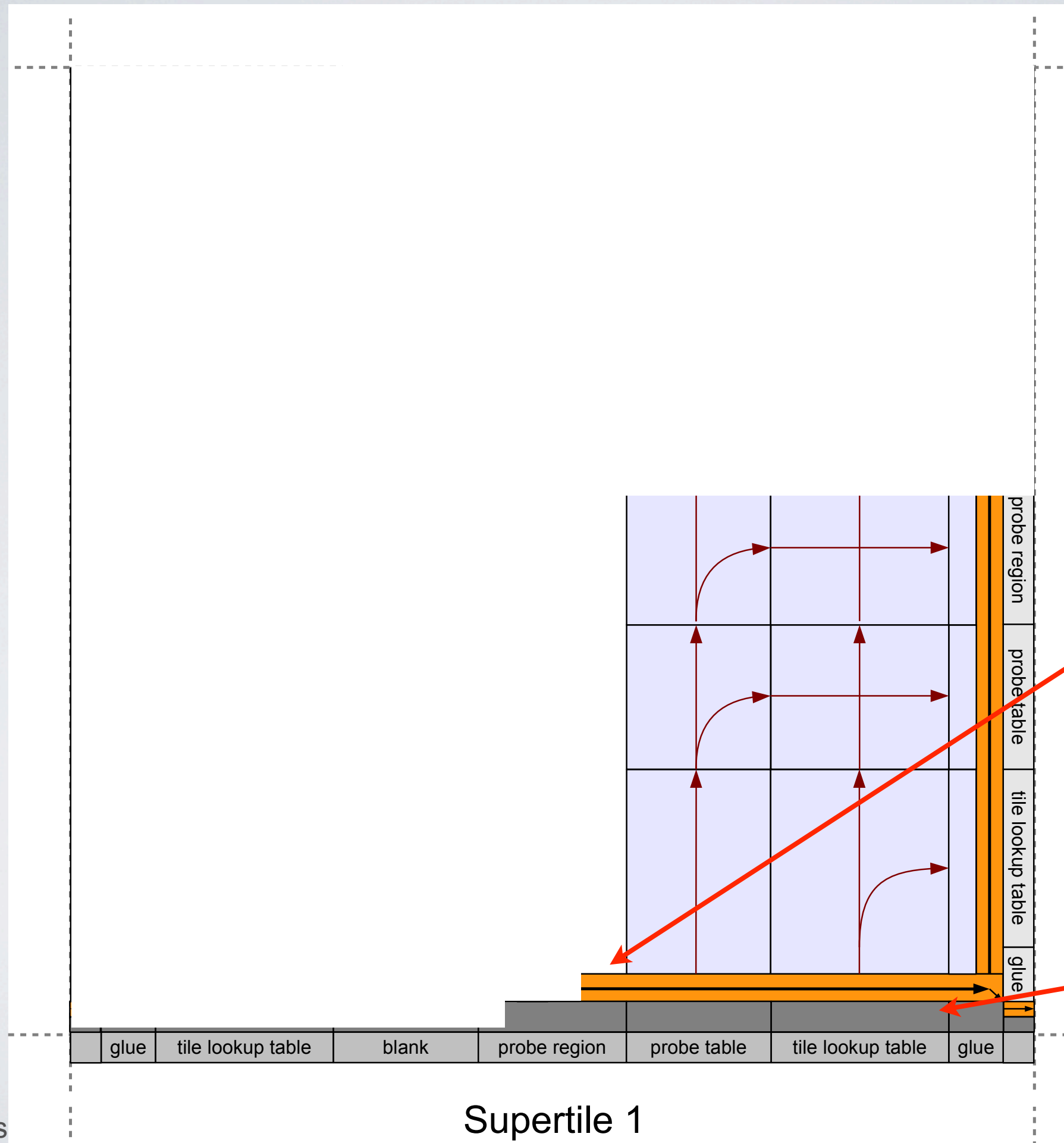
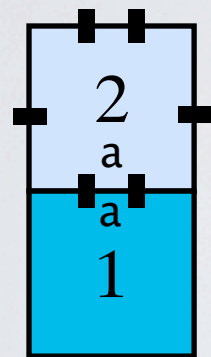
	glue	tile lookup table	blank	probe region	probe table	tile lookup table	glue	
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Supertile 1

One-sided binding with a single strength- τ south sup erside



One-sided binding with a single strength- τ south superside

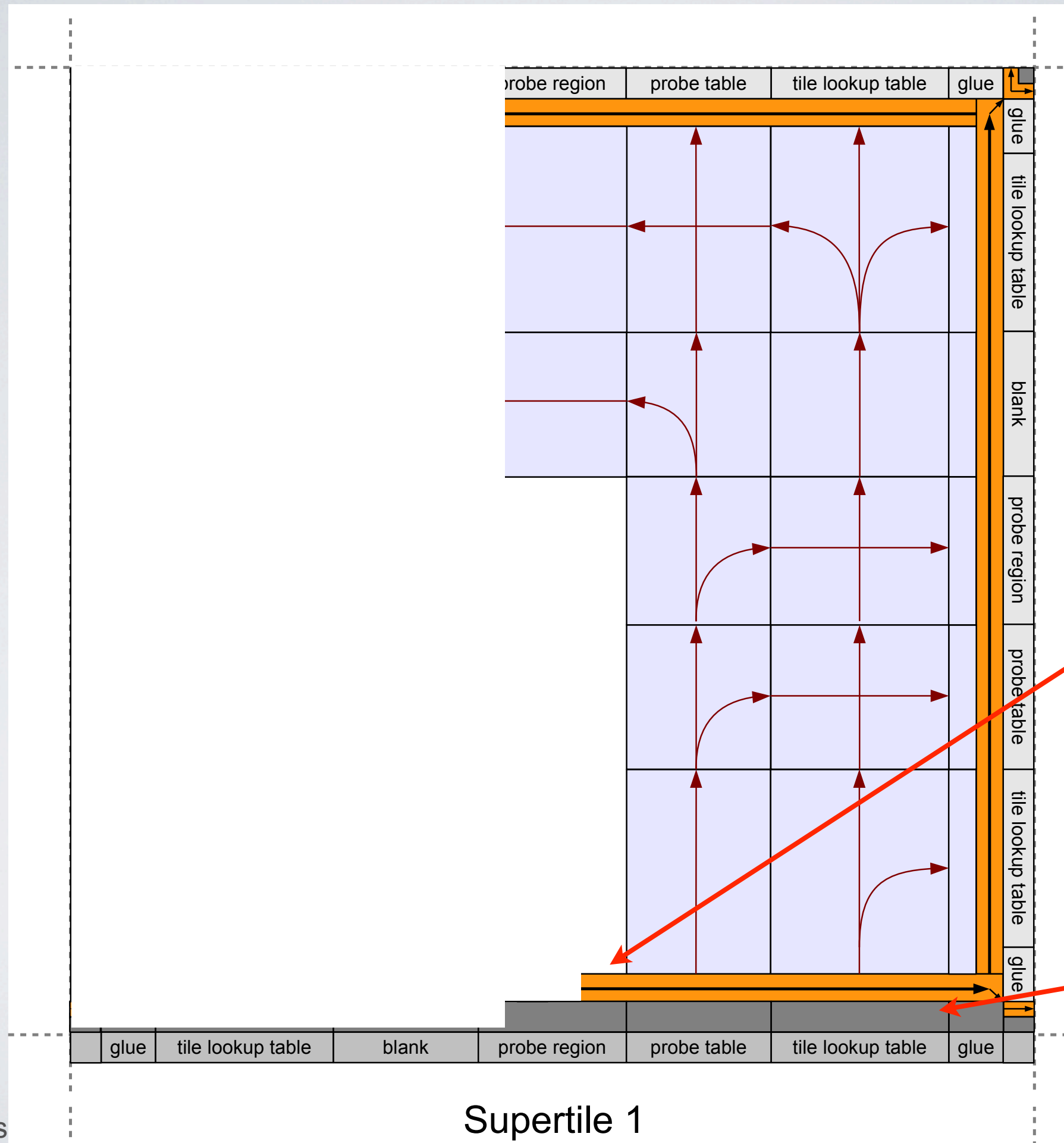
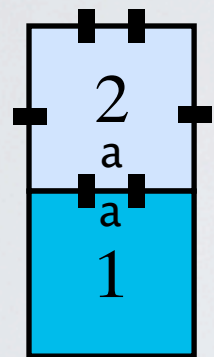


crawler encodes
glue of south
superside

“Genome” is
copied

“Genome” is
read

One-sided binding with a single strength- τ south superside



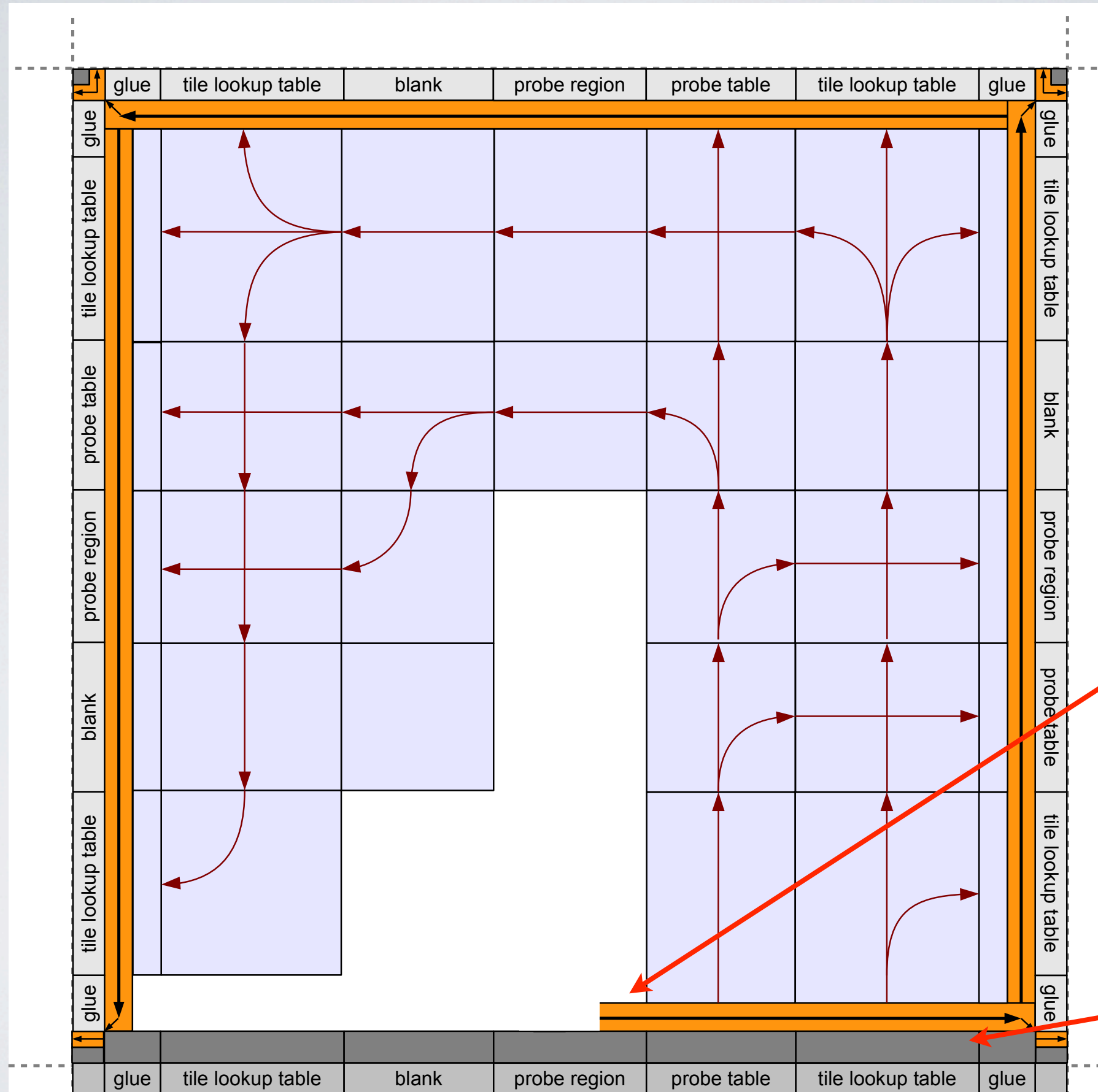
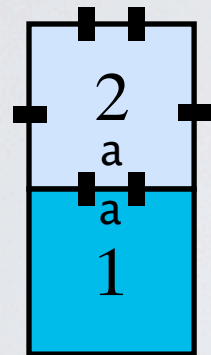
crawler encodes
glue of south
superside

"Genome" is
copied

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read

Supertile 1

One-sided binding with a single strength- τ south superside



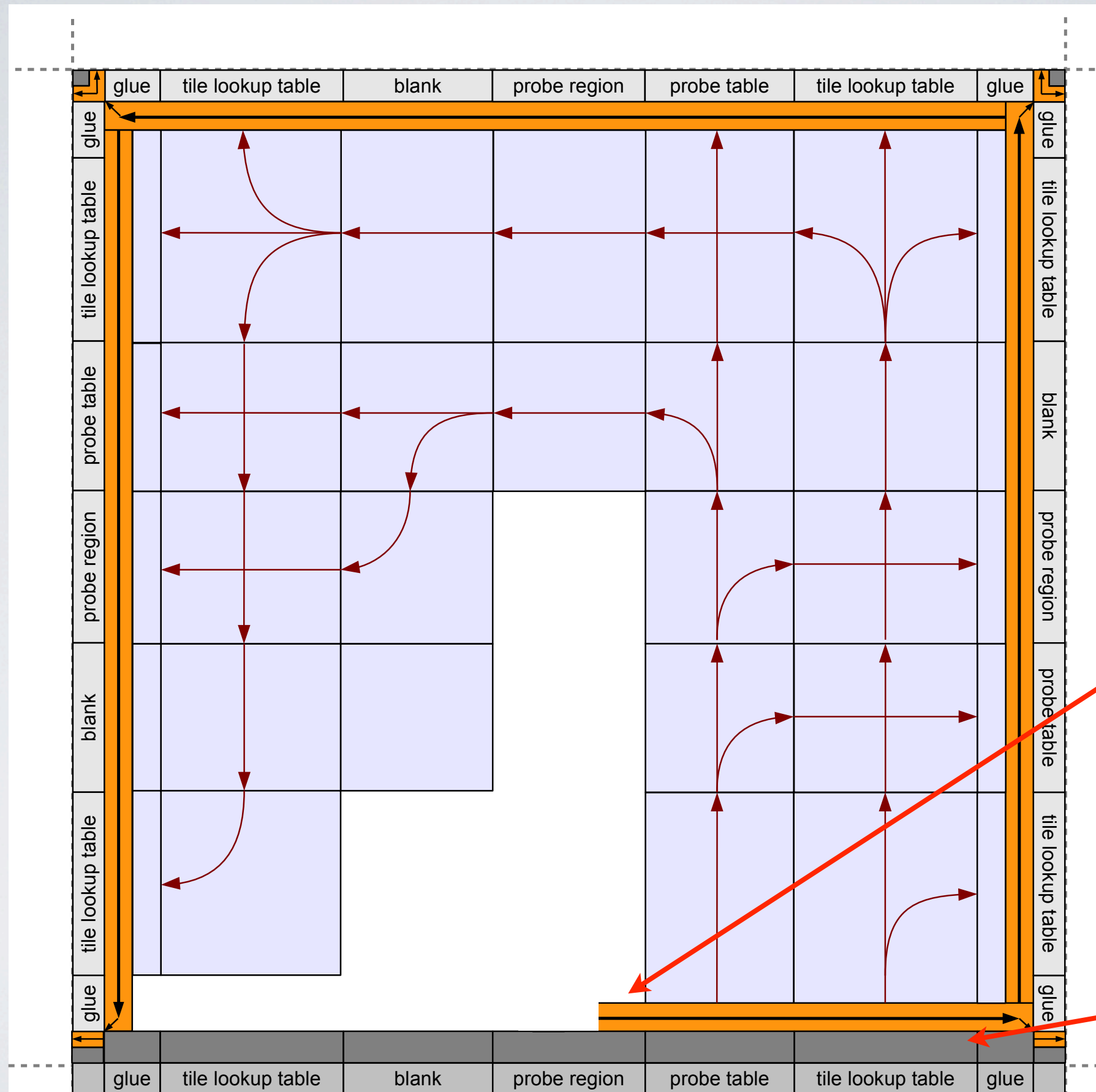
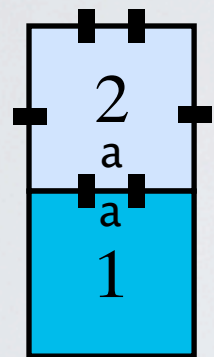
crawler encodes
glue of south
superside

“Genome” is
copied

“Genome” is
read

Supertile 1

One-sided binding with a single strength- τ south superside



Nondeterminism
Rotations

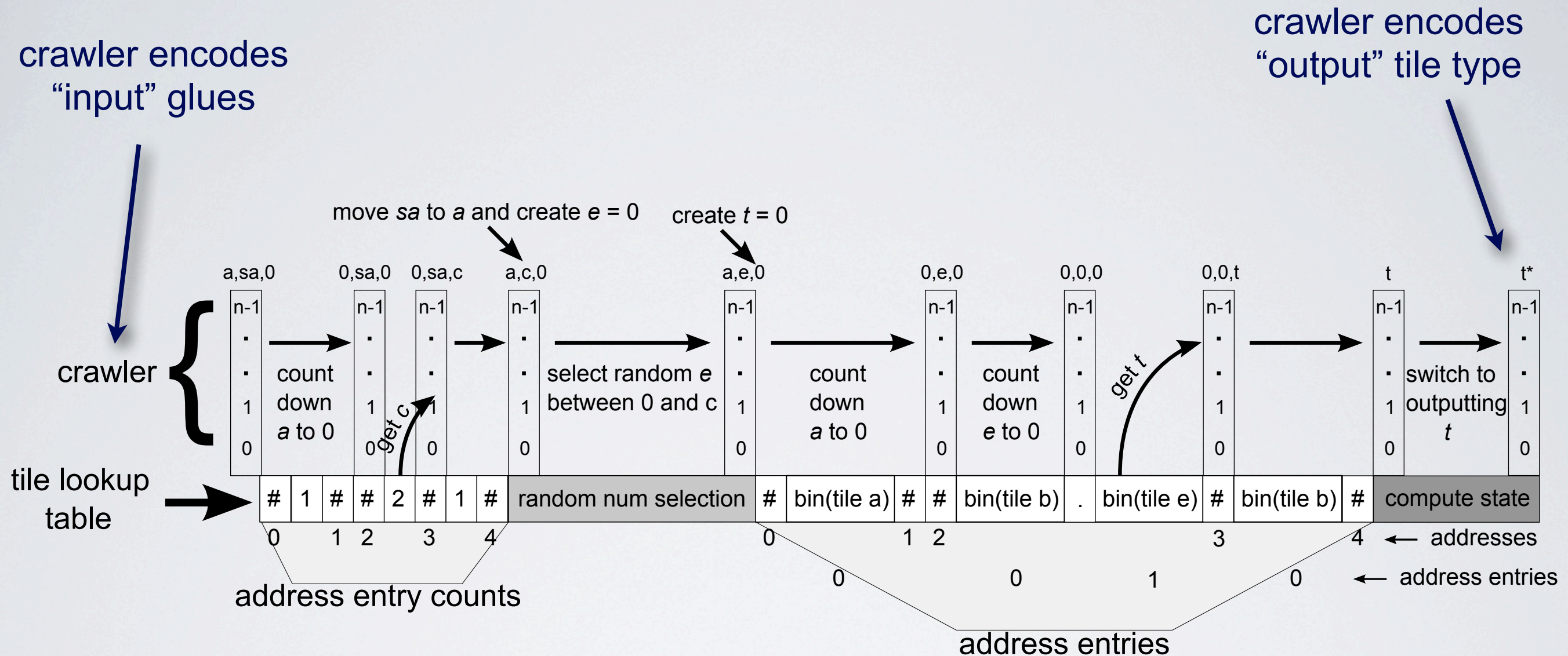
crawler encodes
glue of south
superside

“Genome” is
copied

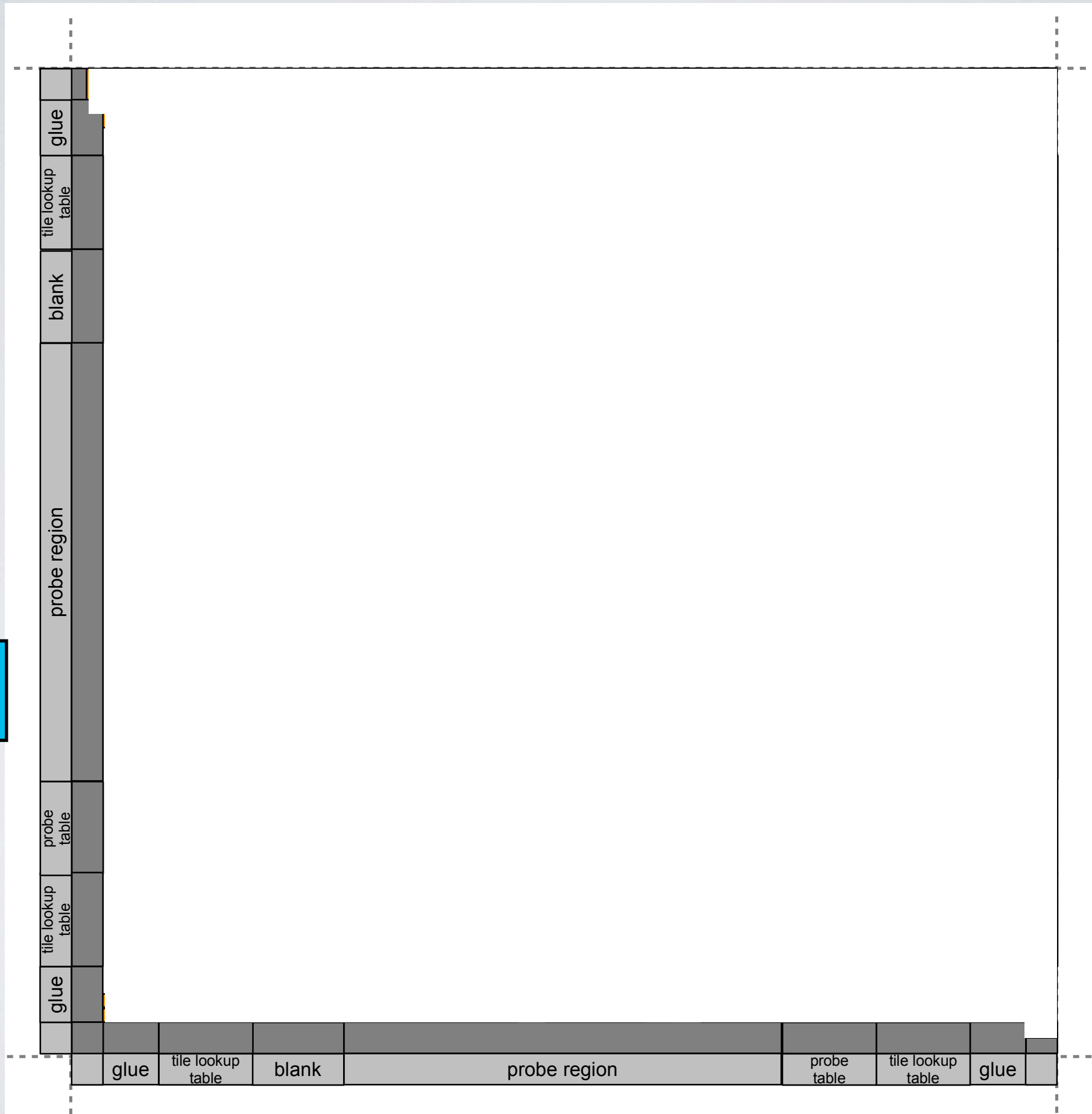
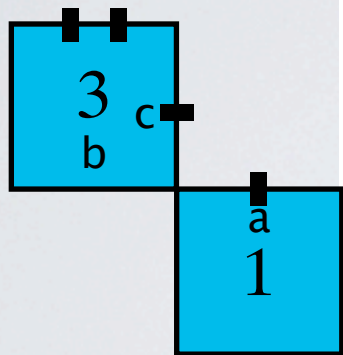
“Genome” is
read

Supertile 1

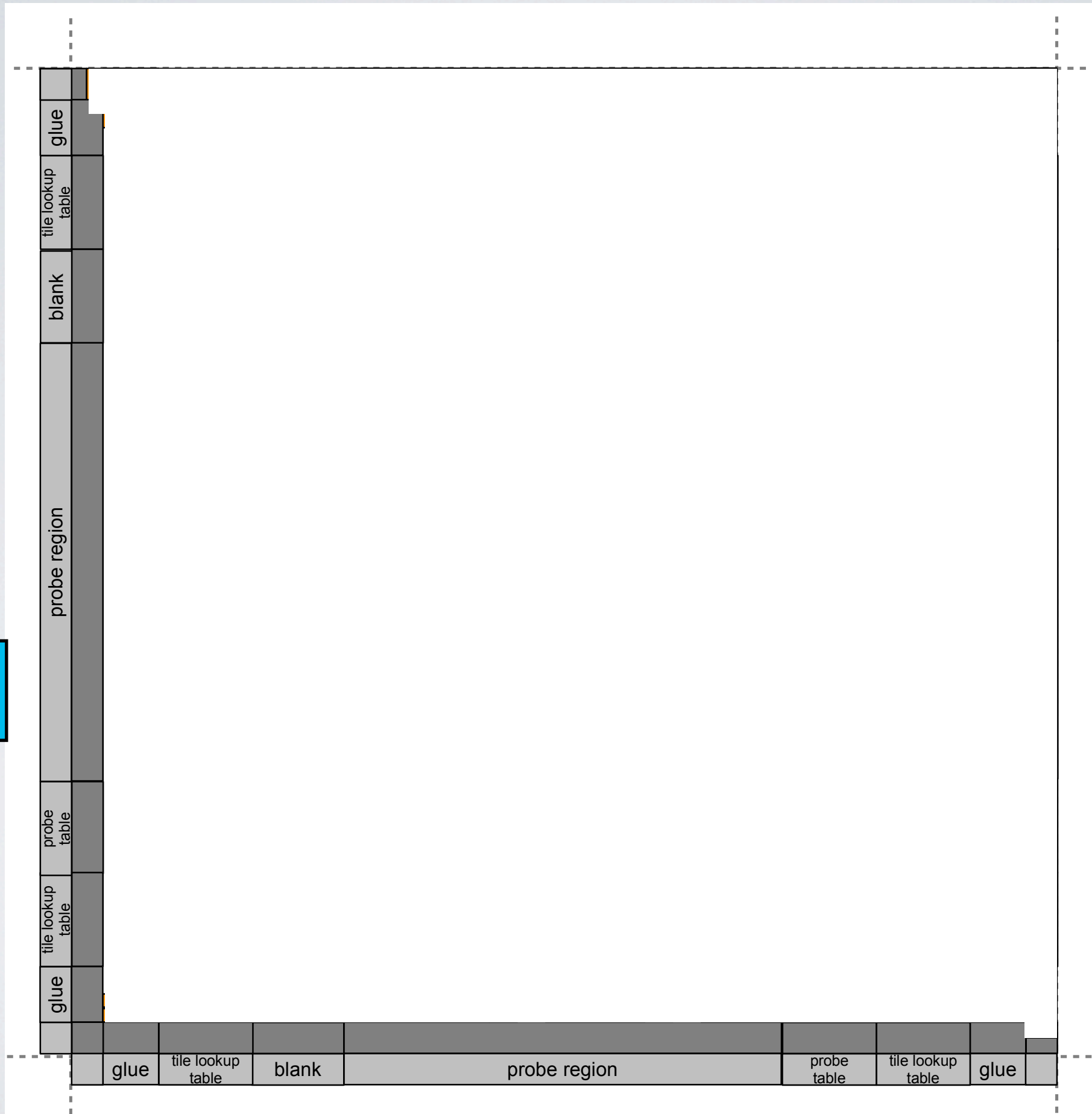
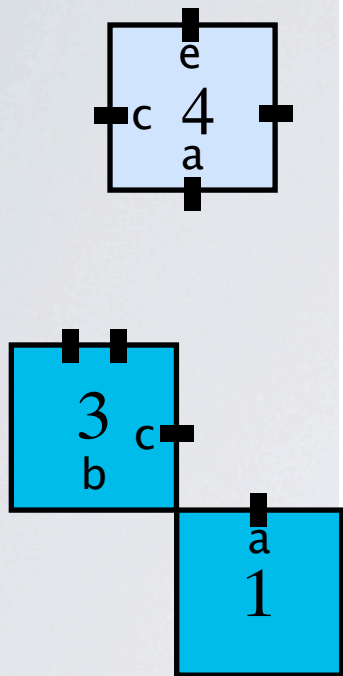
Crawler doing a tile lookup

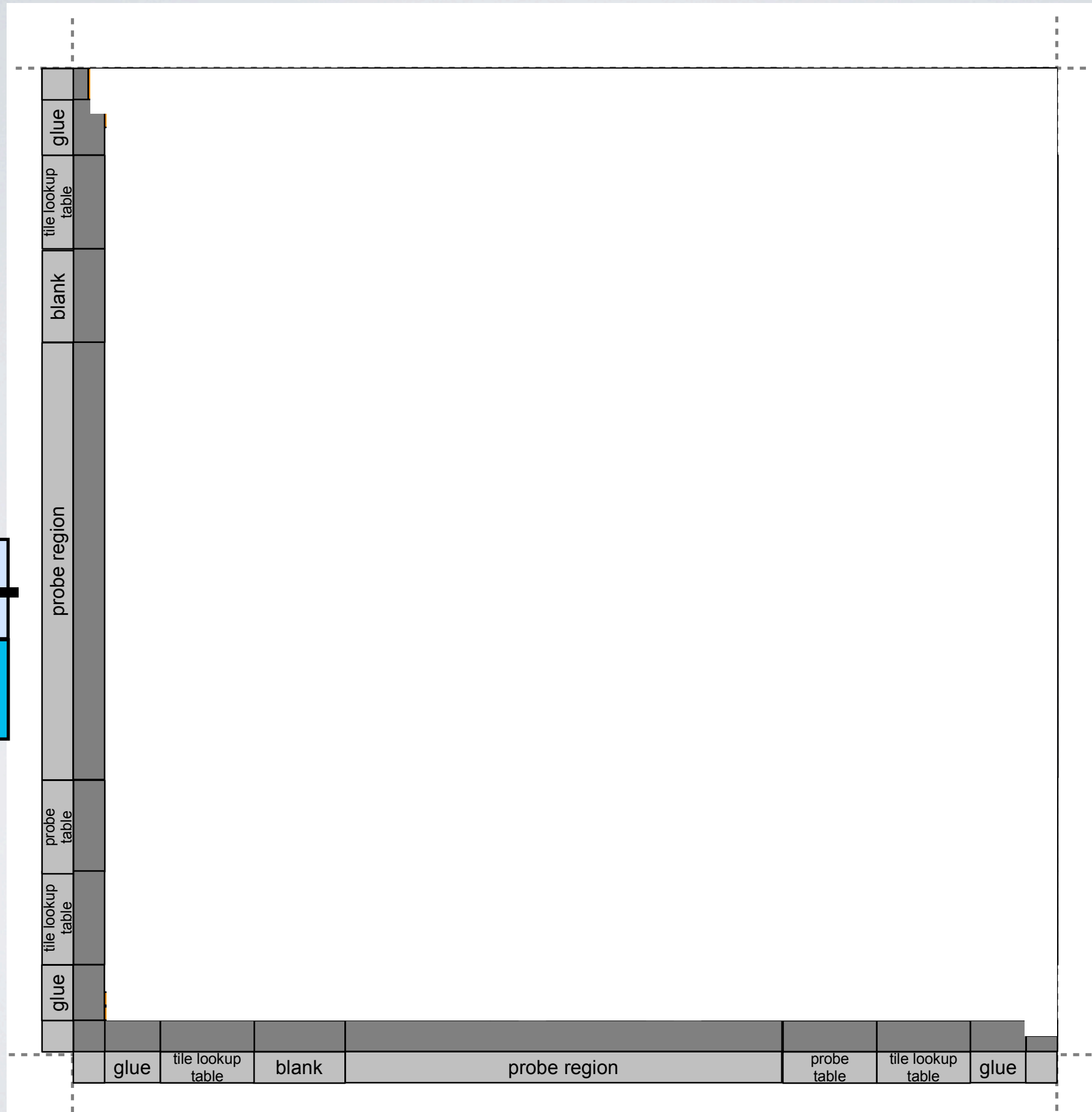
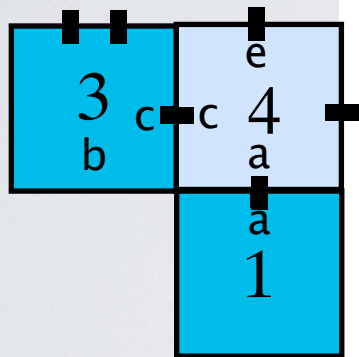


Two-sided binding with adjacent cooperating supersides

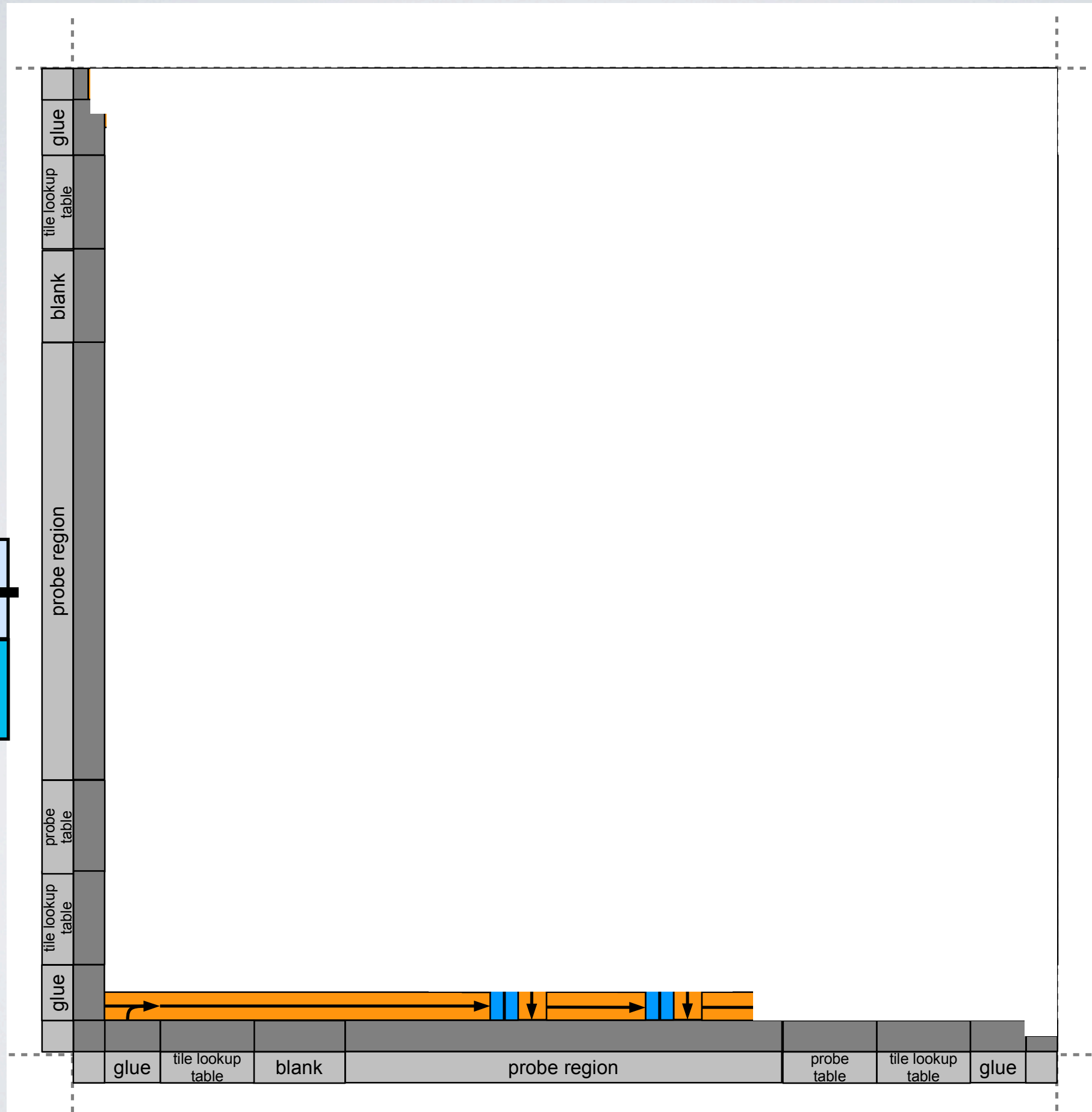


Two-sided binding with adjacent cooperating supersides

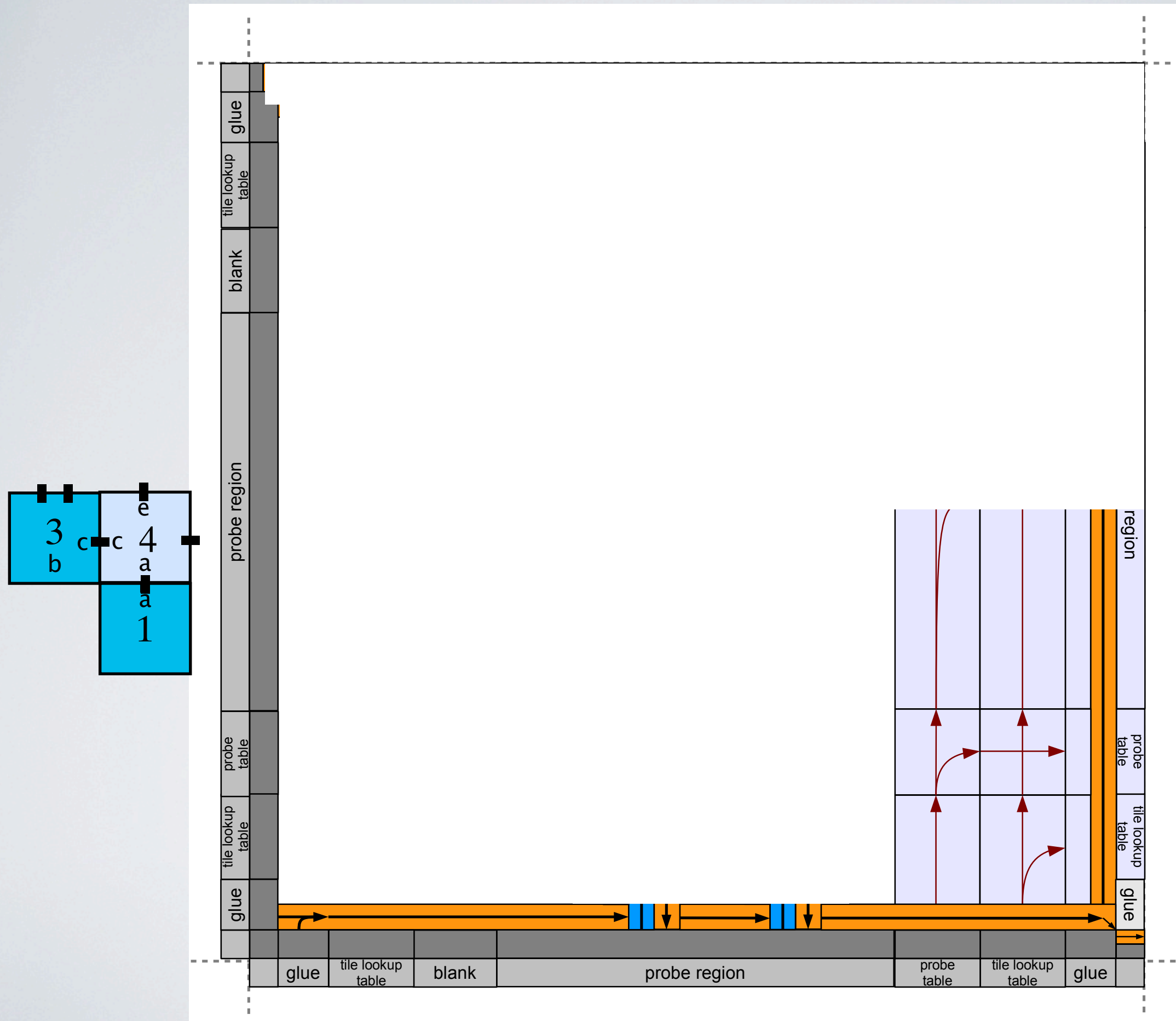




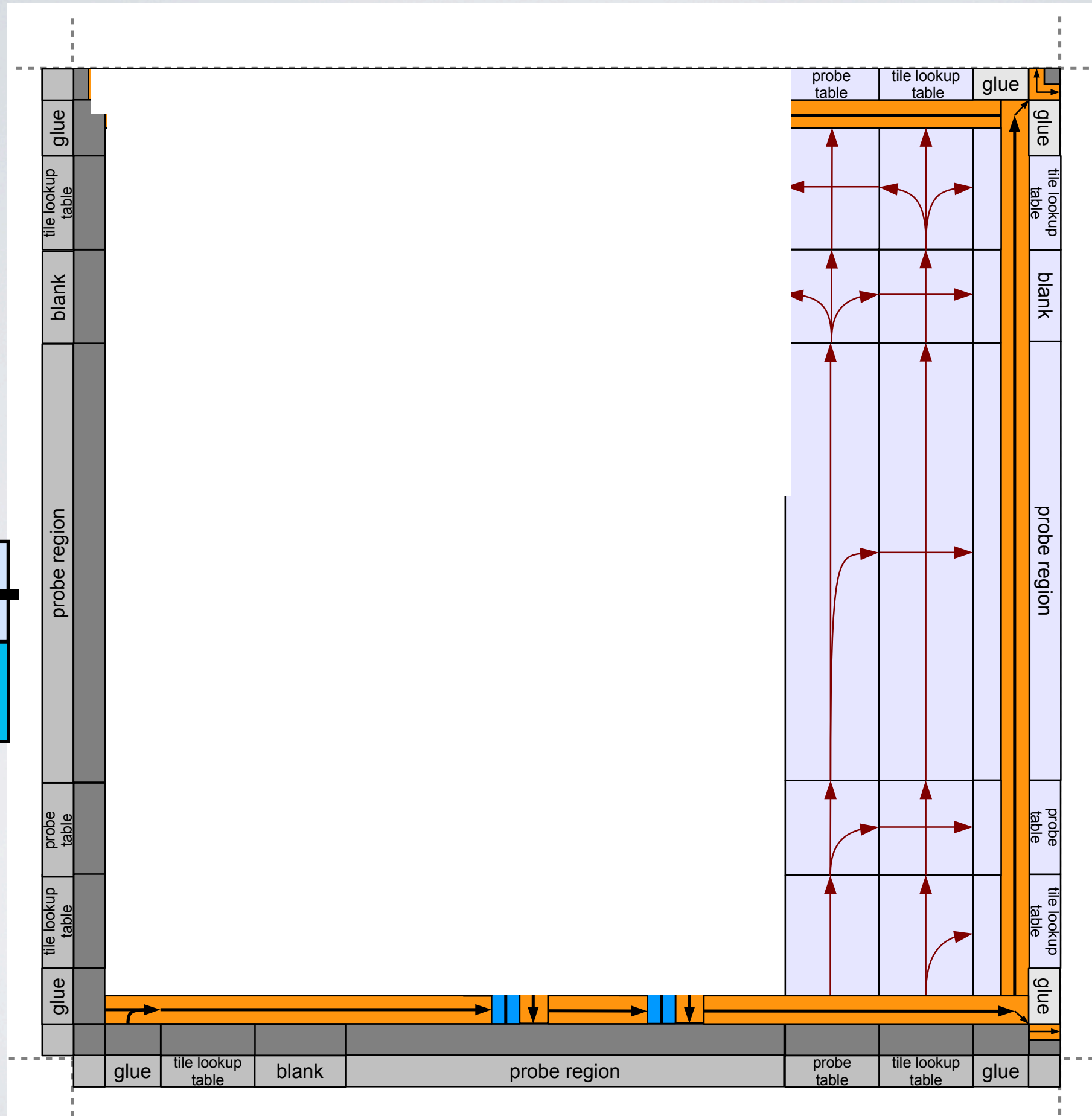
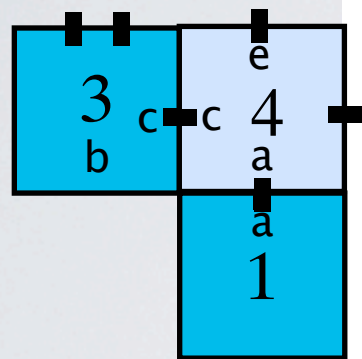
Damien Woods, FOCS 2012



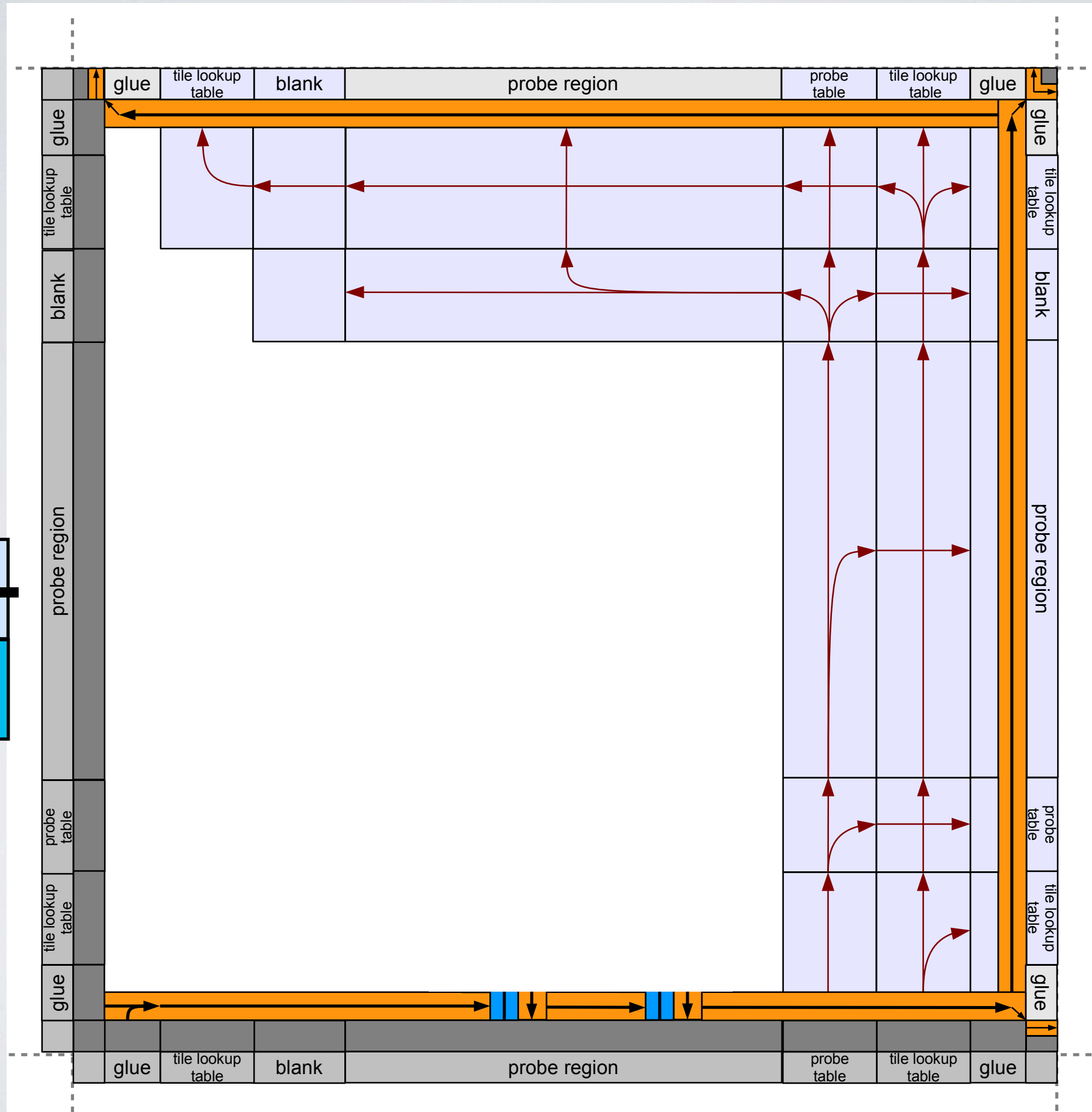
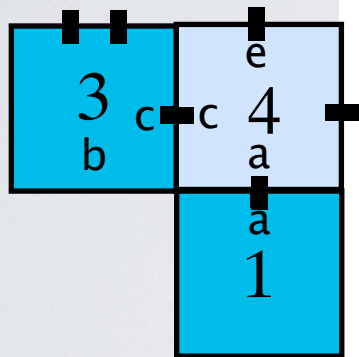
Two-sided binding with adjacent cooperating supersides



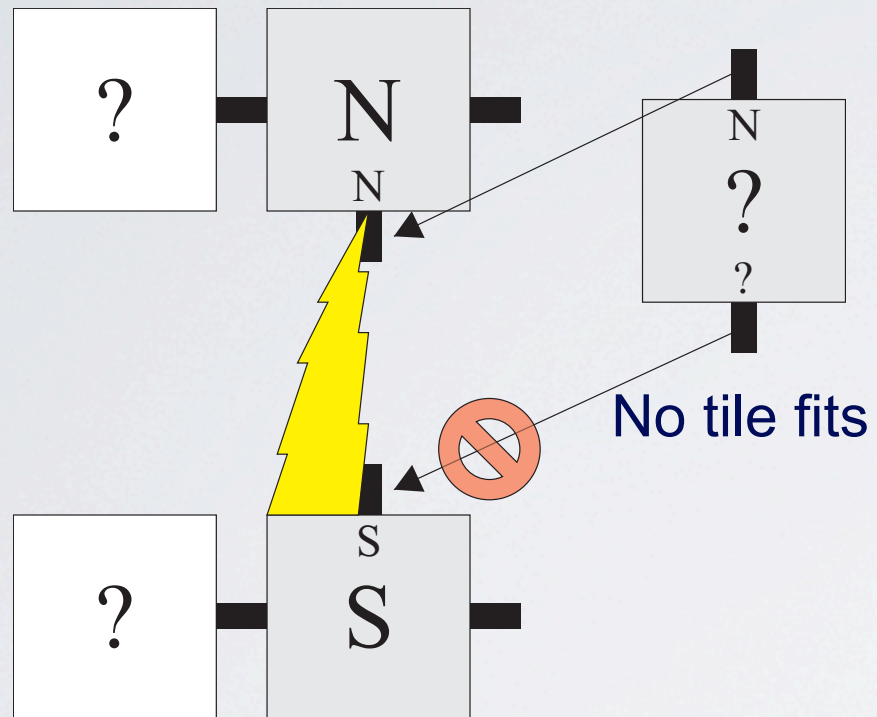
Two-sided binding with adjacent cooperating supersides



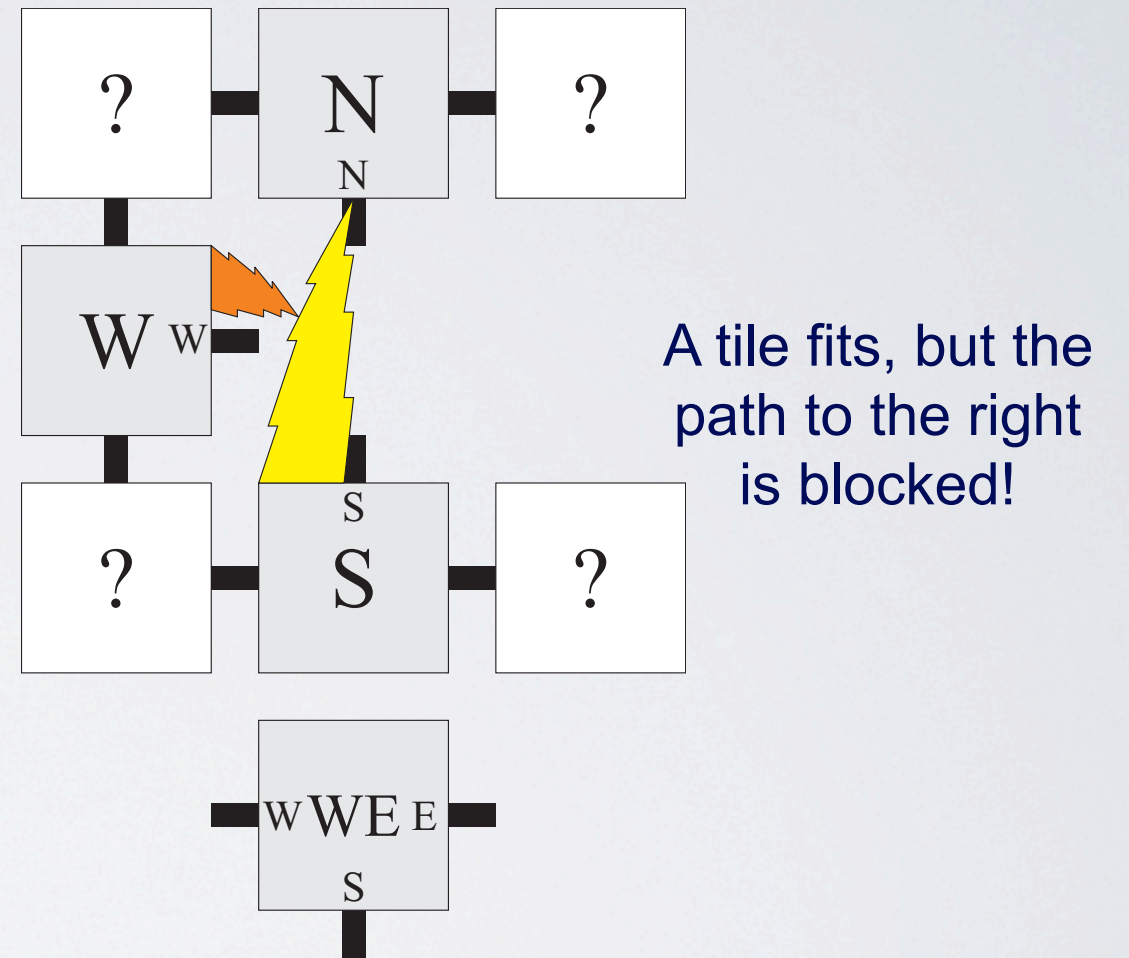
Two-sided binding with adjacent cooperating supersides



A key problem

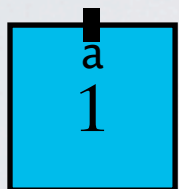
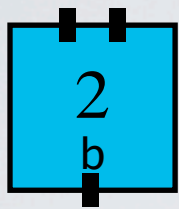
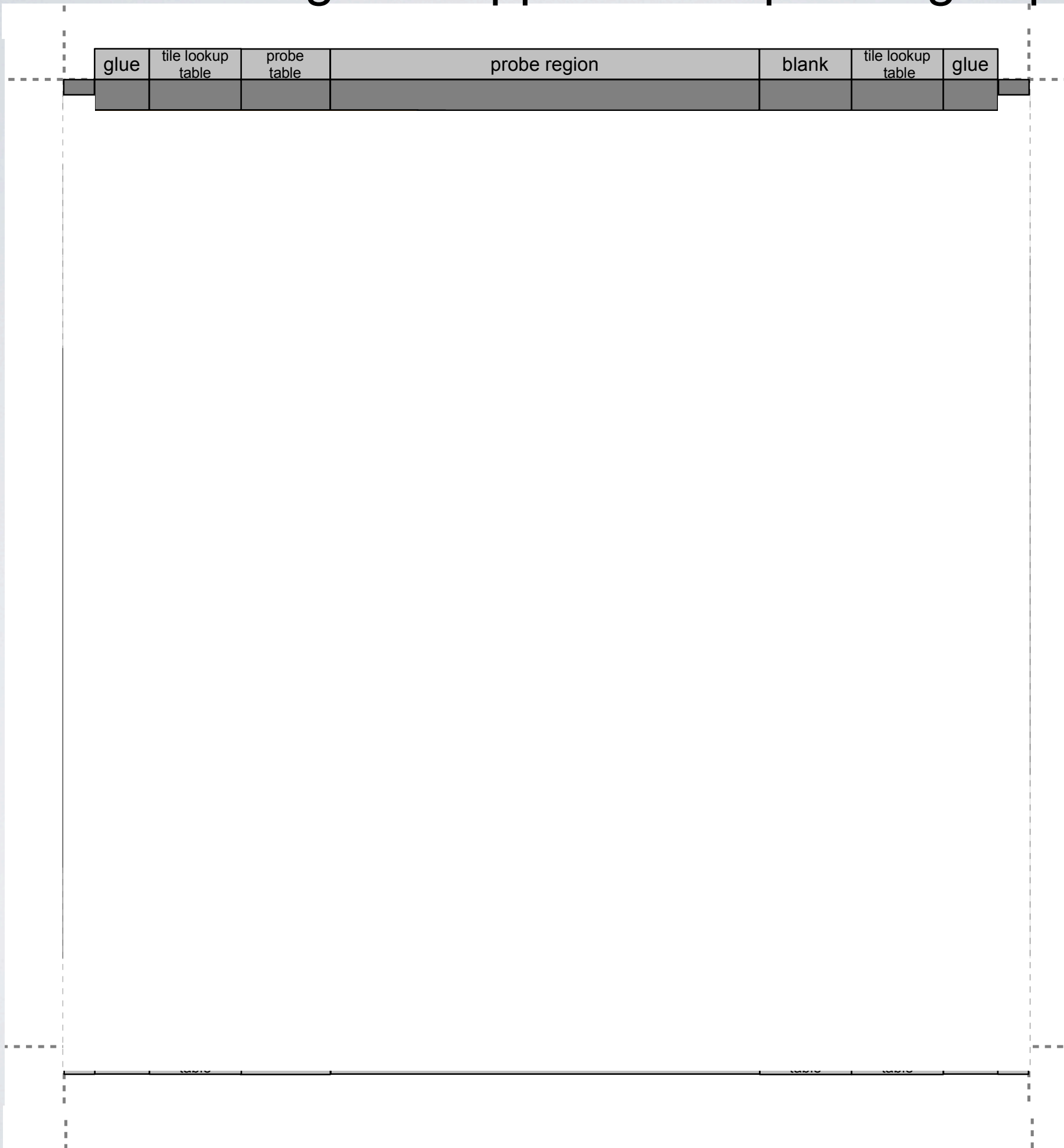


Better luck next time!

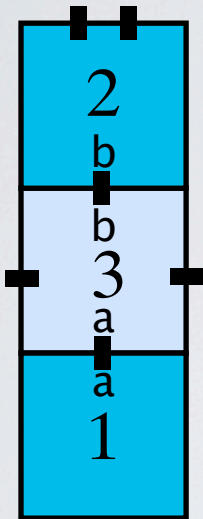
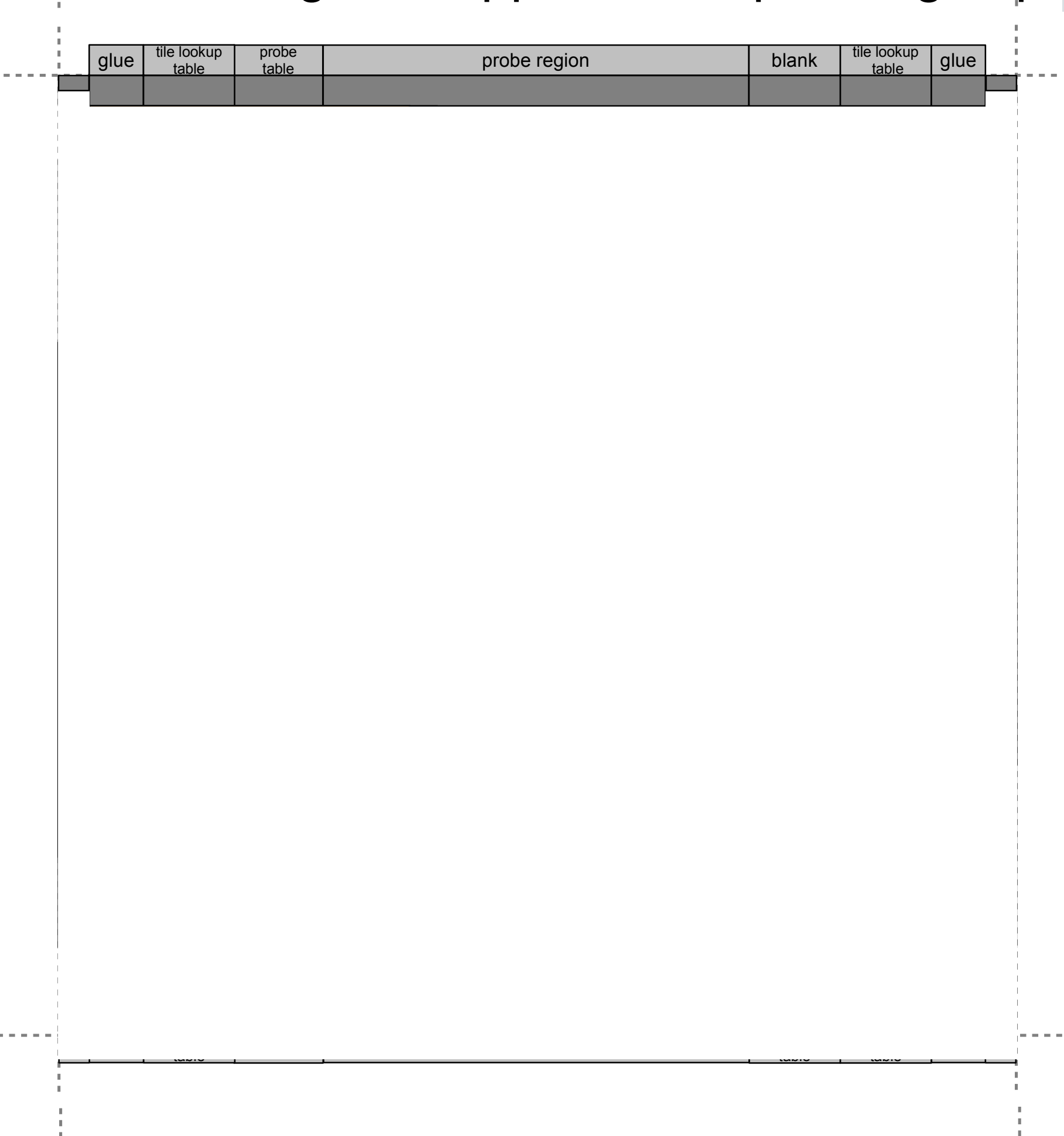


Uh oh!

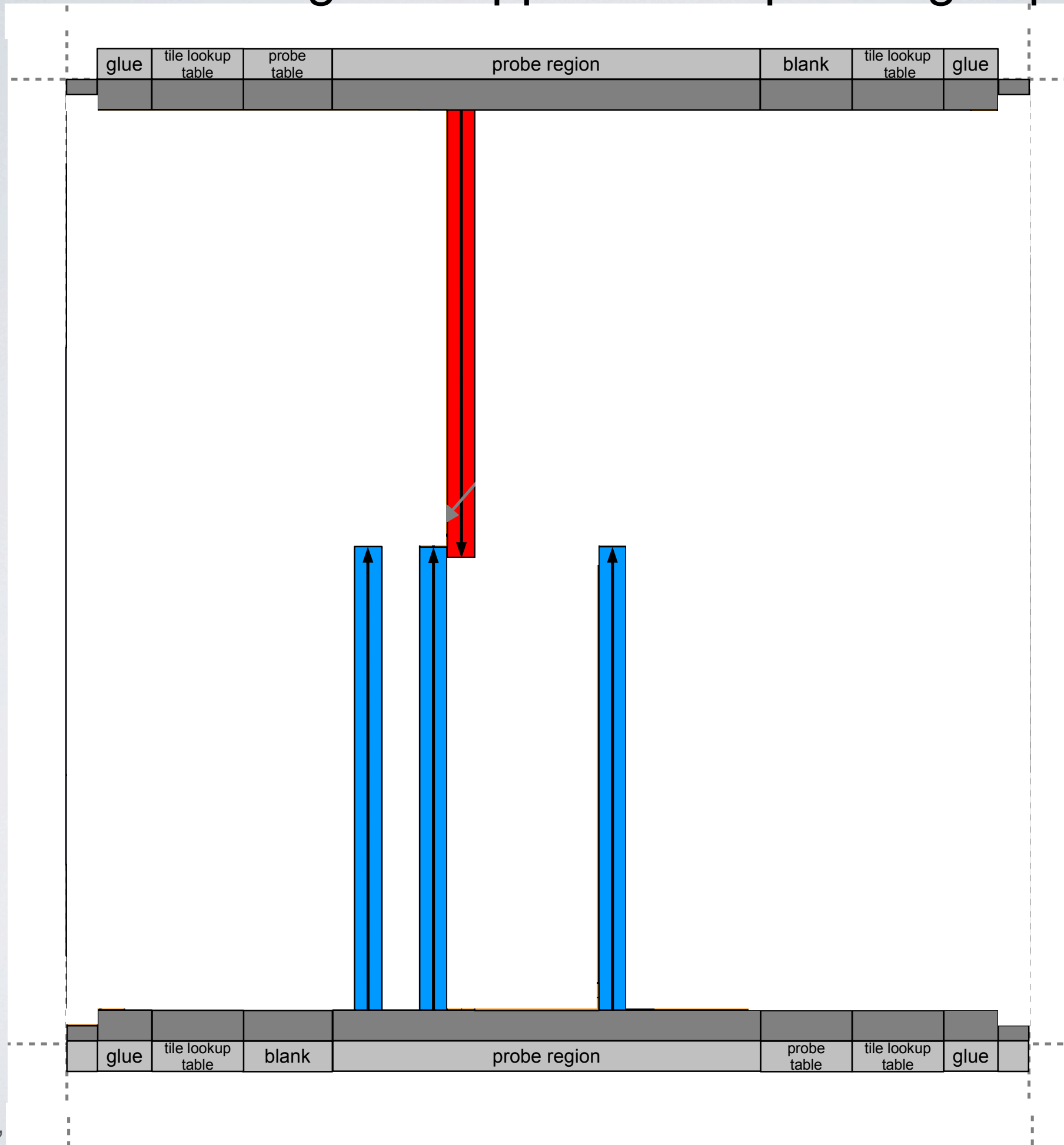
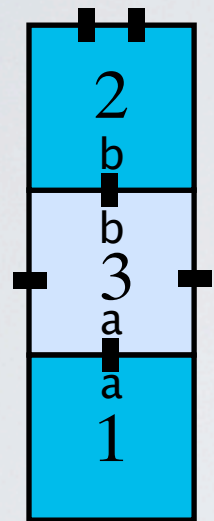
Two-sided binding with opposite cooperating supersides



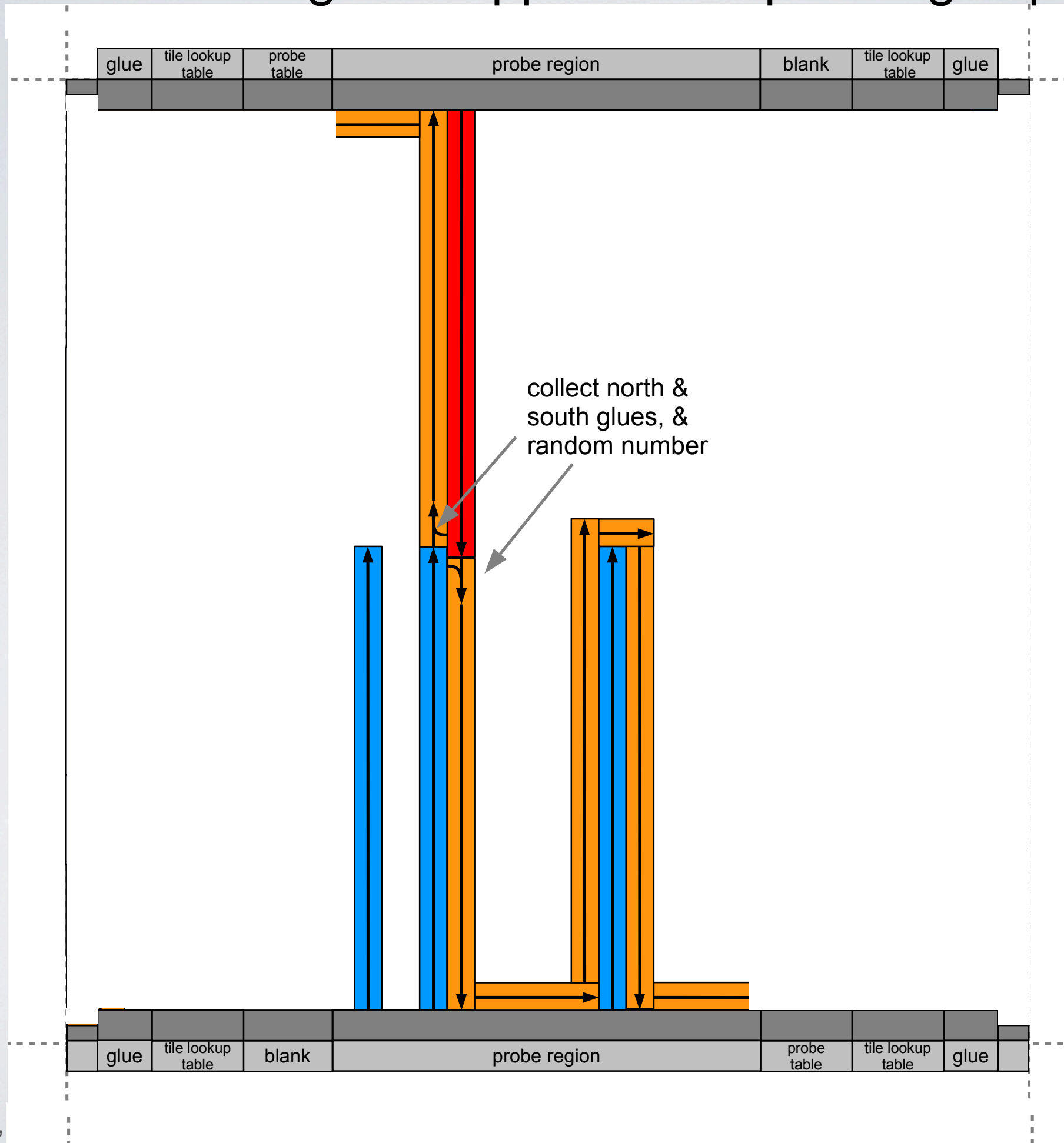
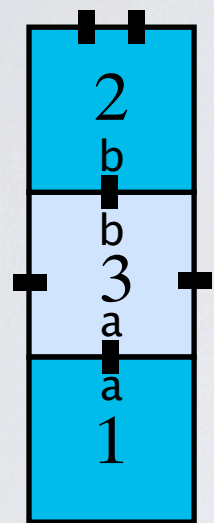
Two-sided binding with opposite cooperating supersides



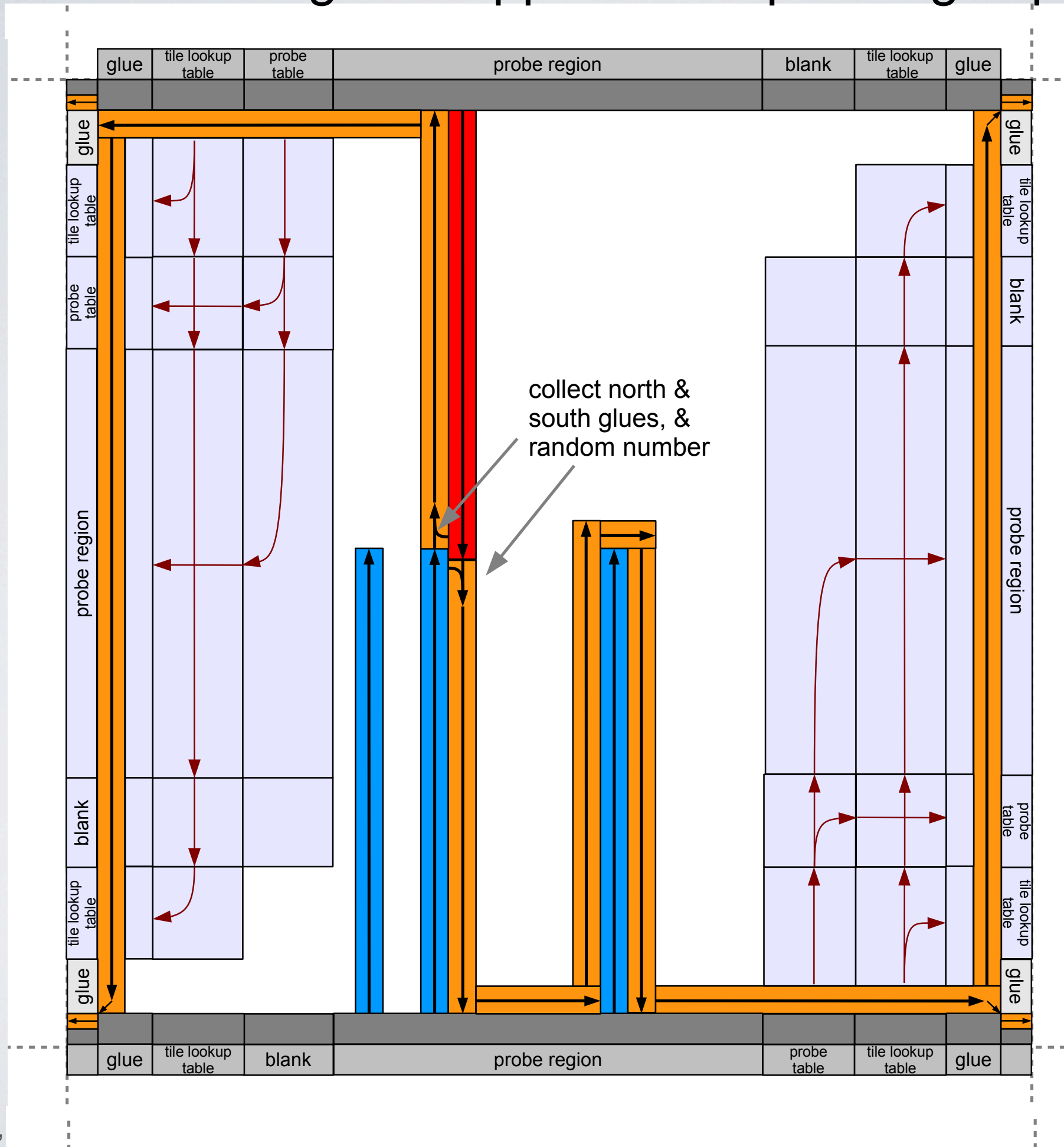
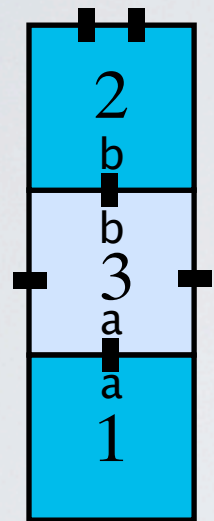
Two-sided binding with opposite cooperating supersides



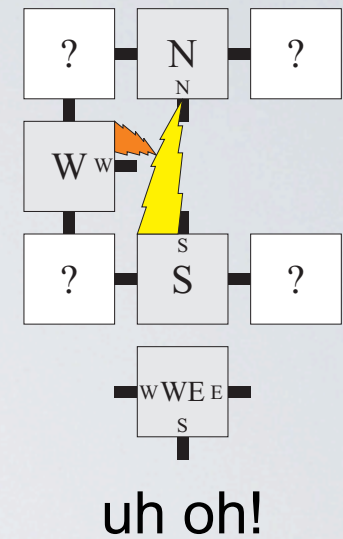
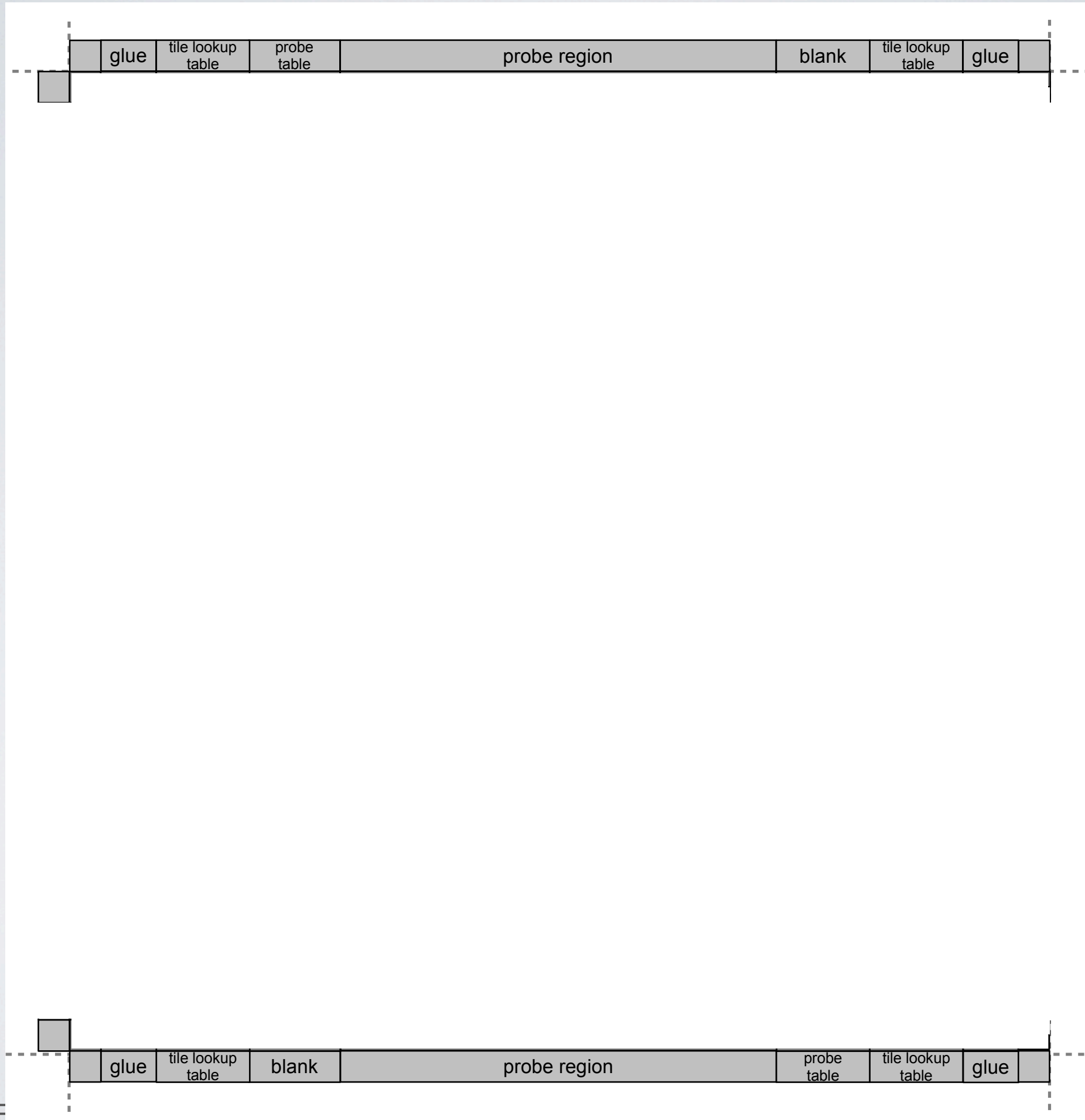
Two-sided binding with opposite cooperating supersides



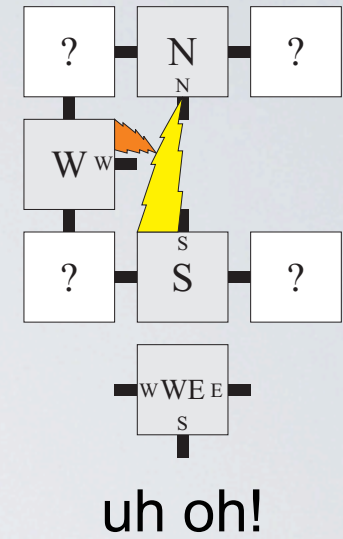
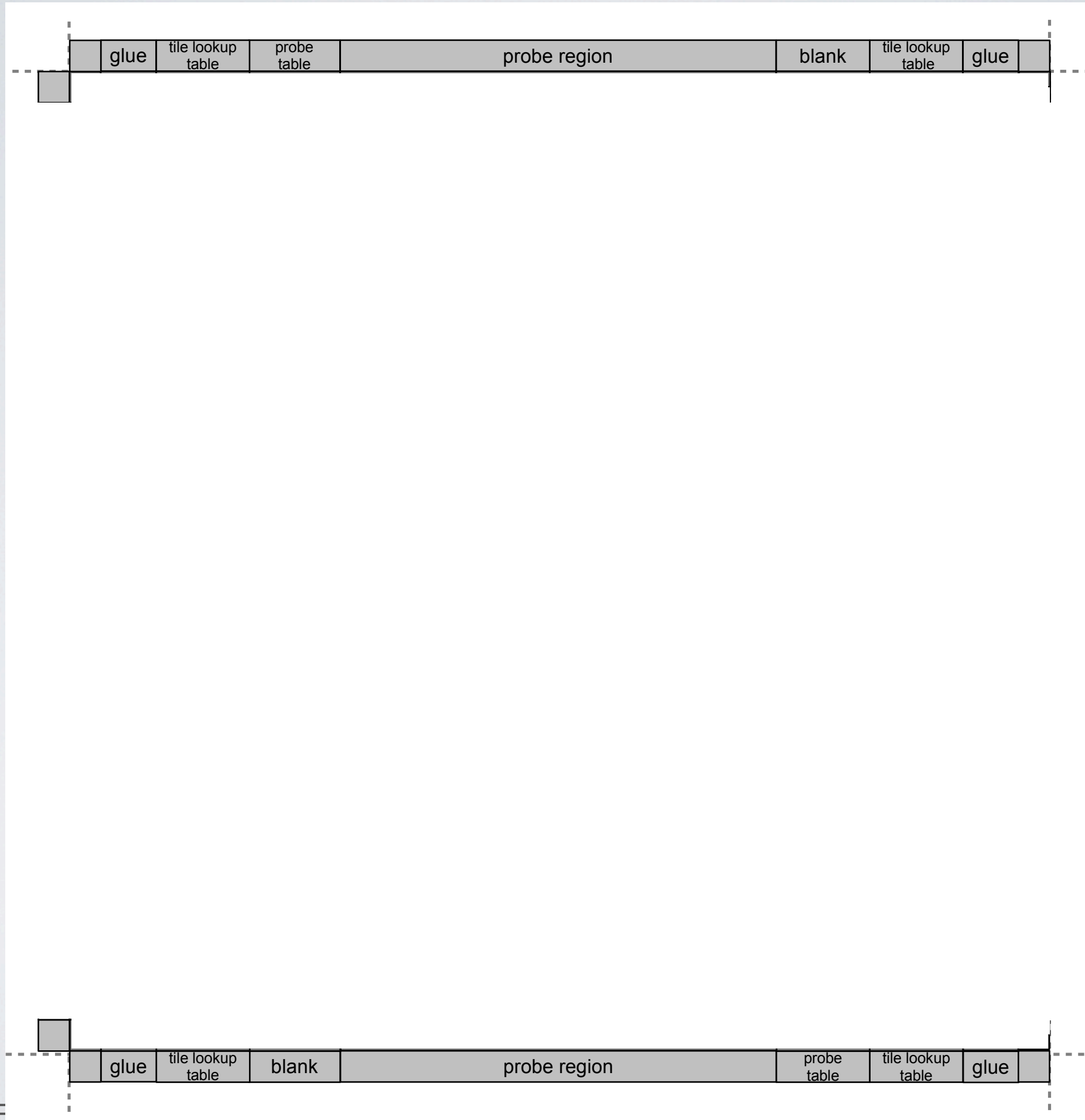
Two-sided binding with opposite cooperating supersides



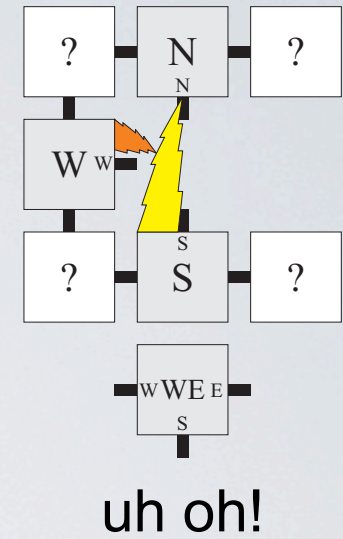
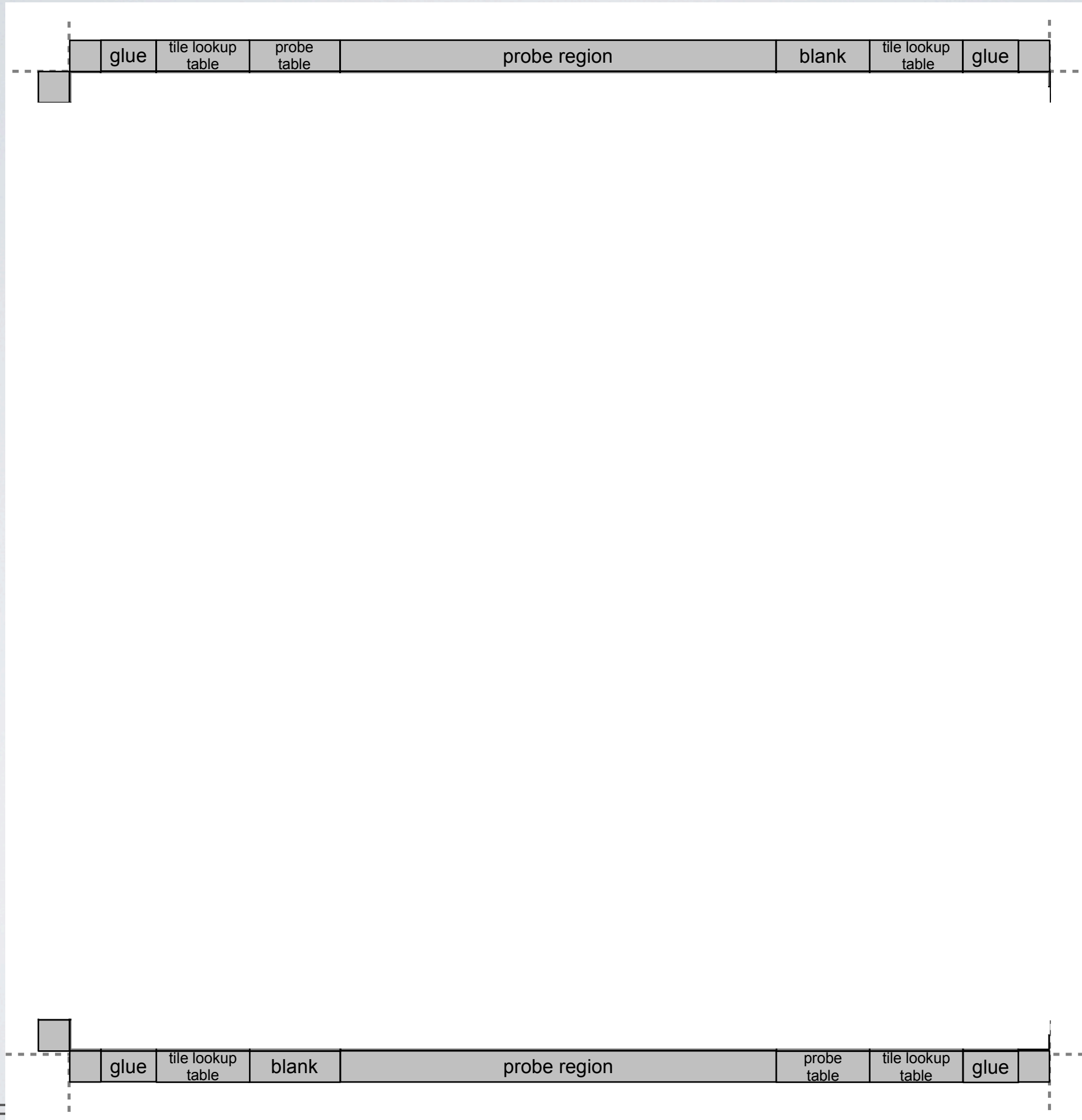
3-sided “uh-oh” example: probes miss each other



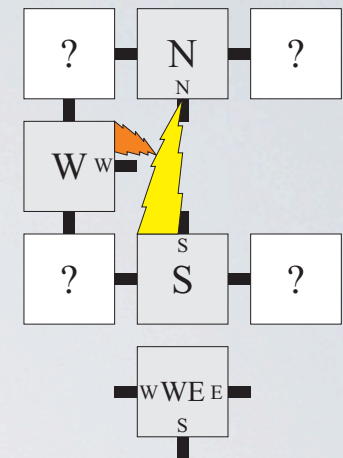
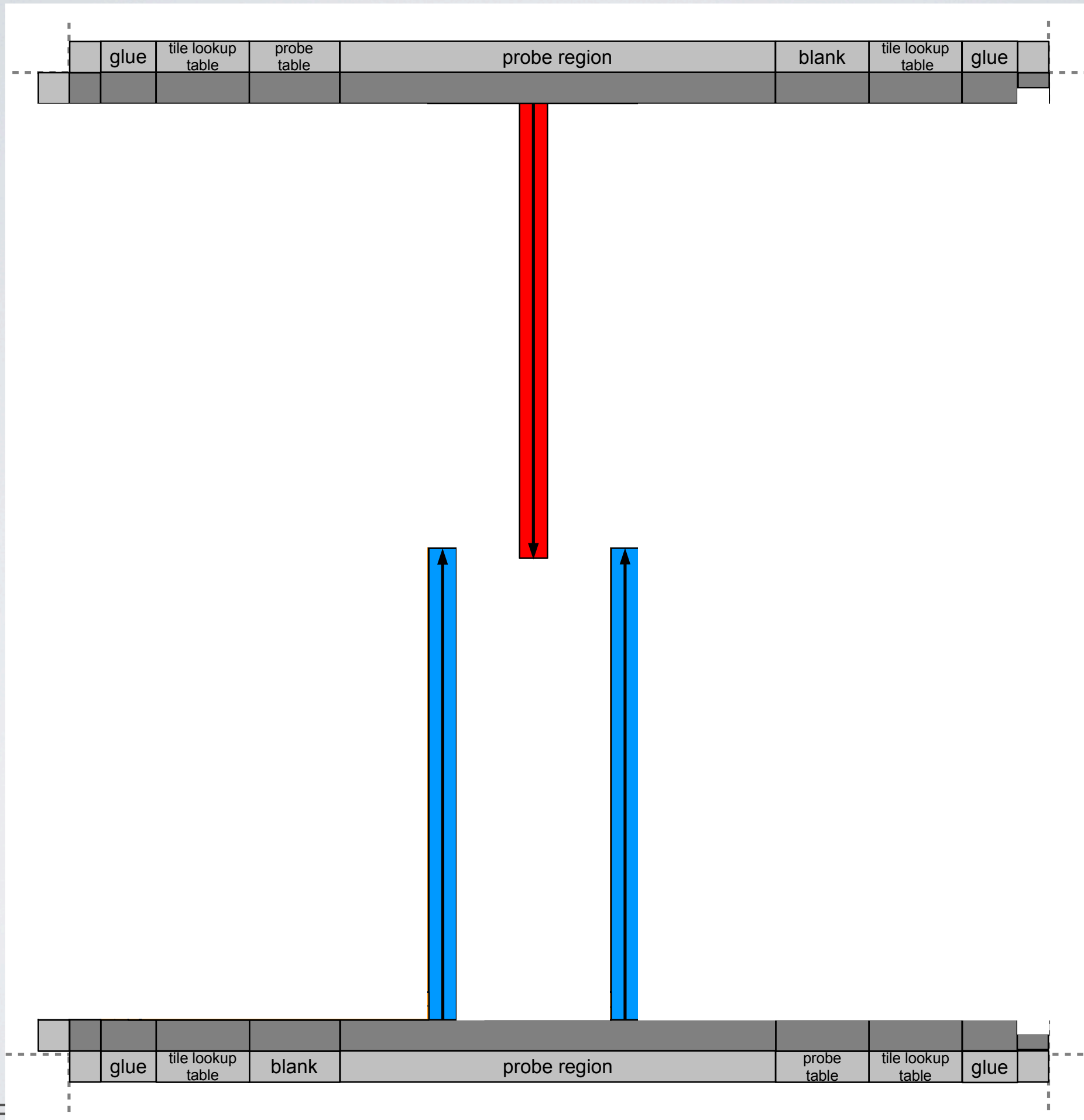
3-sided “uh-oh” example: probes miss each other



3-sided “uh-oh” example: probes miss each other

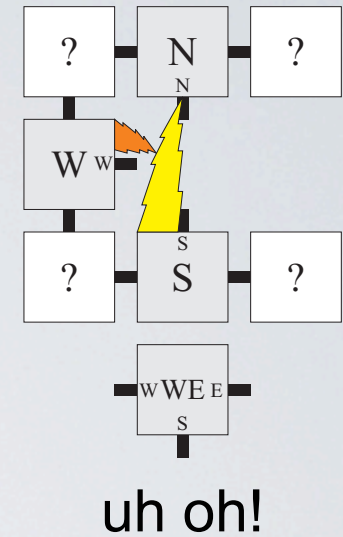
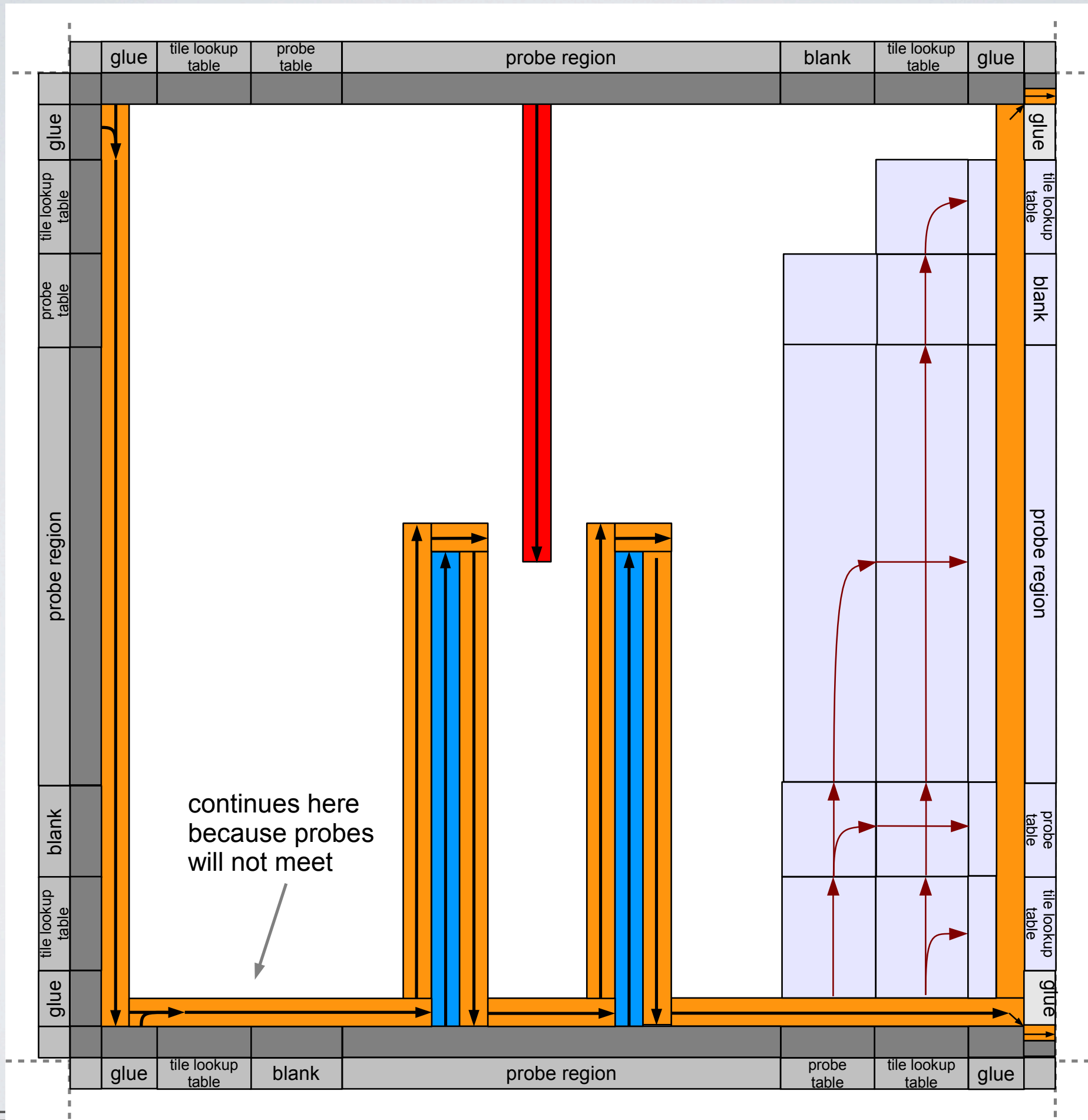


3-sided “uh-oh” example: probes miss each other

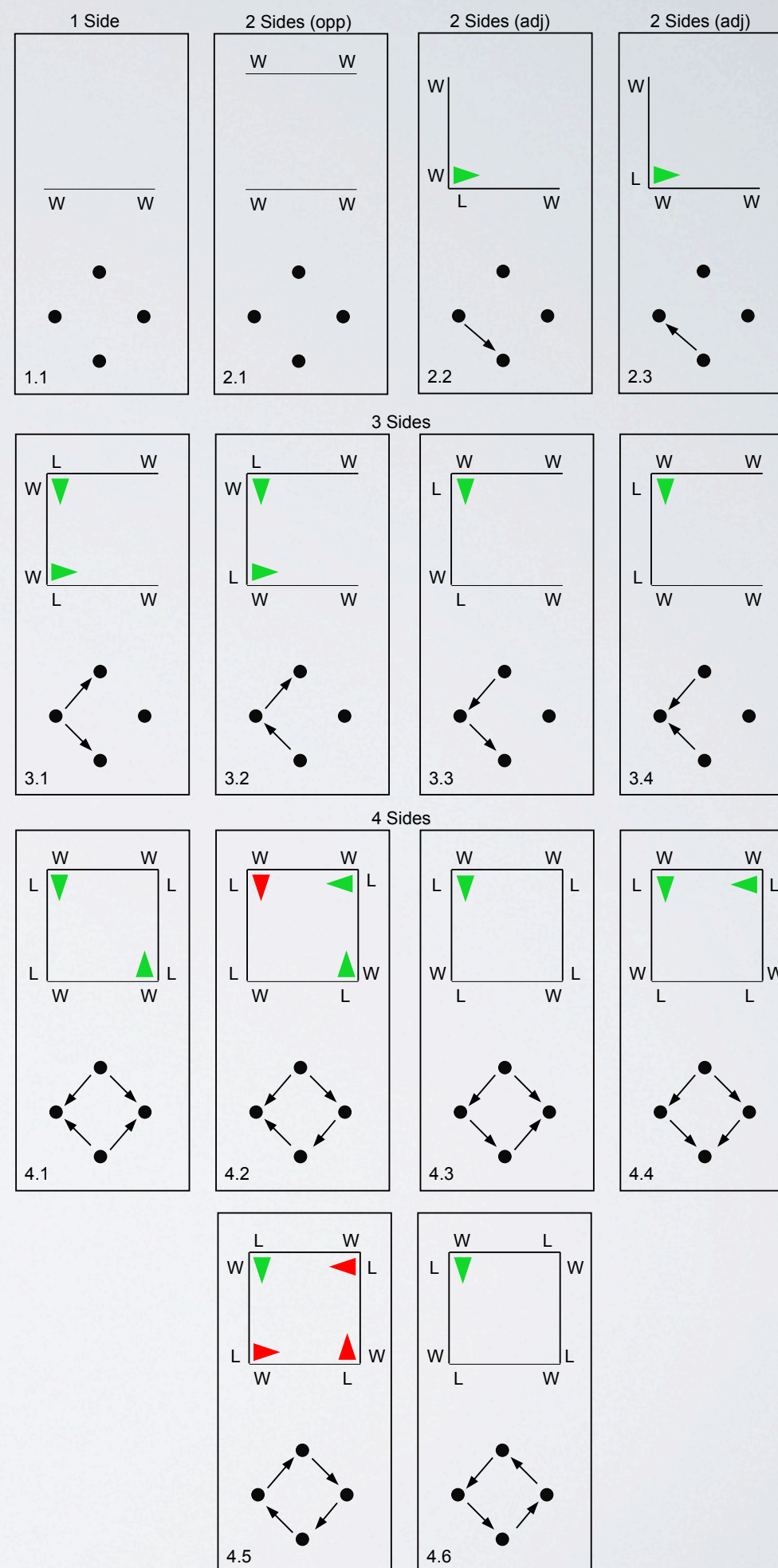


uh oh!

3-sided "uh-oh" example: probes miss each other



- Variety of cases for different orders of superside arrival
- Superside win/lose configurations and crawler initiation locations (green)
- Proof analogy: Distributed game, computation & geometry



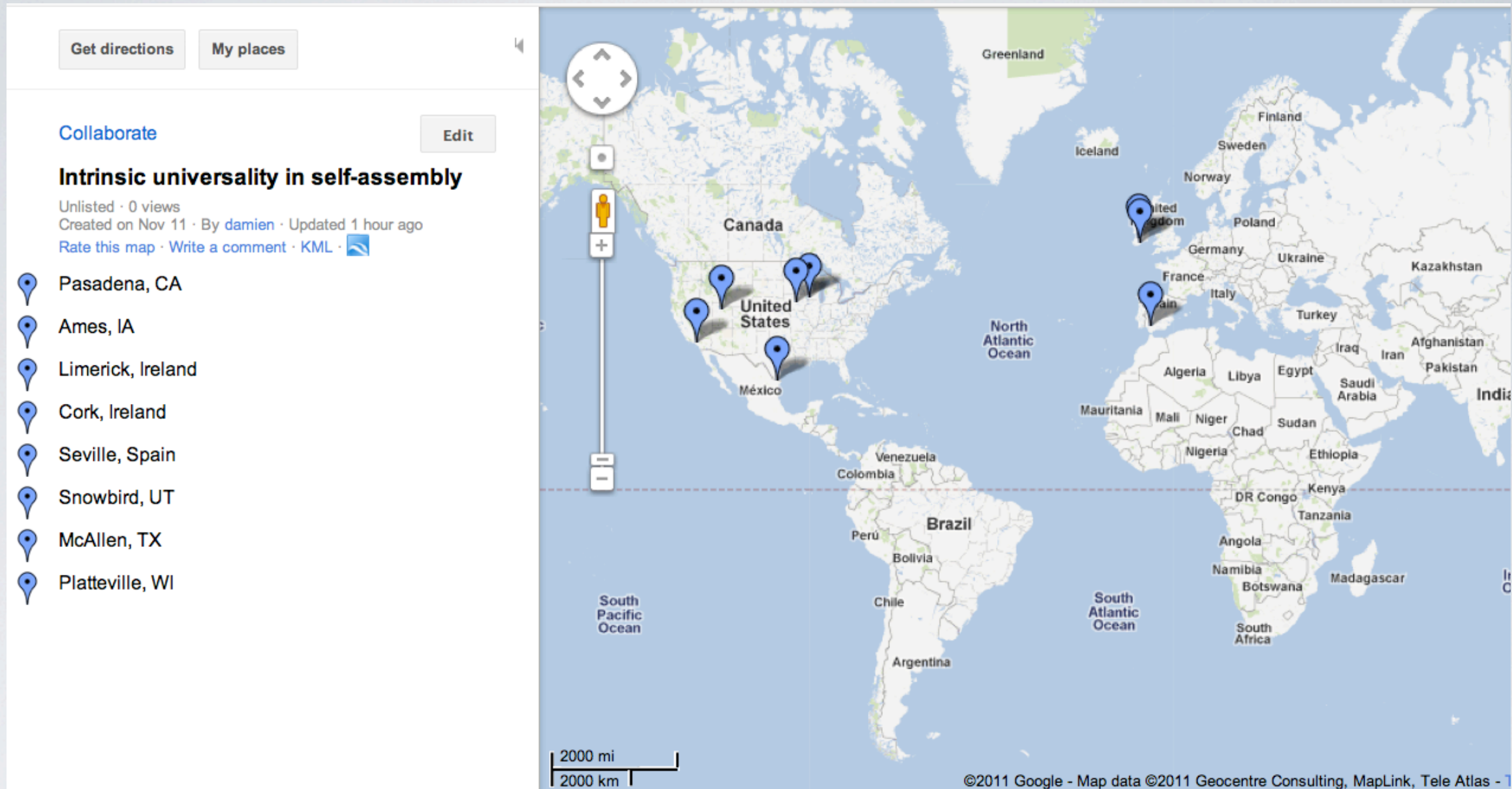
Future work & conclusions

- Can we use intrinsic universality as a tool in tile assembly to prove negative results on the power of tile assembly systems?
 - Conjecture: The temperature 1 tile assembly model is not intrinsically universal for all tile assembly systems
- As has been done in the theory of cellular automata, can we use intrinsic universality as a tool to classify tile assembly systems into hierarchies based on their ability to simulate each other?
- Application to other tile assembly models: roughly speaking, in any tile assembly model that simulates the aTAM (in a ‘for all, exists’ sense), our intrinsically universal tile set immediately implies that there is *one* tile set in *that* model that simulates all aTAM tile assembly systems

Intrinsic universality: Who dunnit?



Where?



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