Tales of the Collatz process in binary

(and ternary) (and senary)

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We present two papers

"Binary representation of ancestors in the Collatz graph",
T. Stérin, RP 2020

"The Collatz process embeds a base conversion algorithm",
T. Stérin and D. Woods, RP 2020

Orthogonal results but same motivation

I. Motivation: why study the Collatz process in binary?
II. Results:
    A. Characterizing the binary representation of ancestors in the Collatz process
    B. The Collatz process can solve base conversion 3 → 2 (not in AC0)

Slides, papers and code: https://dna.hamilton.ie/tsterin/

The Collatz map

- One of the simplest example of *Switched Affine Maps*. Formally introduced in the 60s based on Lothar Collatz's work from the 30s.

J. C. Lagarias. "The 3x+1 problem: an annotated blibliography". 2003, 2006

$$C(x) = \begin{cases} x/2 & \text{if } x \equiv 0 \mod 2 \\ 3x+1 & \text{if } x \equiv 1 \mod 2 \end{cases}$$

$$x \in \mathbb{N}$$

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Examples: $C(8) = 4 \qquad C(3) = 10$

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$C(5) = 16$

The Collatz map, slightly fast forwarded

- One of the simplest example of *Switched Affine Maps*. Formally introduced in the 60s based on Lothar Collatz's work from the 30s.

$$T(x) = \begin{cases} x/2 & \text{if } x \equiv 0 \bmod 2 \\ (3x+1)/2 & \text{if } x \equiv 1 \bmod 2 \end{cases}$$

$$x \in \mathbb{N}$$

The Collatz process

$$T(x) = \begin{cases} x/2 & \text{if } x \equiv 0 \bmod 2 \\ (3x+1)/2 & \text{if } x \equiv 1 \bmod 2 \end{cases}$$

Let’s iterate: 34,

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Let's iterate: 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, **1**

The Collatz process

$$T(x) = \begin{cases} x/2 & \text{if } x \equiv 0 \bmod 2 \\ (3x+1)/2 & \text{if } x \equiv 1 \bmod 2 \end{cases}$$

Let's iterate: 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, **1**, 2, **1**, 2, **1**, ...

The "trivial" cycle.

The Collatz conjecture

$$T(x) = \begin{cases} x/2 & \text{if } x \equiv 0 \bmod 2 \\ (3x+1)/2 & \text{if } x \equiv 1 \bmod 2 \end{cases}$$

Let’s iterate: 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, **1**, 2, **1**, 2, **1**, ...

Collatz conjecture: All strictly positive integers reach **1** under the action of the Collatz process.

The Collatz conjecture

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Let’s iterate: 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, **1**, 2, **1**, 2, **1**, ...

Collatz conjecture: All strictly positive integers reach **1** under the action of the Collatz process.

Two seemingly independent components:

1. **Cyclic conjecture:** the only strictly positive cycle is **2,1,2,1,**...
2. **Non-divergence conjecture:** No Collatz trajectory on the strictly positive integers diverges.

As of 2020, **computer tested** up to 2^67

D. Barina. *The Journal of Supercomputing*, 2020.

Is the Collatz problem really about natural numbers?

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$$T(x) = \begin{cases} x/2 & \text{if } x \equiv 0 \mod 2 \\ (3x+1)/2 & \text{if } x \equiv 1 \mod 2 \end{cases}$$

$$x \in \mathbb{N}$$

- $\mathbb{N}(+, \times)$ is a semiring
- There is a "well-defined" **parity function** $\epsilon : \mathbb{N} \to \mathbb{Z}/2\mathbb{Z}$
  - It is an homomorphism: $\epsilon(1) = 1$ $\epsilon(x+y) = \epsilon(x) + \epsilon(y)$
    $\epsilon(x \times y) = \epsilon(x) \times \epsilon(y)$
  - It satisfies:

$$\epsilon(x) = 0 \Leftrightarrow 2 \mid x$$

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Parity function:

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Three known strictly negative cycles:

-1, -1, ...
-5, -7, -10, **-5,** …
-17, -25, -37, -55, -82, -41, -61, -91, -136, -68, -34, **-17,** ...

No known divergent integers

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$$x \in \{ \tfrac{a}{b} \mid b \text{ odd } \} \subset \mathbb{Q}$$

Parity function:
$\epsilon(x) = a \bmod 2$

Examples:

-2/7, -1/7, 2/7, 1/7, **5/7**, 11/7, 20/7, 10/7, **5/7**, …

5/3, 3, 5, 8, 4, **2**, 1, **2**, 1, **2**, …

Lagarias' Periodicity Conjecture: J. C. Lagarias. *The American Mathematical Monthly,* 1985.

Every odd-denominator rational reaches a cycle.

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Binary decomposition algorithm
(i.e., on N, the standard binary encoding)

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while true {
Store $\epsilon(x)$
$x := (x - \epsilon(x))/2$
}

Binary decomposition algorithm (unbounded)

Is the Collatz problem really about natural numbers?

$$x \in \mathbb{N}$$
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26

$$x \in \mathbb{Z}$$
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$$x \in \mathbb{Z}$$
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$$x \in \left\{ \tfrac{a}{b} \mid b \text{ odd} \right\} \subset \mathbb{Q}$$
$$\epsilon(x) = a \bmod 2$$

26

$\downarrow$

...0000011010

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$$x \in \mathbb{Z}$$
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-2/7

...(100)(100)10

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26 → ...0000011010

-1 → ...111111111

-2/7 → ...(100)(100)10

$\mathbb{Z}_2$ The set of “2-adic integers” = semi-infinite binary strings

Is the Collatz problem really about natural numbers?

$$x \in \mathbb{N}$$
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26
...0000011010
$$\{(0)^\infty w \mid w \in \{0,1\}^*\}$$

$$x \in \mathbb{Z}$$
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-1
...111111111
$$\{(1)^\infty w \mid w \in \{0,1\}^*\}$$

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$$\epsilon(x) = a \bmod 2$$
-2/7
...(100)(100)10
Eventually repeating strings

$$\mathbb{Z}_2$$

"2-adic integers" = the set of semi-infinite binary strings

- $\mathbb{Z}_2(+,\times)$ is a ring (uncountable! unordered!)

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...11111111
+ …0000001

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- $\mathbb{Z}_2(+, \times)$ is a ring (uncountable!, unordered!)
- $\epsilon(x) = \mathrm{LSB}(x)$ is a suitable parity function

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We can run the Collatz process in $\mathbb{Z}_2$

Lagarias Periodicity Conjecture:

In $\mathbb{Z}_2$, an **eventually periodic** input to the Collatz process yields to an **evenutally periodic** Collatz sequence.

J. C. Lagarias, *The American Mathematical Monthly,* 1985.

Is the Collatz problem really about natural numbers?

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No, the Collatz problem is about half-infinite binary strings. (for the scope of this talk)

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In that context, it is *crucial* to understand the action of the Collatz process in binary.

Computer Science w.r.t the Collatz problem

Three fields (at least) are concerned by the Collatz problem:

Number Theory **Computer Science** **Logic**

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- By its formulation
- The approach led to numerous results:

J. C. Lagarias. “The 3x+1 problem: an annotated blibliography”. 2003, 2006

- ❏ No strictly positive cycle of length < 17.026.679.261
  S. Eliahou. *Discrete Mathematics*, 1993.

- ❏ “Almost all strictly positive integers almost lead to 1”
  T. Tao. *preprint*, 2019.

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Computer Science

- By its formulation in $\mathbb{Z}_2$
- Generalised Collatz Maps are Turing complete (exponential slow down):

J.H Conway. *Number Theory Conference*, 1972.

$$G(x) = \begin{cases} x/5+2 & \text{if } x \equiv 0 \bmod 5 \\ 7(x-1)/5+3 & \text{if } x \equiv 1 \bmod 5 \\ 2(x-2)/5+1 & \text{if } x \equiv 2 \bmod 5 \\ 6(x-3)/5+4 & \text{if } x \equiv 3 \bmod 5 \\ (x-4)/5 & \text{if } x \equiv 4 \bmod 5 \end{cases}$$

P. Koiran and C. Moore. *TCS*, 199.

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- Peano-independence in discrete dynamical systems. Ex: Goodstein sequences.

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Is the Collatz conjecture true?

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Computational power of the Collatz process?

Is it Turing complete?

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Peano or ZFC independence?

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- Computational power of the Collatz process?
  Is it Turing complete?
- Complexity of prediction problem?

Computer Science w.r.t the Collatz problem

Computer Science

- By its formulation in $\mathbb{Z}_2$
- Generalised Collatz Map e
  Turing complete (exponential own)

- Computational power of the Collatz process?
  Is it Turing complete?
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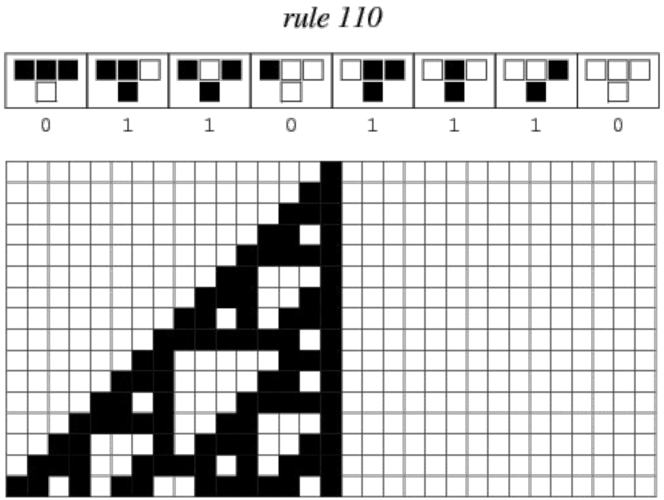
Very minimalistic discrete dynamical systems are known to be efficient Turing complete models:



M. Cook. *Complex Systems*, 2004

T. Neary and D. Woods. ICALP, 2006.

Motivation

- Computational power of the Collatz process?
  Is it Turing complete?
- Complexity of prediction problem?

The Collatz process in binary

Two operations to understand in binary:

- $x/2$
- $3x+1$

The Collatz process in binary

Two operations to understand in binary:

- x/2
- 3x+1

x/2 corresponds to shifting the binary representation to the right:

...111001010 → ...11100101

The Collatz process in binary

What about 3x + 1 in binary?

$$3x + 1 = x + (2x + 1)$$

Meaning: “x + (**left_shift**(x) + 1)”

Take x = 9 (1001 in binary):

$$\begin{array}{r} 1001 \\ + \quad \end{array}$$

The Collatz process in binary

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What about 3x + 1 in binary?

$$3x + \mathbf{1} = x + (\mathbf{2x} + \mathbf{1})$$

Meaning: “x + (**left_shift**(x) + **1**)”

Take x = 5 (1001 in binary):

$$\begin{array}{r} 1001 \\ + \;\mathbf{10011} \end{array}$$

The Collatz process in binary

What about 3x + 1 in binary?

$$3x + 1 = x + (\mathbf{2x} + 1)$$

Meaning: “x + (**left_shift**(x) + **1**)”

Take x = 9 (1001 in binary):

$$\begin{array}{r} 1001 \\ + \; \mathbf{10011} \\ = \; 11100 \end{array}$$

3*9 + 1 = 28

The Collatz process in binary

$3x + 1 = x + (2x + 1)$

$$
\begin{array}{r}
1001 \\
+\ 10011 \\
=\ 11100
\end{array}
$$

Meaning:

"Each bit of the input gets added to its right neighbour and the potential carry on that neighbour."

1001

The Collatz process in binary

$$3x + 1 = x + (2x + 1)$$

$$\begin{array}{rr} & 1001 \\ + & 10011 \\ = & 11100 \end{array}$$

Meaning:

"Each bit of the input gets added to its right neighbour and the potential carry on that neighbour."

10010

The Collatz process in binary

$3x + 1 = x + (2x + 1)$

  1001
+ 10011
= 11100

Meaning:

"Each bit of the input gets added to its right neighbour and the potential carry on that neighbour."

100010

The Collatz process in binary

$3x + 1 = x + (2x + 1)$

```
   1001
+ 10011
= 11100
```

Meaning:

“Each bit of the input gets added to its right neighbour and the potential carry on that neighbour.”

$$1001\boxed{\bar{0}}$$

$$0$$

The Collatz process in binary

$3x + 1 = x + (2x + 1)$

$$\begin{array}{r} 1001 \\ +\ 10011 \\ =\ 11100 \end{array}$$

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$$10\bar{0}\boxed{\bar{1}}\bar{0}$$

$$00$$

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$$00$$

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$$1\,0\,\boxed{\bar{0}}\,\bar{1}\,\bar{0}$$

$$100$$

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$$100$$

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$$1\boxed{0}\overline{0}\,\overline{1}\,\overline{0}$$

$$1100$$

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$3x + 1 = x + (2x + 1)$

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$$
\begin{array}{rr}
 & 1001 \\
+ & 10011 \\
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$$

$$..00\boxed{1}00\bar{0}1\bar{1}\bar{0}$$

$$1100$$

The Collatz process in binary

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$$11100$$

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= 11100
```

Meaning:

"Each bit of the input gets added to its right neighbour and the potential carry on that neighbour."

..0010010

..011100

The Collatz process in binary

$3x + 1 = x + (2x + 1)$

Meaning:

“Each bit of the input gets added to its right neighbour and the potential carry on that neighbour.”

$$
\begin{array}{r}
1001 \\
+ \quad 10011 \\
= \quad \mathbf{11100}
\end{array}
$$

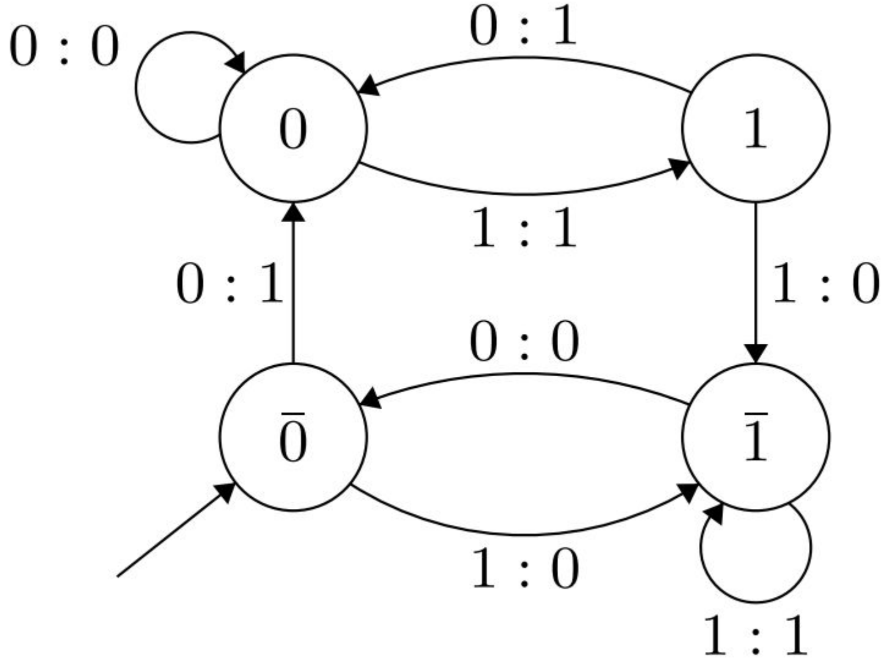
..00100010

..011100

3*9 + 1 = 28

The 3x+1 operation is 2-automatic

Caption
read : write

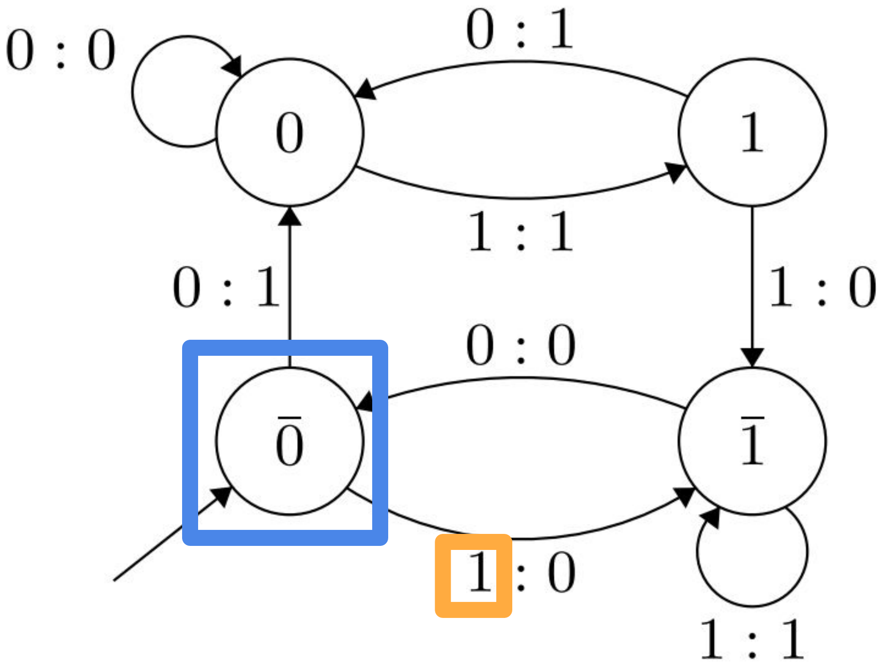


The 3x+1 binary FST

The 3x+1 operation is 2-automatic

Caption
read : write

100010



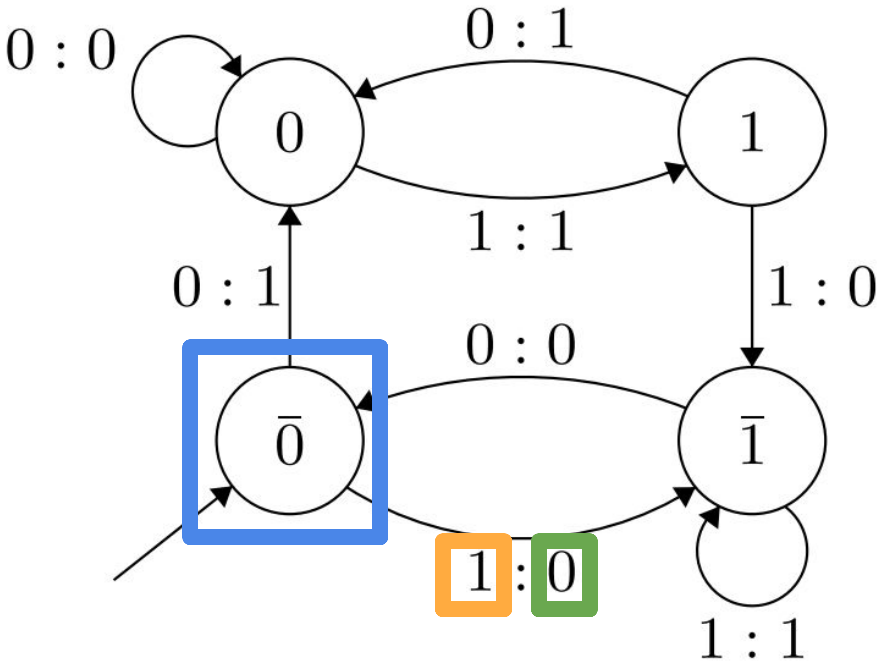
The 3x+1 binary FST

The 3x+1 operation is 2-automatic

Caption
read : write

1001$\bar{0}$

0



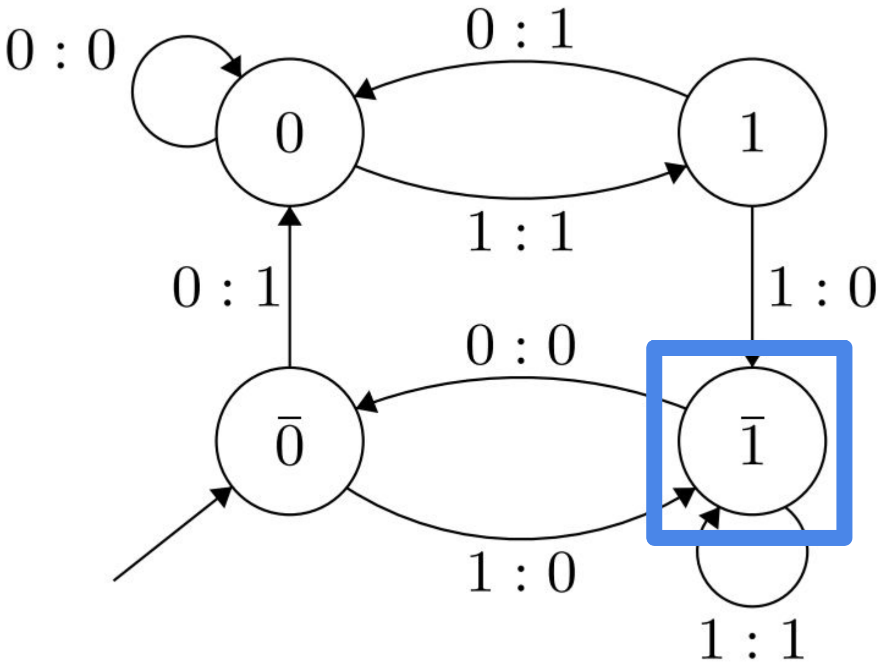
The 3x+1 binary FST

The 3x+1 operation is 2-automatic

Caption
read : write

$100\bar{1}\bar{0}$

0



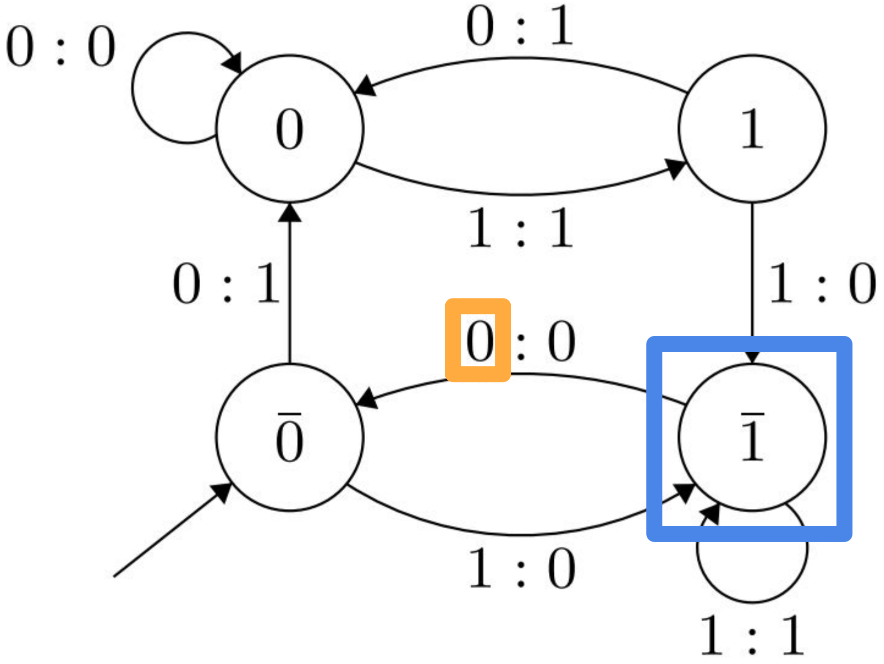
The 3x+1 binary FST

The 3x+1 operation is 2-automatic

Caption
read : write

$100\bar{1}\bar{0}$

0



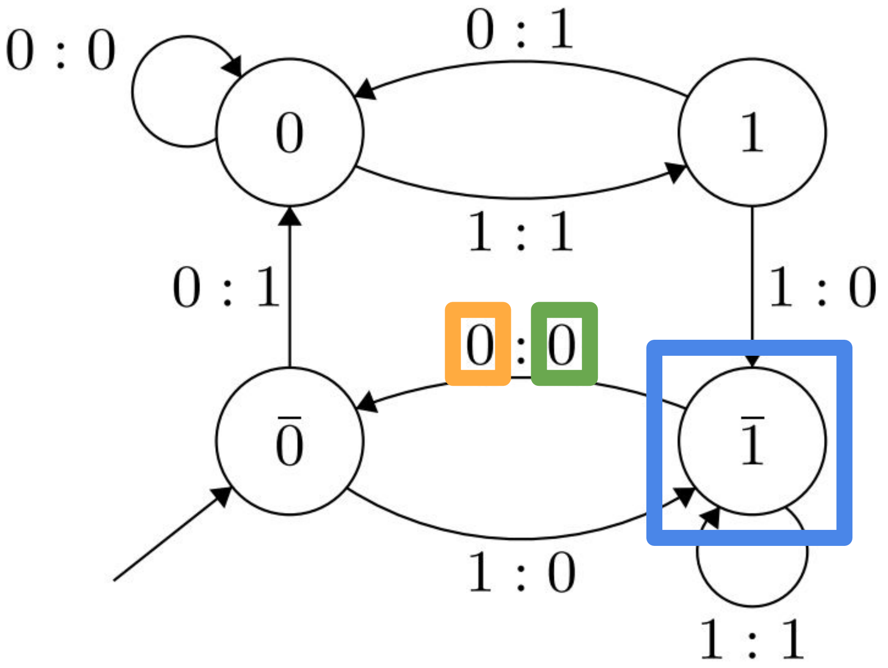
The 3x+1 binary FST

The 3x+1 operation is 2-automatic

Caption
read : write

$$100\bar{1}\bar{0}$$

$$00$$



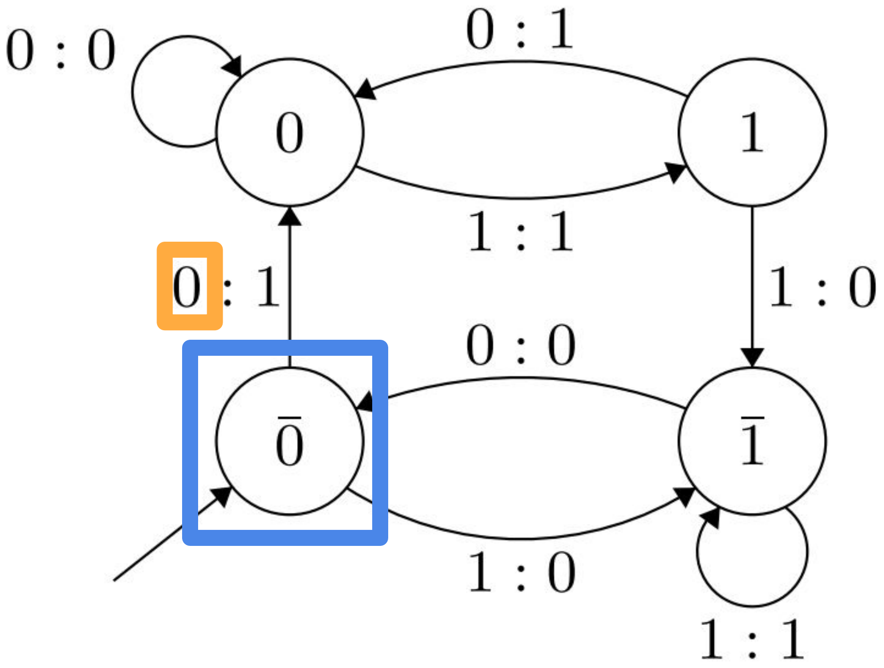
The 3x+1 binary FST

The 3x+1 operation is 2-automatic

Caption
read : write

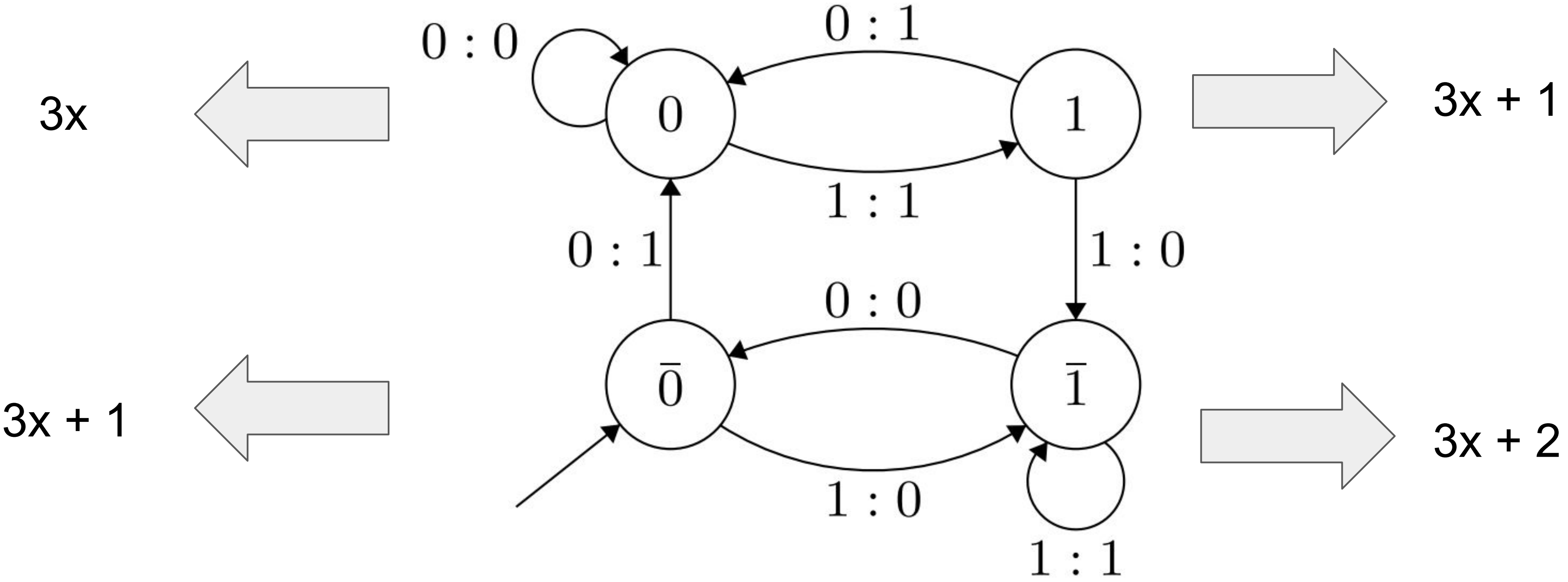
$1\,0\,\bar{0}\,\bar{1}\,\bar{0}$

00

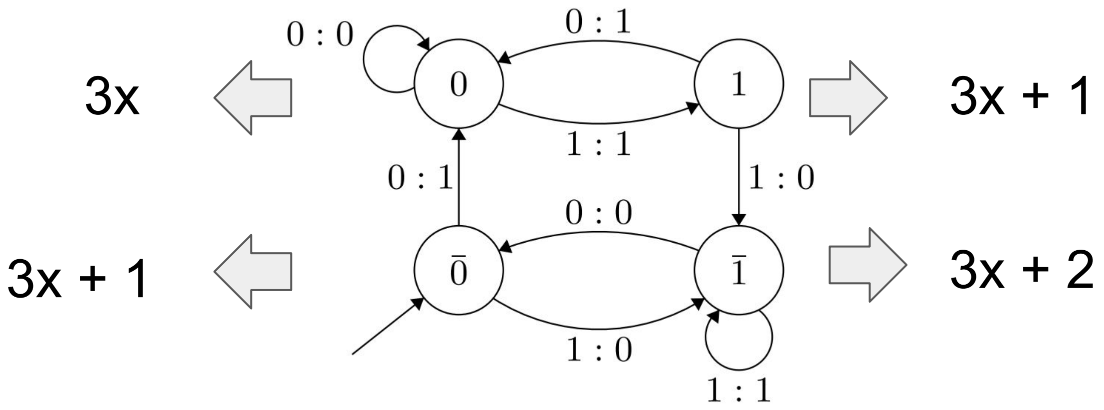


The 3x+1 binary FST

The 3x, 3x+1 and 3x+2 operations are 2-automatic



The 3x, 3x+1 and 3x+2 operations are 2-automatic



“Binary representation of ancestors in the Collatz graph”,
T. Stérin, RP 2020 → Regular expressions

“The Collatz process embeds a base conversion algorithm”,
T. Stérin and D. Woods, RP 2020 → Base 2 → Base 3

Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Let’s run the Collatz process on 642:

642 1010000010

Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Let’s run the Collatz process on 642:

321 1010000010

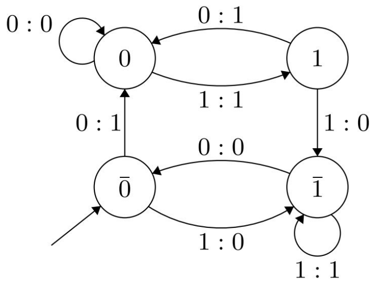
Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Let’s run the Collatz process on 642:

321   1010000010

964   1111000100

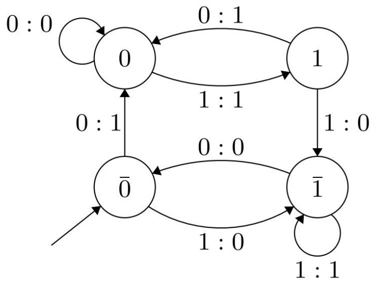


Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Let’s run the Collatz process on 321:

321	1010000010
241	1111000100



Binary representation of ancestors

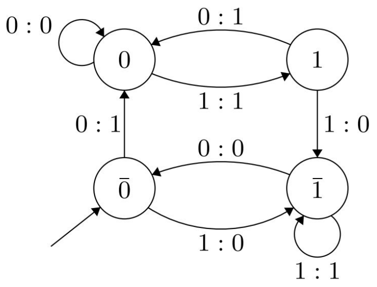
“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Let’s run the Collatz process on 642:

321 1010000010

241 1111000100

724 1011010100



Binary representation of ancestors

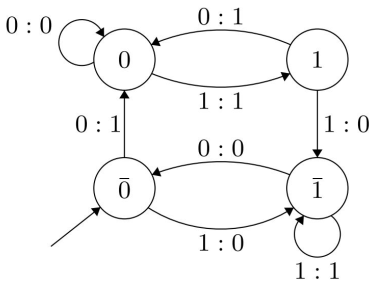
"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Let's run the Collatz process on 642:

321 1010000010

241 1111000100

181 1011010100

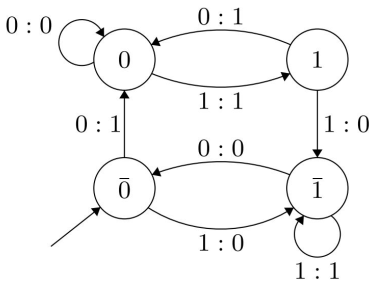


Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Let's run the Collatz process on 642:

321	1010000010
241	111100010 0
181	1011010100
544	1000100000



Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

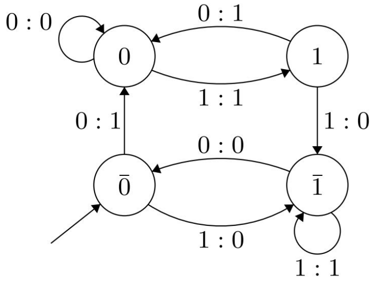
Let's run the Collatz process on 642:

321 1010000010

241 1111000100

181 1011010100

17 1000100000



642 is an ancestor of **17**

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

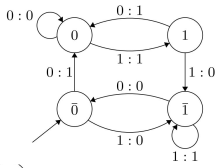
Let's run the Collatz process on 642:

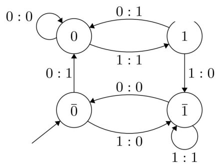
321 1010000010

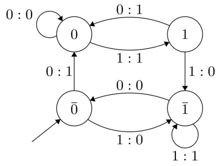
241 1111000100

181 1011010100

17 1000100000







642 is an ancestor of 17 at **"odd-distance"** k = 3

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

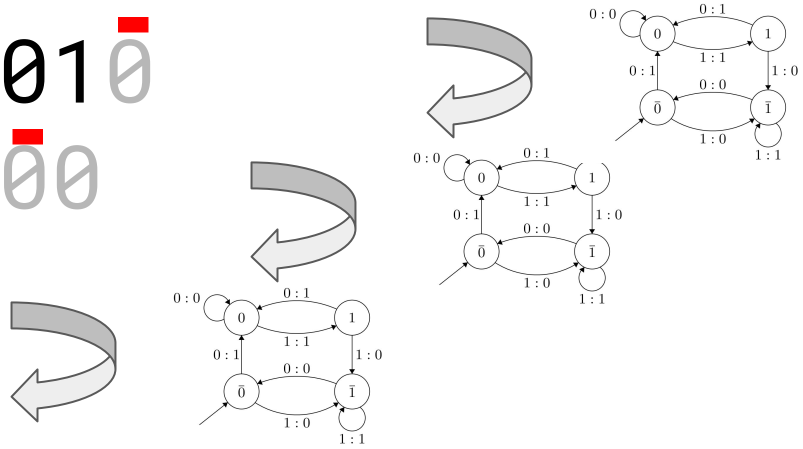
Let's run the Collatz process on 642:

321 1010000010

241 1111000100

181 1011010100

17 1000100000



Question:

What is the structure of all ancestors of 17 at "odd-distance" k = 3?

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

APPLE PATENT

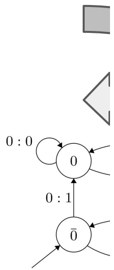
Let's run the Collatz process on 642:

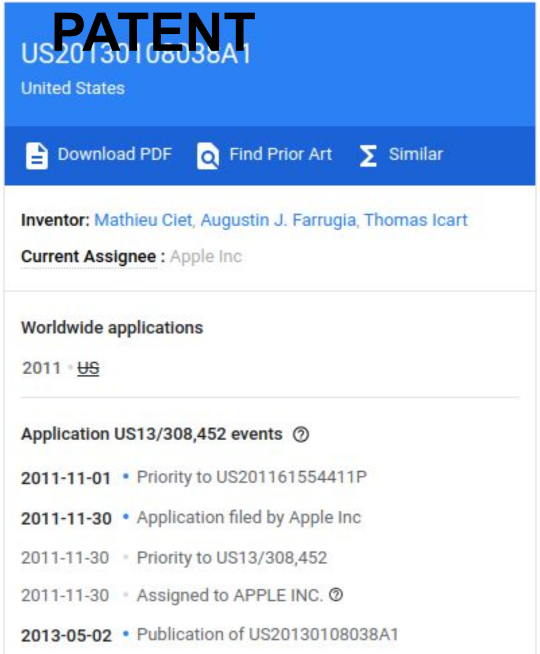
321 1010000010

241 1111000100

181 1011010100

17 1000100000





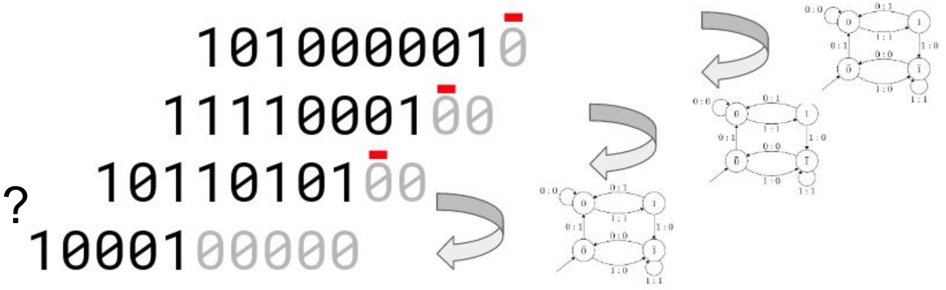
Question:

What is the structure of all ancestors of 17 at "odd-distance" k = 3?

Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Question:
What is the structure of all ancestors of 17 at “odd-distance” k = 3?



What is the structure of all ancestors of x at “odd-distance” k?

Let’s denote this set by **Pred(k,x)**

THEOREM:

Pred(k,x) is regular

J. Shallit and D. A. Wilson. *Bulletin of the EACTS*, 1992.

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Question:

What is the structure of all ancestors of x at "odd-distance" k?

THEOREM:

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J. Shallit and D. A. Wilson. *Bulletin of the EACTS*, 1992. ⇨ Doubly exponential regular expressions

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Question:
What is the structure of all ancestors of x at "odd-distance" k?

THEOREM:

Pred(k,x) is regular

J. Shallit and D. A. Wilson. *Bulletin of the EACTS*, 1992. → Doubly exponential regular expressions

Our Work → Simply exponential regular expressions +

Structure of these regular expressions tightly related to the structure of the Collatz process's **Parity Vectors**

Our work generalises: L. Colussi. *TCS*, 2011.
P. C. Hew. *TCS*, 2016.

Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Example: ancestors of 17 at distance 3.

(1010000100101111101)*10100(000111)*00(01)*01(0)*

642 1010000010

1111000100

1011010100

1000100000

17

Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Example: ancestors of 17 at distance 3.

(10100000100101111101)*10100(000111)*00(01)*01(0)*

10100000010
1111000100
1011010100
1000100000
17

10100000111000010
1111000101010100
1011010100000000
10001000000
17

Binary representation of ancestors

“Binary representation of ancestors in the Collatz graph”, T. Stérin, RP 2020

Example: ancestors of 17 at distance 3. There are **12 families.**

```
(101000010010111101)*10100(000111)*00(01)*01(0)*
(101000010010111101)*1010000(100011)*100(01)*01(0)*
(101000010010111101)*101000010010111110(110001)*1100(01)*01(0)*
(101000010010111101)*1010000100101(111000)*11100(01)*01(0)*
(101000010010111101)*10100001001(011100)*011100(01)*01(0)*
(101000010010111101)*1(001110)*0(01)*01(0)*
(101000010010111101)*10100(000111)*0001(10)*1(0)*
(101000010010111101)*1010000(100011)*10001(10)*1(0)*
(101000010010111101)*101000010010111110(110001)*110001(10)*1(0)*
(101000010010111101)*1010000100101(111000)*1(10)*1(0)*
(101000010010111101)*10100001001(011100)*01(10)*1(0)*
(101000010010111101)*1(001110)*001(10)*1(0)*
```

Code: https://github.com/tcosmo/coreli

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Result: ancestors of x at distance k. There are $2^{k-1}3^{\frac{(k-1)(k-2)}{2}}$ **families.**

▶ **Theorem 1.** *For all $x \in \mathbb{N}$, for all $k \in \mathbb{N}$ there exists a regular expression $\mathtt{reg}_k(x)$ that defines $\mathcal{E}Pred_k(x)$. The regular expression $\mathtt{reg}_k(x)$ is structured as a tree with $2^k 3^{k(k-1)/2}$ branches, alphabetic width $O(2^k 3^{k(k+1)/2})$ and star height equal to 1.*

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

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Erratum:
k := k-1

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Example: ancestors of 17 at distance 3. There are **12 families.**

```
(101000010010111101)*10100(000111)*00(01)*01(0)*
(101000010010111101)*1010000(100011)*100(01)*01(0)*
(101000010010111101)*10100001001011110(110001)*1100(01)*01(0)*
(101000010010111101)*1010000100101(111000)*11100(01)*01(0)*
(101000010010111101)*10100001001(011100)*011100(01)*01(0)*
(101000010010111101)*1(001110)*0(01)*01(0)*
(101000010010111101)*10100(000111)*0001(10)*1(0)*
(101000010010111101)*1010000(100011)*10001(10)*1(0)*
(101000010010111101)*10100001001011110(110001)*110001(10)*1(0)*
(101000010010111101)*1010000100101(111000)*1(10)*1(0)*
(101000010010111101)*10100001001(011100)*01(10)*1(0)*
(101000010010111101)*1(001110)*001(10)*1(0)*
```

- Complexity: at least as hard as solving **discrete logarithm** in Z/3^kZ. I.e: find \`i\` such that x = 2^-i mod [3^k].

  Take: **len**()-2, here 5-2 = 3

  2^-(3) = 14^3 = 2744 = 17 mod 3^3

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Example: ancestors of 17 at distance 3. There are **12 families.**

```
(101000010010111101)*10100(000111)*00(01)*01(0)*
(101000010010111101)*1010000(100011)*100(01)*01(0)*
(101000010010111101)*10100001001011110(110001)*1100(01)*01(0)*
(101000010010111101)*1010000100101(111000)*11100(01)*01(0)*
(101000010010111101)*10100001001(011100)*011100(01)*01(0)*
(101000010010111101)*1(001110)*0(01)*01(0)*
(101000010010111101)*10100(000111)*0001(10)*1(0)*
(101000010010111101)*1010000(100011)*10001(10)*1(0)*
(101000010010111101)*10100001001011110(110001)*110001(10)*1(0)*
(101000010010111101)*1010000100101(111000)*1(10)*1(0)*
(101000010010111101)*10100001001(011100)*01(10)*1(0)*
(101000010010111101)*1(001110)*001(10)*1(0)*
```

- Complexity: at least as hard as solving **discrete logarithm** in Z/3^kZ. I.e: find \`i\` such that x = 2^-i mod [3^k].

- \`**Seeds**\` are mysterious:

  Equivalent definitions:
  - Repetend of 1/3^k in $\mathbb{Z}_2$
  - Parity bits of 1/2^k in $\mathbb{Z}/3^k\mathbb{Z}$
  - Binary expansion of $\dfrac{2^{2\cdot 3^{k-1}}}{3^k}$

Seeds approach **Full Complexity:**

J. Lòpez and P. Stoll. *Integers*, 2012

Binary representation of ancestors

"Binary representation of ancestors in the Collatz graph", T. Stérin, RP 2020

Example: ancestors of 17 at distance 3. There are **12 families.**

```
(101000010010111101)*10100(000111)*00(01)*01(0)*
(101000010010111101)*1010000(100011)*100(01)*01(0)*
(101000010010111101)*10100001001011110(110001)*1100(01)*01(0)*
(101000010010111101)*1010000100101(111000)*11100(01)*01(0)*
(101000010010111101)*10100001001(011100)*011100(01)*01(0)*
(101000010010111101)*1(001110)*0(01)*01(0)*
(101000010010111101)*10100(000111)*0001(10)*1(0)*
(101000010010111101)*1010000(100011)*10001(10)*1(0)*
(101000010010111101)*10100001001011110(110001)*110001(10)*1(0)*
(101000010010111101)*1010000100101(111000)*1(10)*1(0)*
(101000010010111101)*10100001001(011100)*01(10)*1(0)*
(101000010010111101)*1(001110)*001(10)*1(0)*
```

Futur work:

Efficiently finding the smallest ancestor at distance k?

Base conversion: The Collatz Quasi CA (CQCA)

“The Collatz process embeds a base conversion algorithm”, T. Stérin and D. Woods, RP 2020

Finite state automaton

Base conversion: The Collatz Quasi CA (CQCA)

"The Collatz process embeds a base conversion algorithm", T. Stérin and D. Woods, RP 2020

~~Finite state automaton~~

↓

2D Cellular automaton

Base conversion: The Collatz Quasi CA (CQCA)

“The Collatz process embeds a base conversion algorithm”, T. Stérin and D. Woods, RP 2020

2D <u>Cellular automaton</u>

- 1D Base 2 and Base 3 CAs

  M. Bruschi. *preprint*, 2005

- 1D Base 6 CA

  I. Korec. *Mathematica Slovaca*, 1992

Ours: **CQCA**, 2D

Runs base 2, 3
and 6 **simulteanously**.

Base conversion: The Collatz Quasi CA (CQCA)

"The Collatz process embeds a base conversion algorithm", T. Stérin and D. Woods, RP 2020

2D <u>Cellular automaton</u>

- 1D Base 2 and Base 3 CAs
  M. Bruschi. *preprint*, 2005.
- 1D Base 6 CA
  I. Korec. *Mathematica Slovaca*, 1992.
- Quasi base 2 CA
  T. Cloney, E. Goles and G. Vichiniac. *Complex Systems*, 1987.

Eric Goles, Nicolas Ollinger,
and Guillaume Theyssier.

Ours: **CQCA**, 2D

Runs base 2, 3
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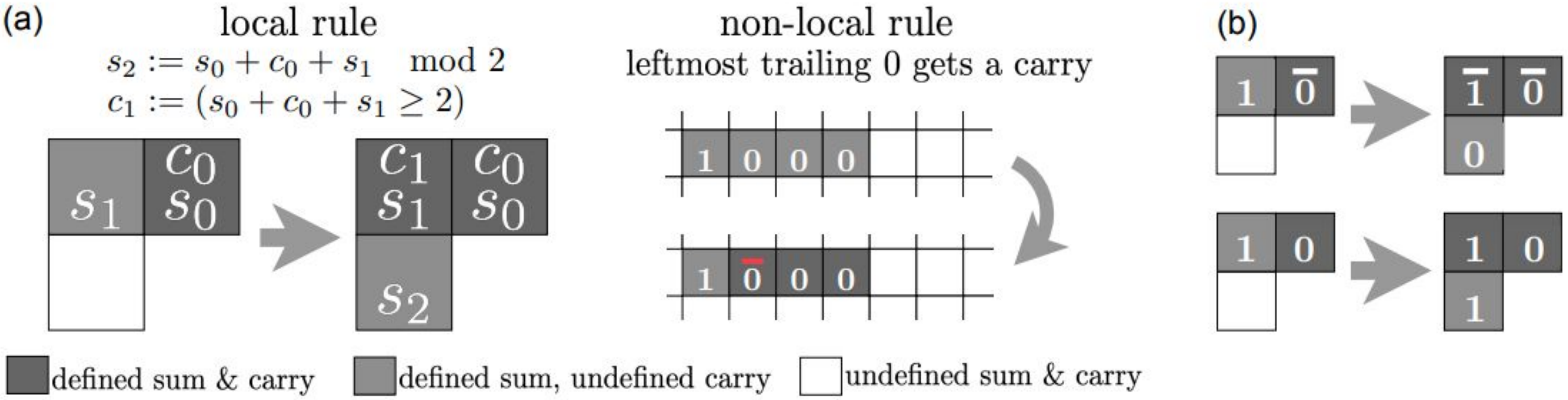
Base conversion: The Collatz Quasi CA (CQCA)

“The Collatz process embeds a base conversion algorithm”, T. Stérin and D. Woods, RP 2020

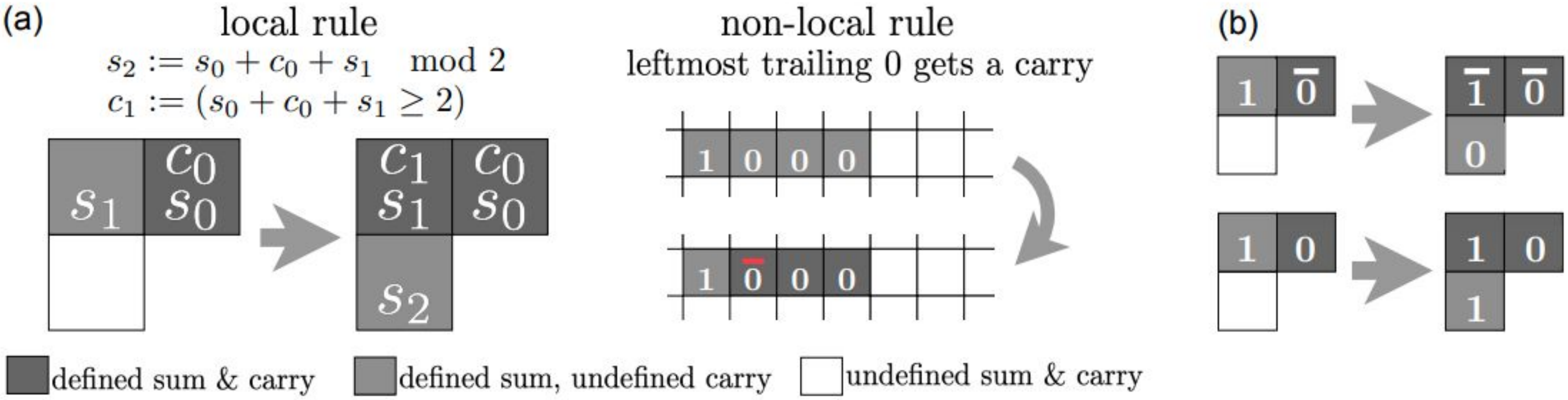
The CQCA: defined in $\mathbb{Z}^2$

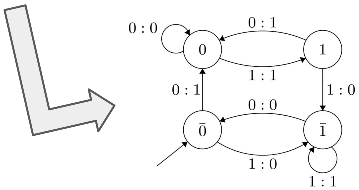


Base conversion: The Collatz Quasi CA (CQCA)

“The Collatz process embeds a base conversion algorithm”, T. Stérin and D. Woods, RP 2020

The CQCA: defined in $\mathbb{Z}^2$





Simulator: `simcqca`

Code: https://github.com/tcosmo/simcqca

Base conversion: The Collatz Quasi CA (CQCA)

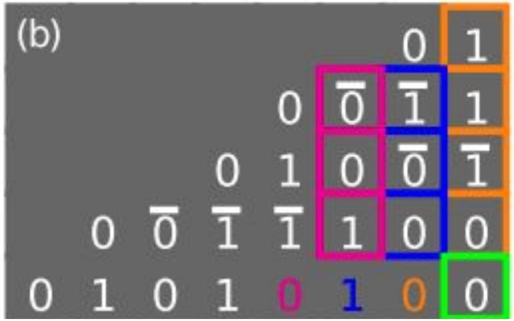
“The Collatz process embeds a base conversion algorithm”, T. Stérin and D. Woods, RP 2020

Base 3’:	Base 3:
0	0
$\overline{0}$	1
1	1
$\overline{1}$	2

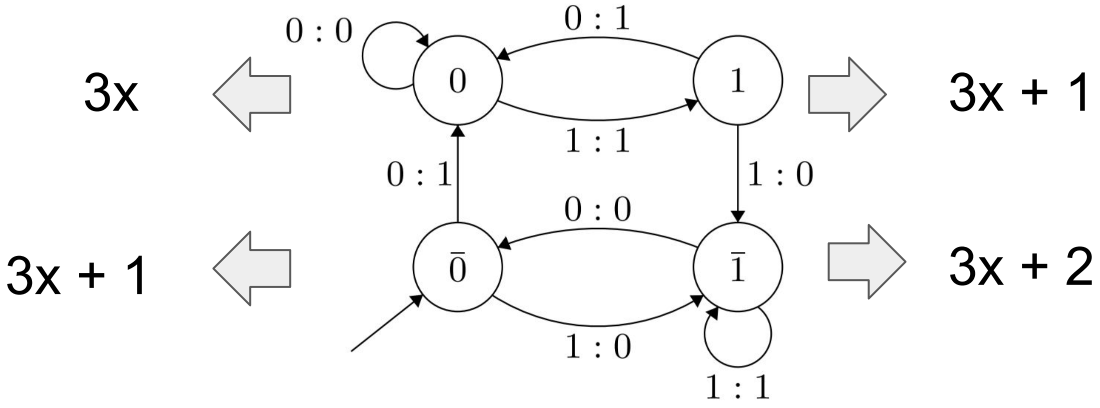
Base conversion: The Collatz Quasi CA (CQCA)

"The Collatz process embeds a base conversion algorithm", T. Stérin and D. Woods, RP 2020

Base 3':	Base 3:
0	0
$\bar{0}$	1
1	1
$\bar{1}$	2



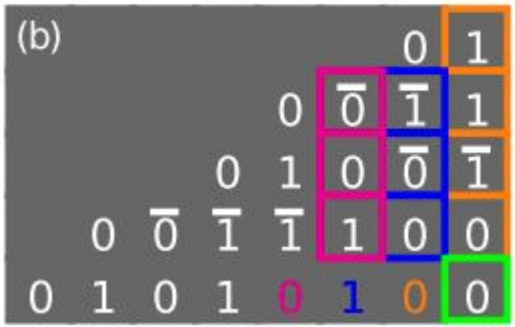
Line by Line

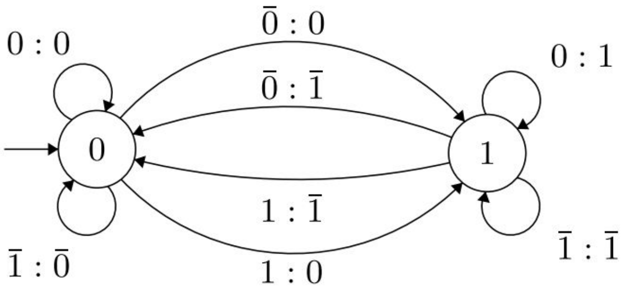


Base conversion: The Collatz Quasi CA (CQCA)

"The Collatz process embeds a base conversion algorithm", T. Stérin and D. Woods, RP 2020

Column by column





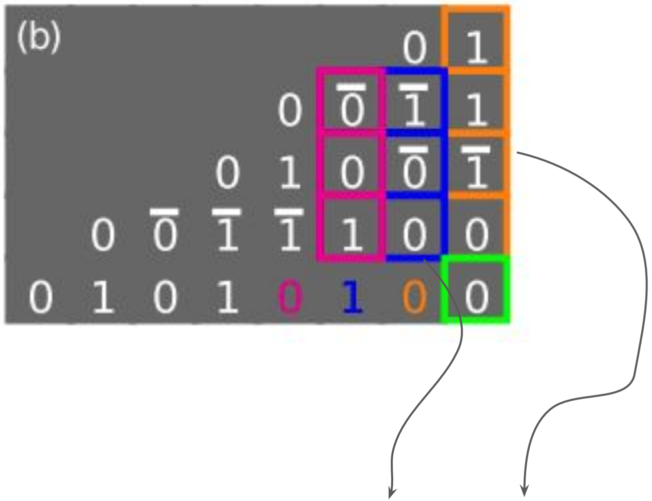
(b) The $x/2$ base $3'$ FST.

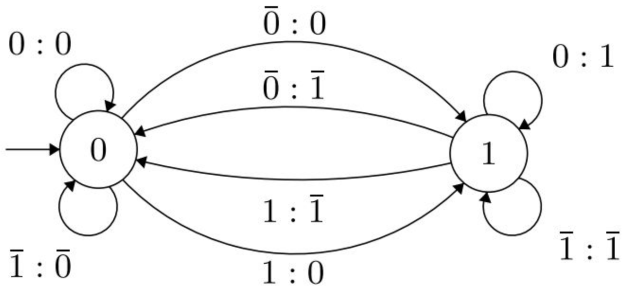
\`1120\` = 42

Base conversion: The Collatz Quasi CA (CQCA)

"The Collatz process embeds a base conversion algorithm", T. Stérin and D. Woods, RP 2020

Column by column





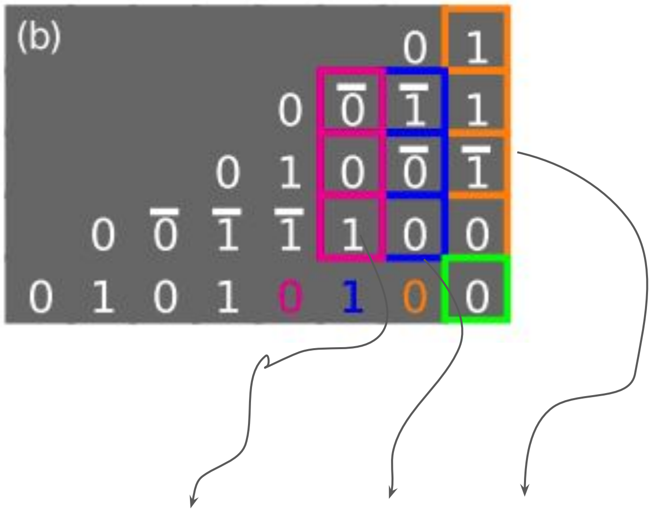
(b) The $x/2$ base $3'$ FST.

\`210\` = 21 \`1120\` = 42

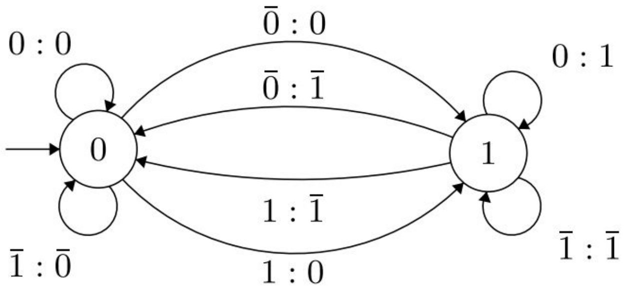
Base conversion: The Collatz Quasi CA (CQCA)

“The Collatz process embeds a base conversion algorithm”, T. Stérin and D. Woods, RP 2020

Column by column



`101` = 10   `210` = 21   `1120` = 42



(b) The $x/2$ base $3'$ FST.

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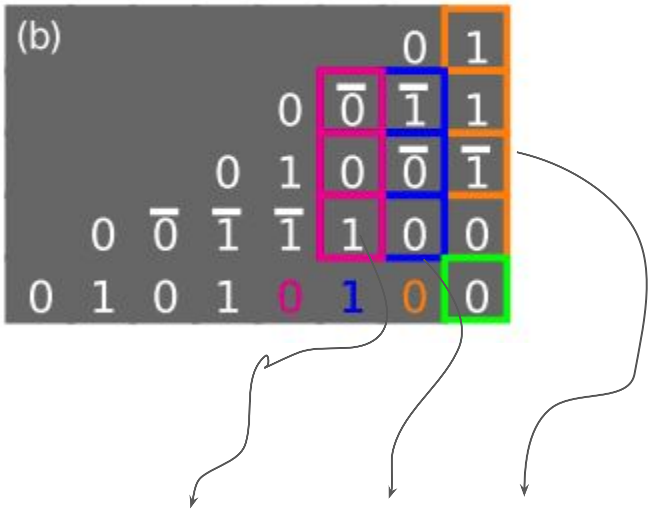

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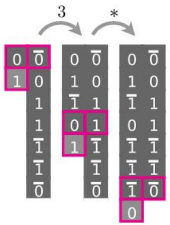
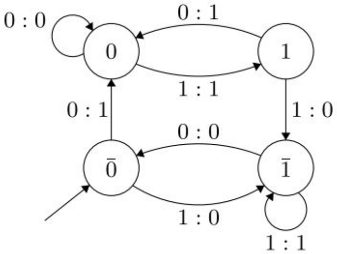
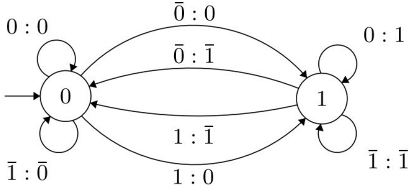
(c) Horizontal applications of the local CQCA rule simulate the $3x + 1$ FST.



(d) Vertical applications of the local rule simulate the $x/2$ FST.



(a) The $3x + 1$ binary FST.



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Dual Transducers

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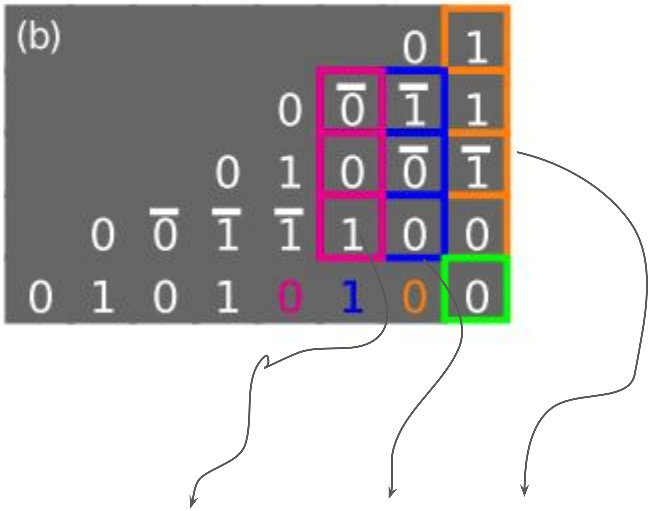

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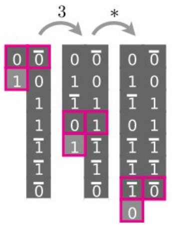

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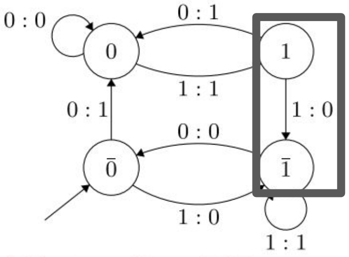

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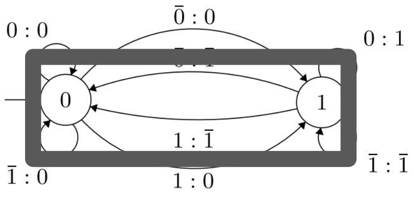

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Motivation

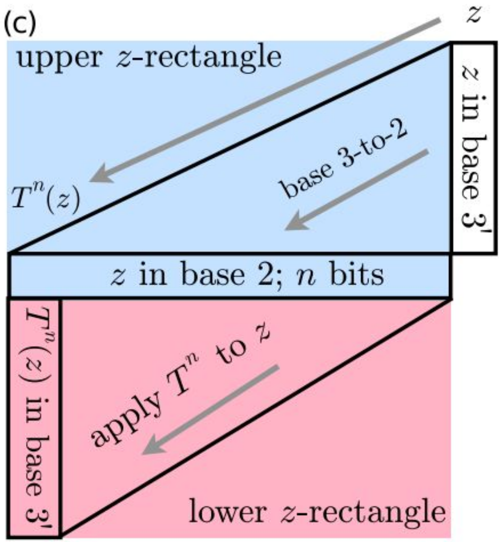
- Computational power of the Collatz process? Is it Turing complete?
- Complexity of prediction problem?

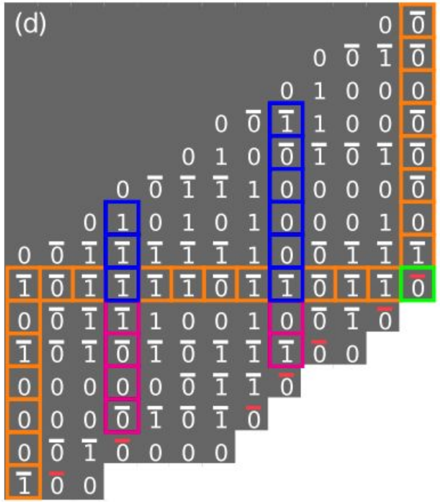
Complexity of prediction:

AC0: polysize, constant depth, arbitrary fanin, Bolean circuits

NC1: polysize, **log** depth, fanin <= 2, Bolean circuits

AC0 != P, problem PARITY not in AC0





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The upper z-rectangle prediction problem is in NC1 and not in AC0.

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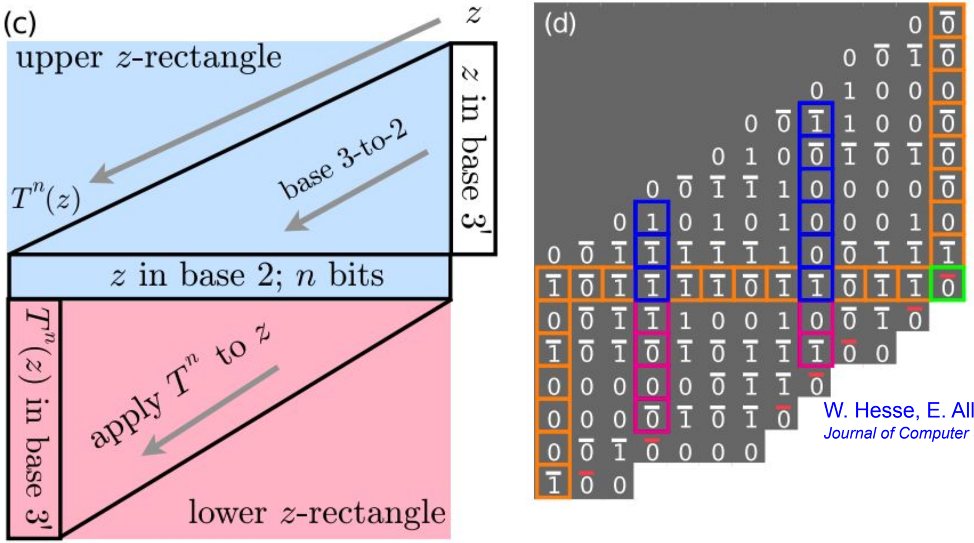

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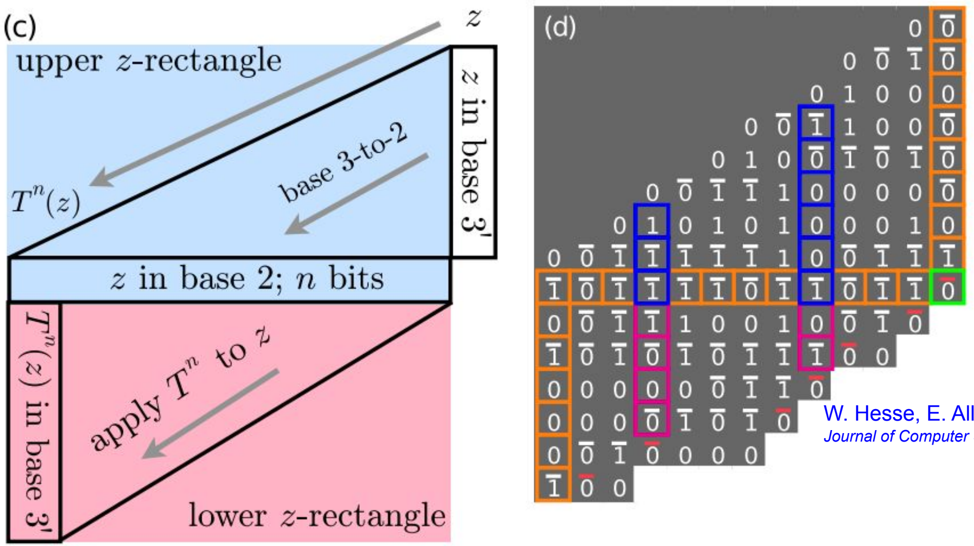

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Answering if a base 3-encoded number is odd or even reduces to PARITY.

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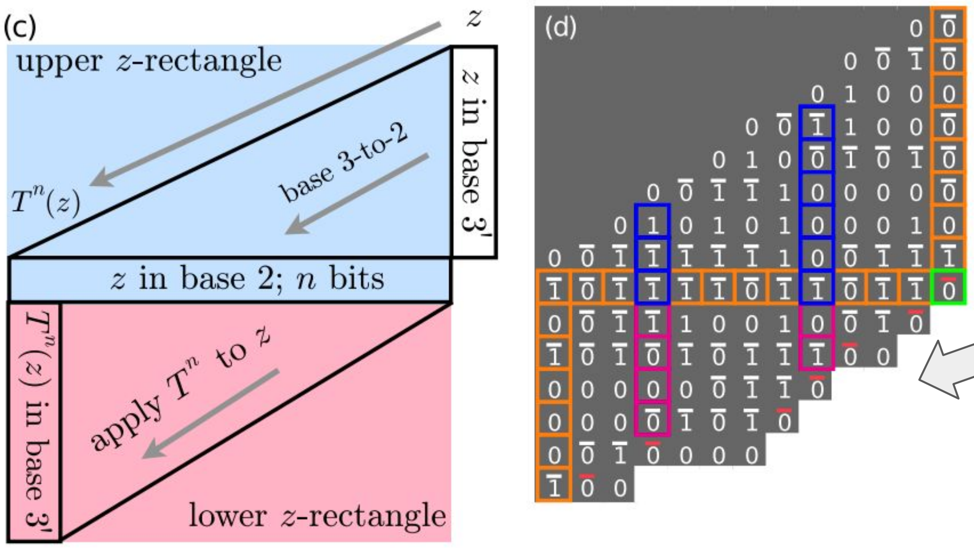
Future Work



Theorem 28 (arxiv numbering):
The upper z-rectangle prediction problem is in NC1 and not in AC0.

What about the lower z-rectangle?

In P, not in AC0. Can we get matching bounds?

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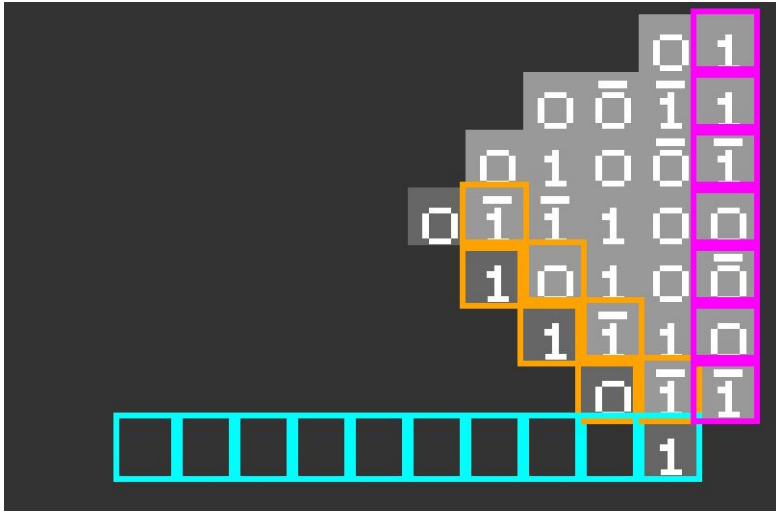

What about this diagonal?

Magenta column: x = \`1120102\` = 1145

Blue line: x = \`10001111001\` = 1145

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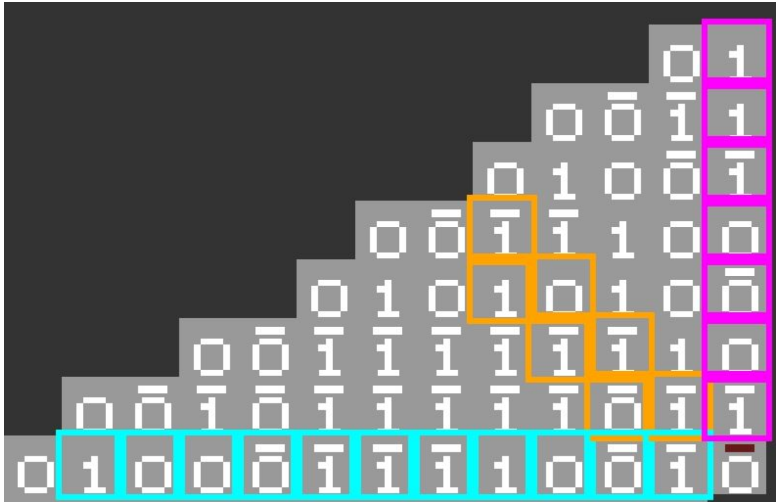
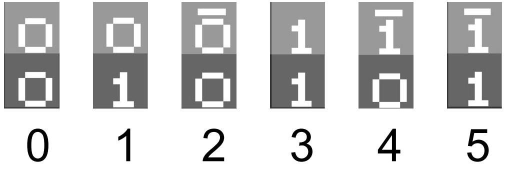


What about this diagonal?

Magenta column: x = `1120102` = 1145
Blue line: x = `10001111001` = 1145

Diagonal is x in base 6

x = `5145` = 1145



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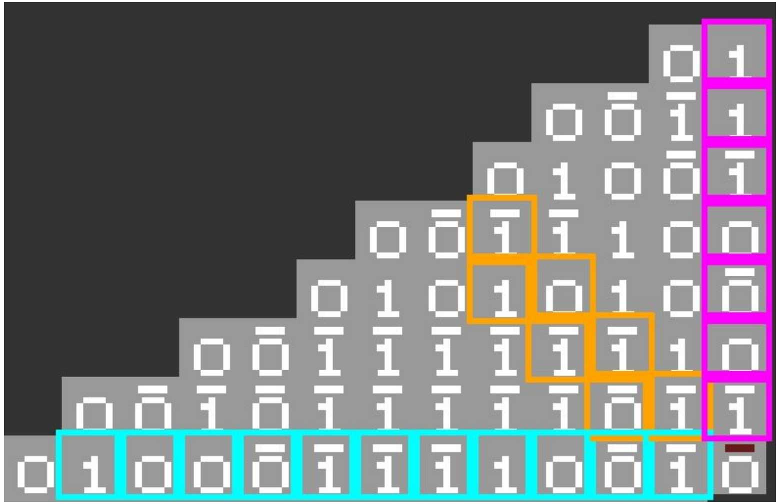
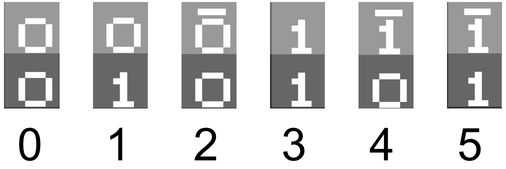


Need to further understand this encoding

- Link with Chinese Remainder
- Link with Z2 x Z3 = Z6

x = \`5145\` = 1145



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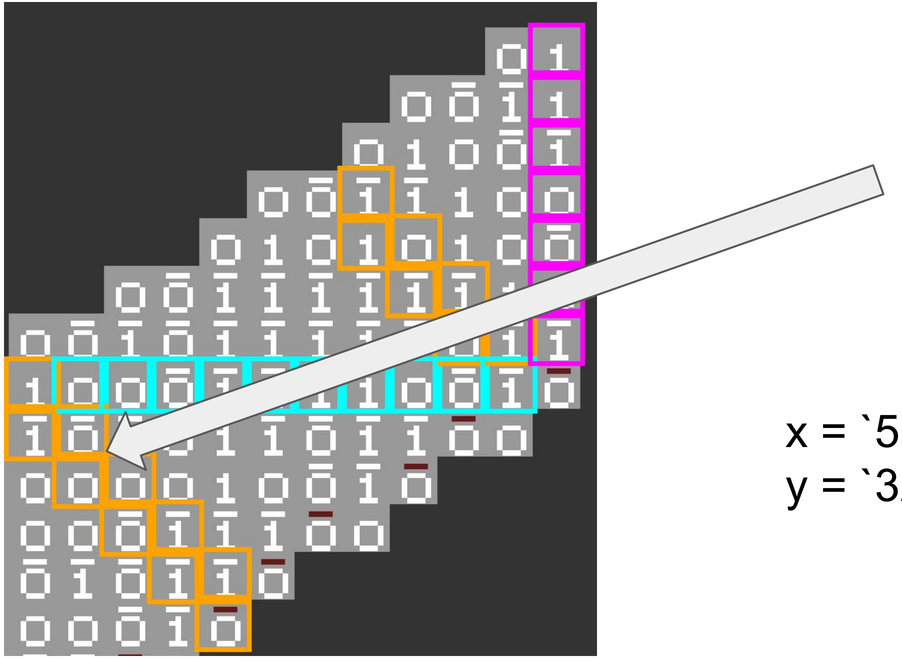
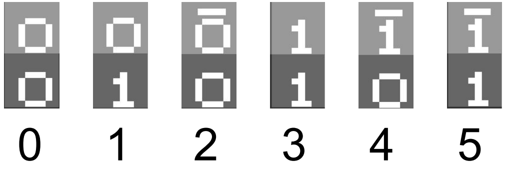


What about that other diagonal?

- What does it compute?

x = \`5145\` = 1145
y = \`32054\` = 4354



Conclusion

- Using base 2/3/6 Finite State Automaton and Cellular Automaton descriptions of the Collatz process brings it in the scope of Computer Science.

- In that context we have a sound framework to ask:

  - What is the computational power of the Collatz process? Is it Turing complete?
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So, what's next?

Acknowledgement

- Damien Woods, Maynooth University, Ireland
- Jose Capco, University of Innsbruck, Austria
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- Olivier Rozier, IPGP, Paris
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Maynooth University National University of Ireland Maynooth

Hamilton Institute

erc European Research Council Established by the European Commission

Science Foundation Ireland SFI For what's next

ERC No 772766, SFI 18/ERCS/5746