# Small tile sets that compute while solving mazes

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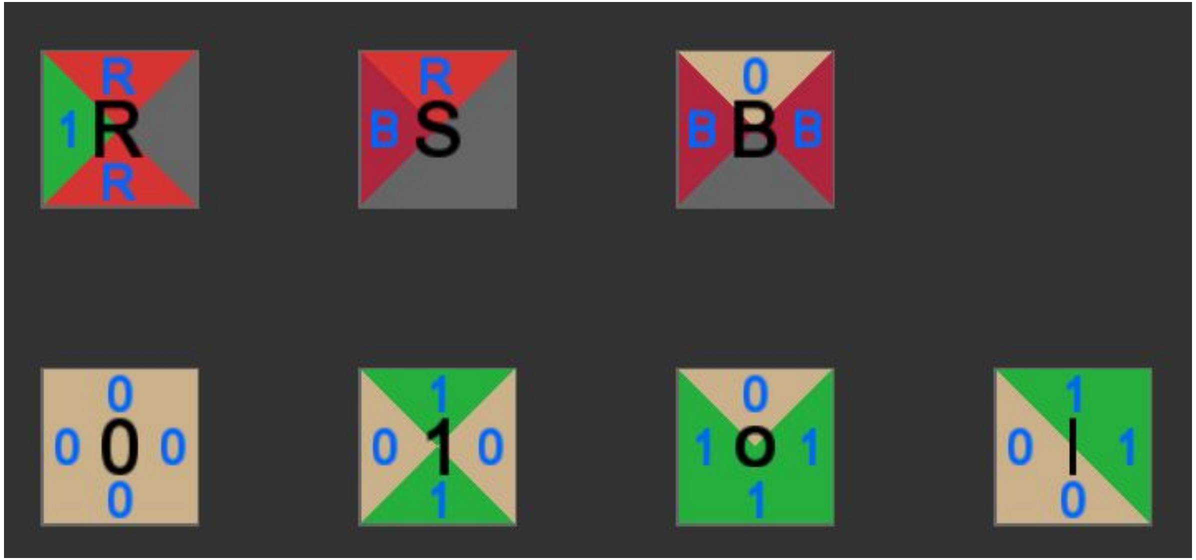
# The abstract Tile Assembly Model (Winfree 1998)

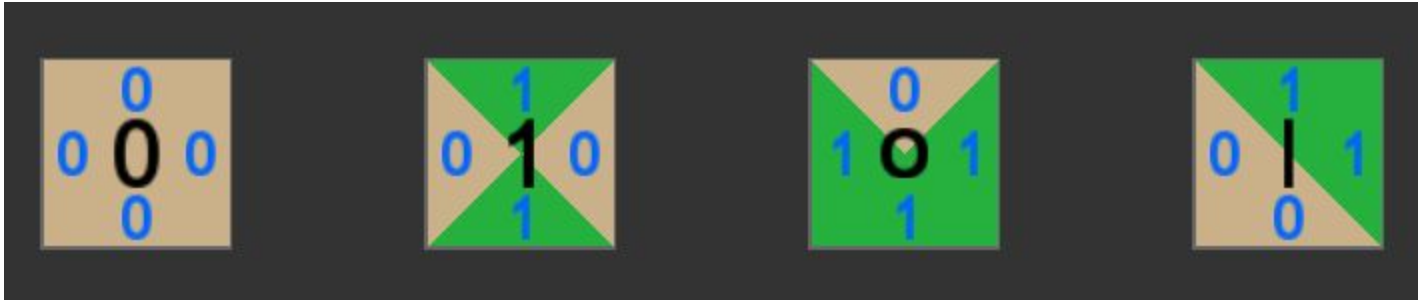
E. Winfree. PhD thesis. (1998)




Assembling a binary counter using 7 tiles and 4 glues (red glues strength 2, beige and green glues strength 1) in the aTAM (temperature 2).

P. W. K. Rothemund and E. Winfree. “The Program-Size Complexity of Self-Assembled Squares”. *STOC* (2000)
M. J. Patitz. “An Introduction to Tile-Based Self-Assembly”. *Natural Computing* 13, 195–224 (2014)

# Introducing the Maze-Walking TAM (MaWaTAM)

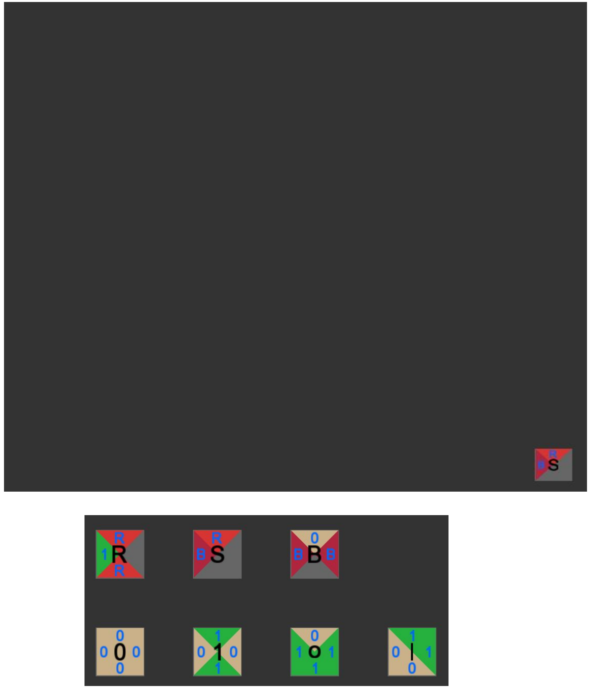



Assembling a binary counter using 4 tiles and 2 glues (strength 1) in the MaWaTAM (temp 2)

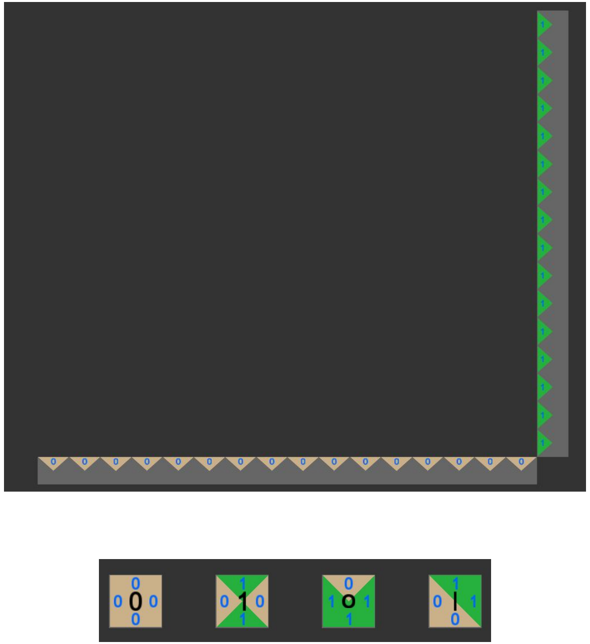
## Key Point

A seed in the MaWaTAM is a **disconnected** set of polyominos that can make use of any glue of the tile set (plus null glue of strength 0).

# Binary counter: aTAM vs MaWaTAM

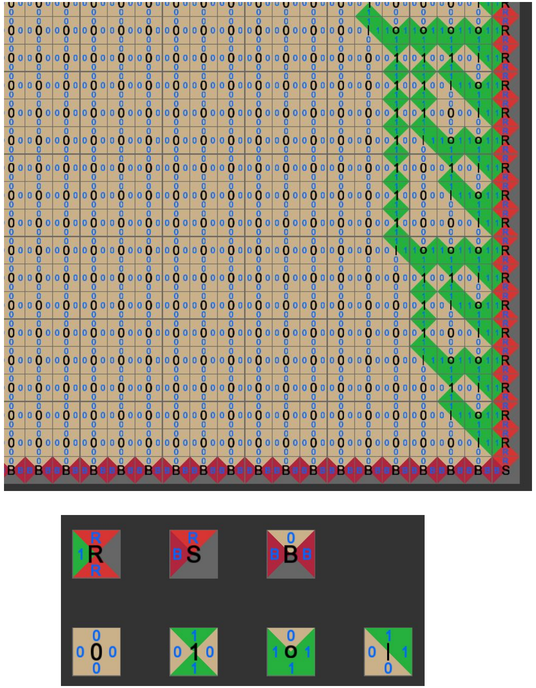


aTAM

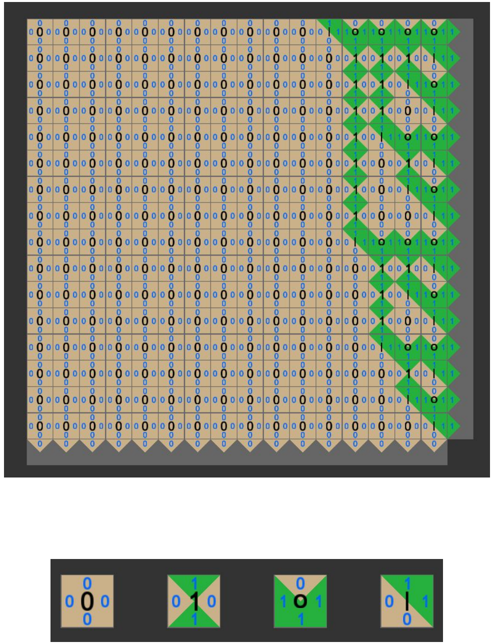


Maze-Walking TAM

# Binary counter: aTAM vs MaWaTAM



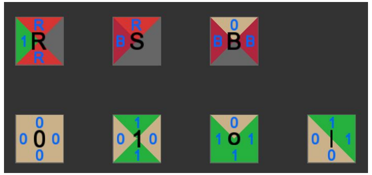
aTAM



MaWaTAM

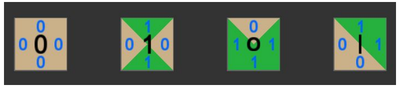
# Binary counter: aTAM vs MaWaTAM

1. Unbounded structure
2. Tile set responsible for structure & computation
3. Seed triggers the computation



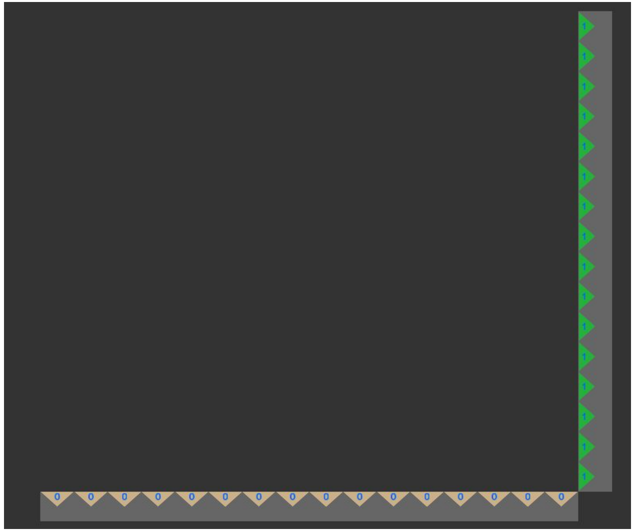
aTAM

1. Bounded structure
2. Tile set responsible for computational power
3. Seed responsible for structure/function to compute

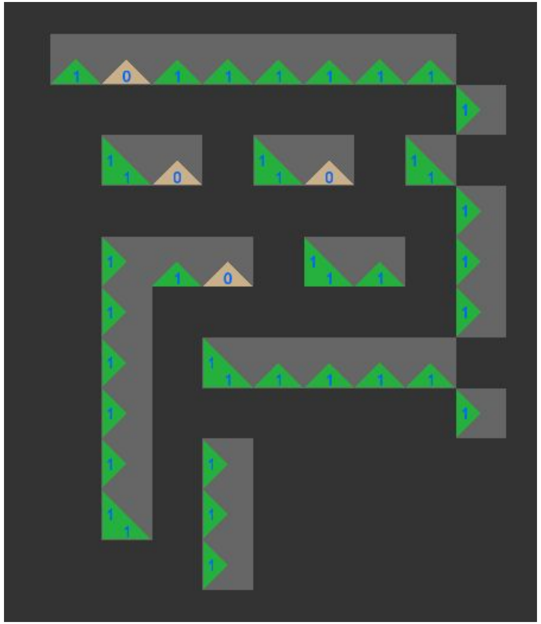


MaWaTAM

# Physical reality behind disconnected seeds



Disconnected MaWaTAM seed for assembling binary counter.



A more complex disconnected MaWaTAM seed. It looks like a **maze**!

# Physical reality behind disconnected seeds

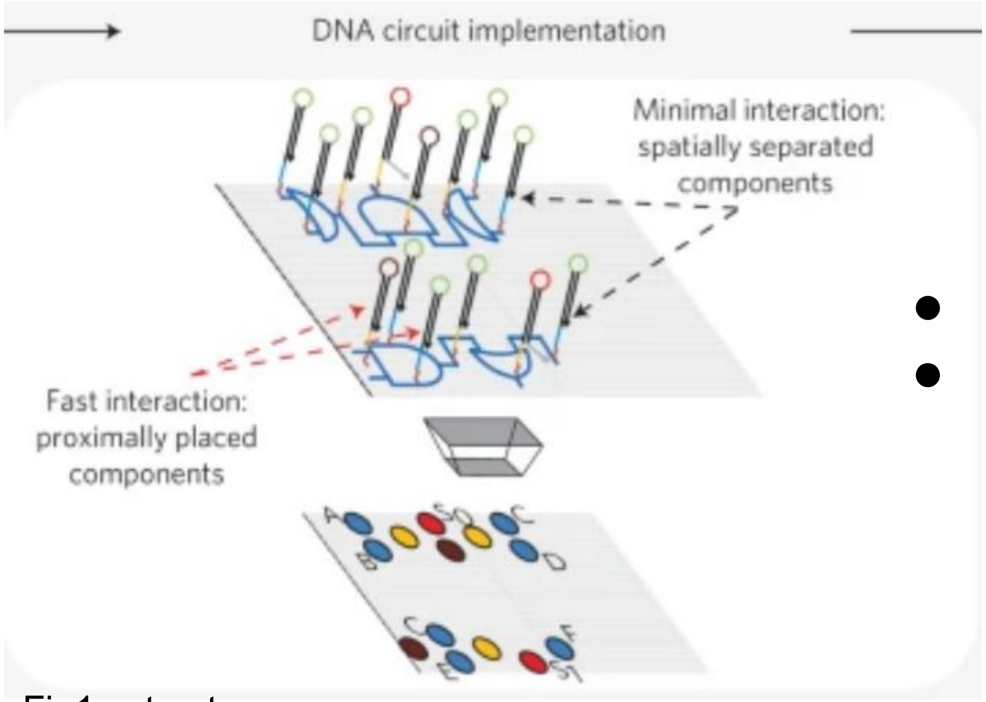



Fig1 extract.

- Underlying DNA origami surface
- Localised computation

Gourab Chatterjee, Neil Dalchau, Richard A. Muscat, Andrew Phillips, and Georg Seelig. “A spatially localized architecture for fast and modular DNA computing”. *Nature Nanotechnology*, 12(9):920–927, Sep 2017.

# Physical reality behind disconnected seeds

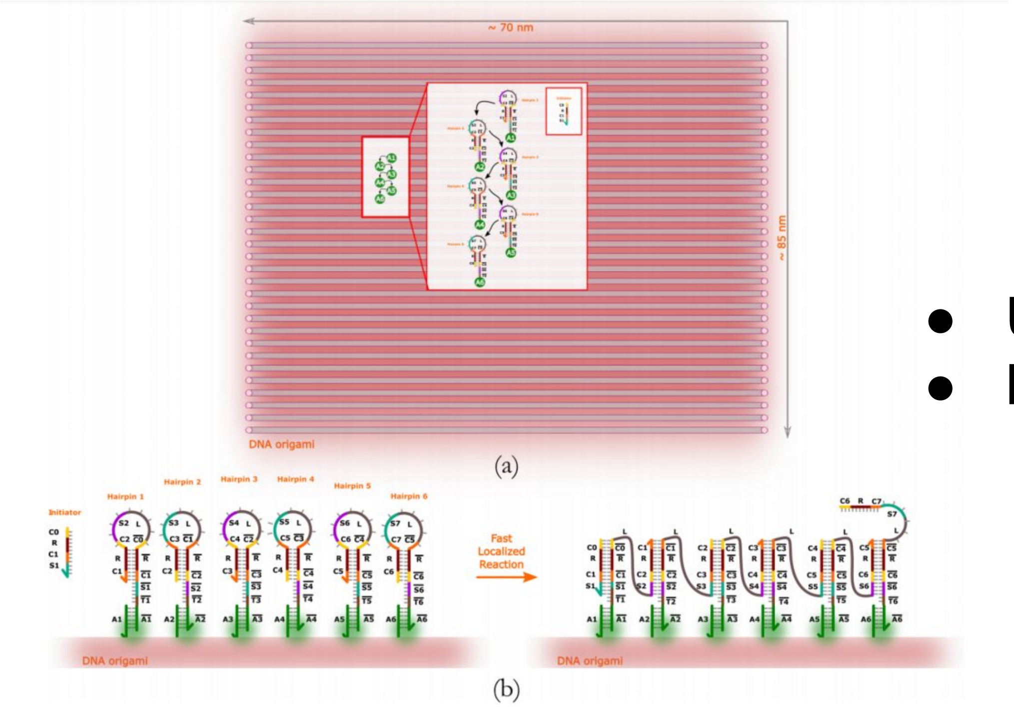
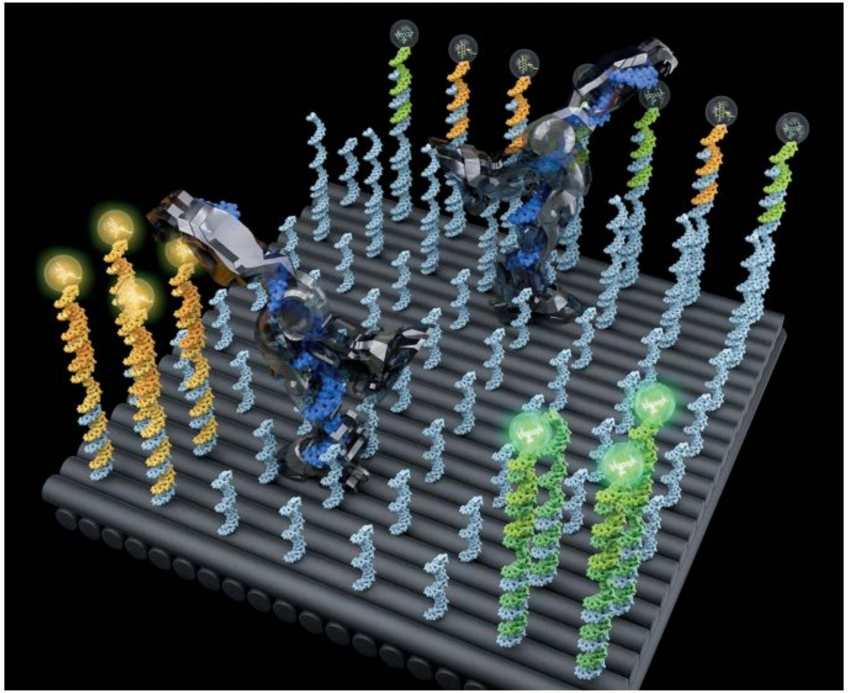



Fig1 extract.

- Underlying DNA origami surface
- Localised computation

Hieu Bui, Shalin Shah, Reem Mokhtar, Tianqi Song, Sudhanshu Garg, and John Reif. “Localized DNA hybridization chain reactions on DNA origami”. *ACS Nano*, 12(2):1146–1155, Feb 2018.

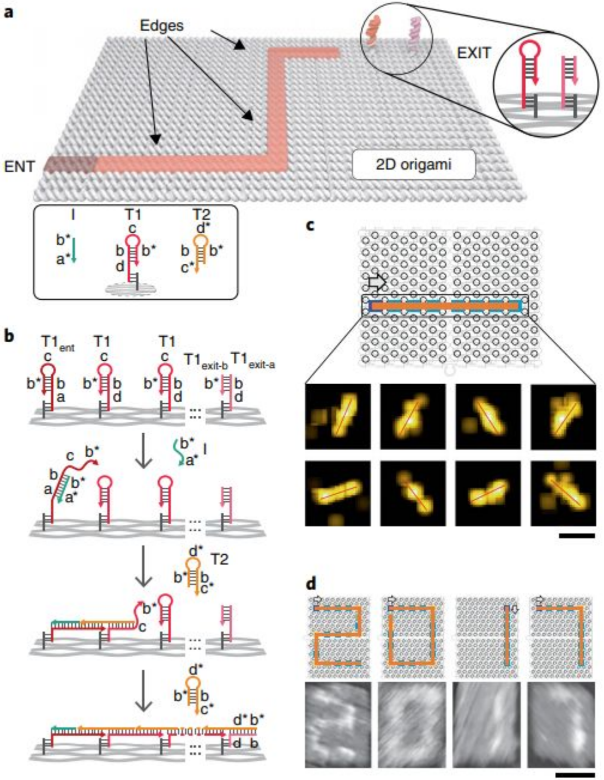
# Physical reality behind disconnected seeds



- Underlying DNA origami surface
- DNA Walker

Anupama J. Thubagere, Wei Li, Robert F. Johnson, Zibo Chen, Shayan Doroudi, Yae Lim Lee, Gregory Izatt, Sarah Wittman, Niranjan Srinivas, Damien Woods, Erik Winfree, and Lulu Qian. “A cargo-sorting DNA robot”. *Science*, 357(6356), 2017.

# Physical reality behind disconnected seeds




- Underlying DNA origami surface
- Solving mazes

Jie Chao, Jianbang Wang, Fei Wang, Xiangyuan Ouyang, Enzo Kopperger, Huajie Liu, Qian Li, Jiye Shi, Lihua Wang, Jun Hu, Lianhui Wang, Wei Huang, Friedrich C. Simmel, and Chunhai Fan. “Solving mazes with single-molecule DNA navigators”. *Nature Materials*, 18(3):273–279, Mar 2019.

# Similar theoretical work

Lulu Qian and Erik Winfree. “Parallel and scalable computation and spatial dynamics with dna-based chemical reaction networks on a surface”. In Satoshi Murata and Satoshi Kobayashi, editors, DNA Computing and Molecular Programming, pages 114–131, Cham, 2014. Springer International Publishing.

Samuel Clamons, Lulu Qian, and Erik Winfree. “Programming and simulating chemical reaction networks on a surface”. Journal of The Royal Society Interface, 17(166):20190790, 2020.

Tatiana Brailovskaya, Gokul Gowri, Sean Yu, and Erik Winfree. “Reversible computation using swap reactions on a surface”. In Chris Thachuk and Yan Liu, editors, DNA Computing and Molecular Programming, pages 174–196, Cham, 2019. Springer International Publishing.

# The question we ask

How small can a tileset be and still provide *interesting* computational power?

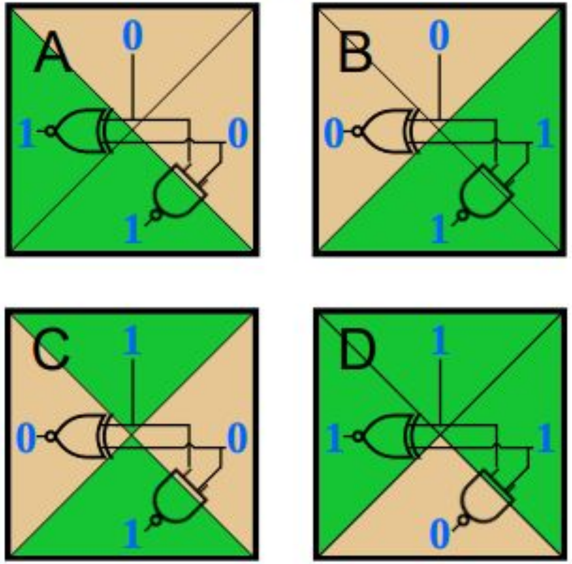
By *interesting* we mean the ability to run any Boolean circuit.

## **Intuition**

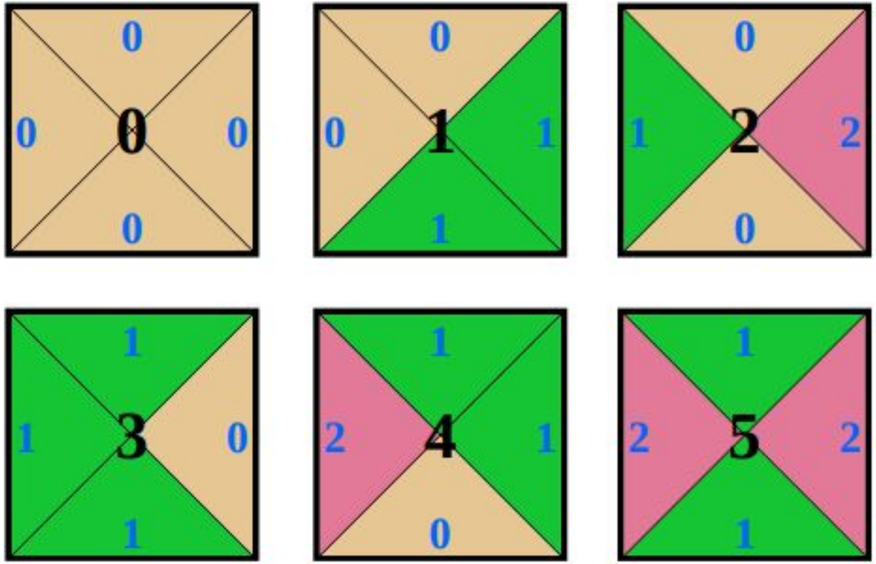
1. Disconnected seeds encode circuits
2. Tile sets give the computational power

# Our result

(a) NAND-NXOR

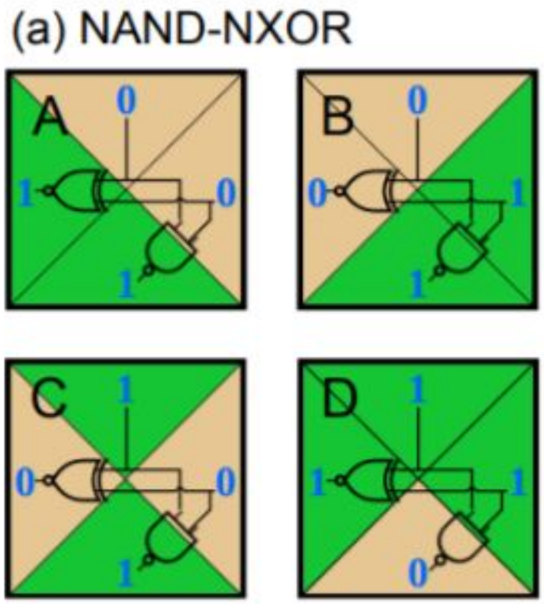


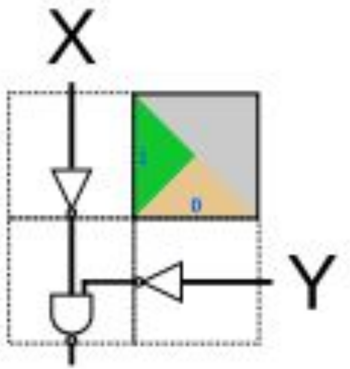

(b) Collatz




We found two small tile sets (4 and 6 tiles) that both can implement **any** Boolean circuit in the MaWaTAM.

# Making gates with the 4 tiles tile set

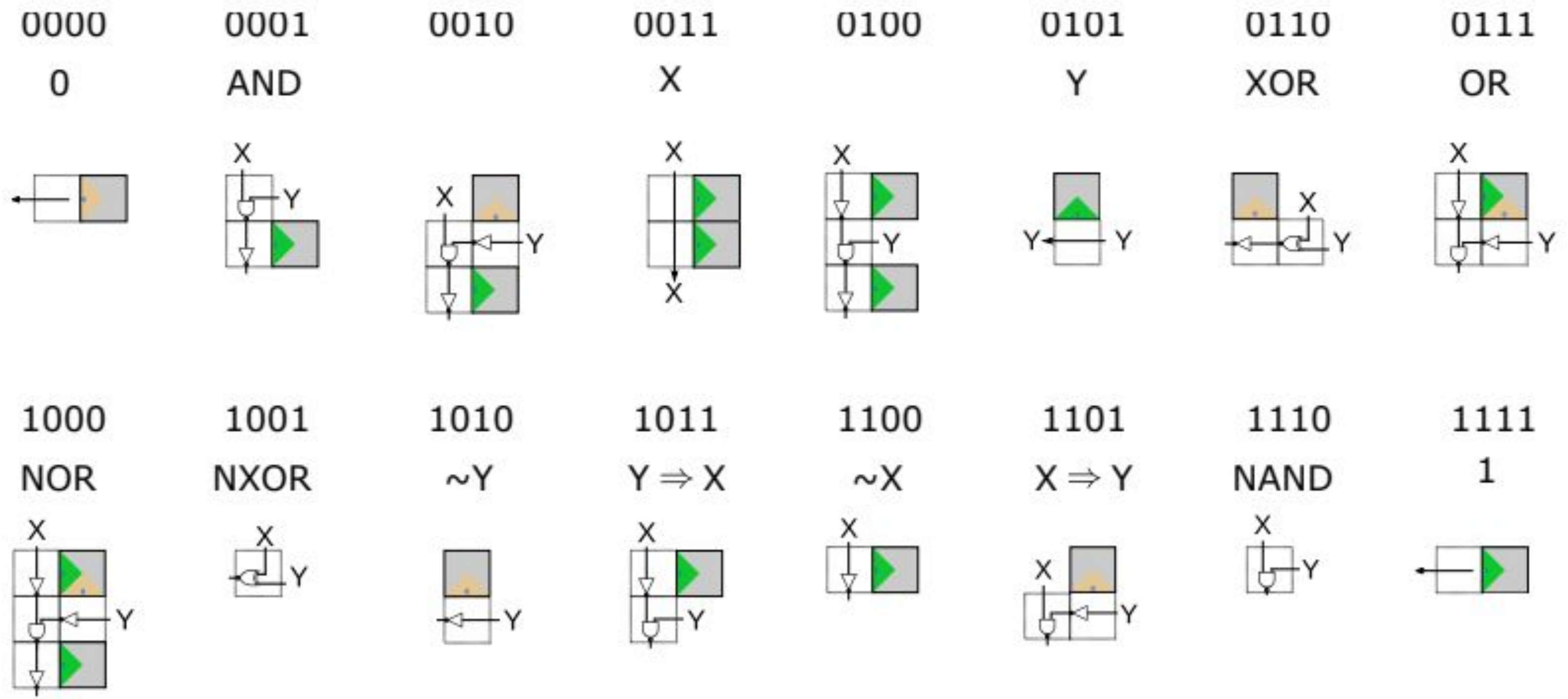



Seed to build a “OR” gate:

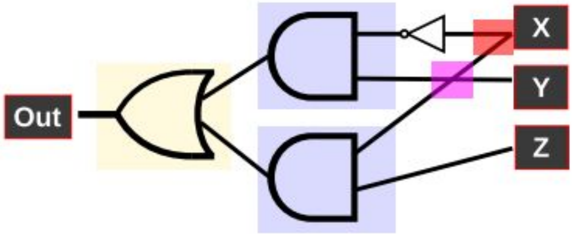
X OR Y = (NOT X) NAND (NOT Y)

# Making gates with the 4 tiles tile set

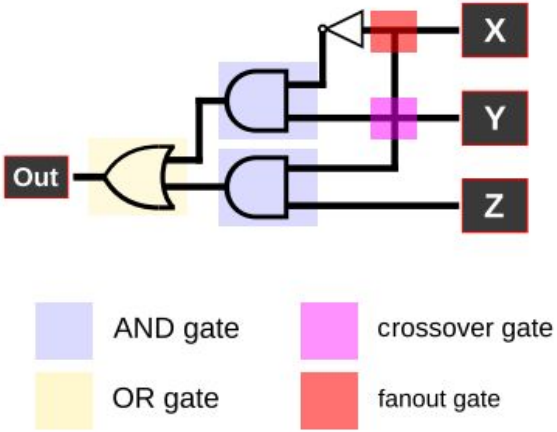


# Making arbitrary circuits with the 4 tiles tile set

(j1) 3-bit prime recognition circuit




(j2) grid layout of the 3-bit prime circuit




(j3) 3-bit prime recognition circuit