On the Computational Power of 6-bit Iterated Boolean Gate Arrays

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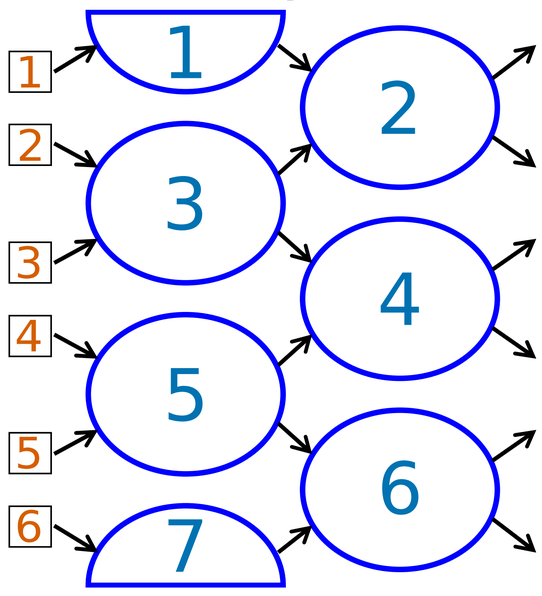
ENS DE LYON — Inria INVENTEURS DU MONDE NUMÉRIQUE

I. The Model

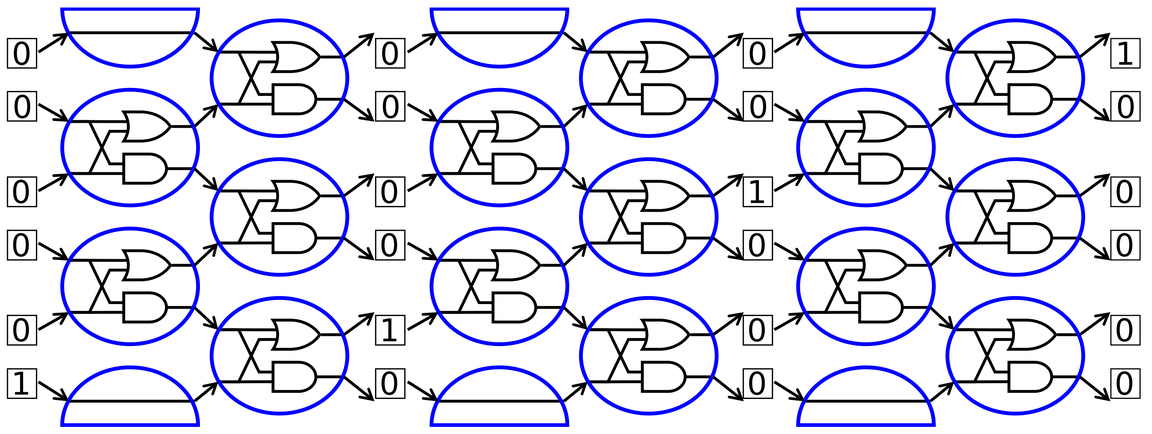
Proposed and implemented by *Woods, Doty, Myhrvold, Hui, Zhou, Yin, Winfree*, DNA 23 Track B, paper in preparation.

1) 6-bit Layer

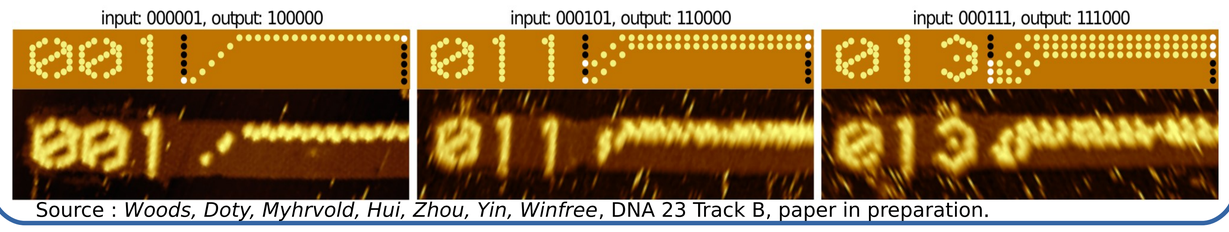


- **Programmer chooses 7 gates**
- **User chooses 6 input bits**
- **System computes by iterating**

2) Example: Sorting Bits



3) Implemented Using DNA Tiles



Source : *Woods, Doty, Myhrvold, Hui, Zhou, Yin, Winfree*, DNA 23 Track B, paper in preparation.

4) n-bit Layer

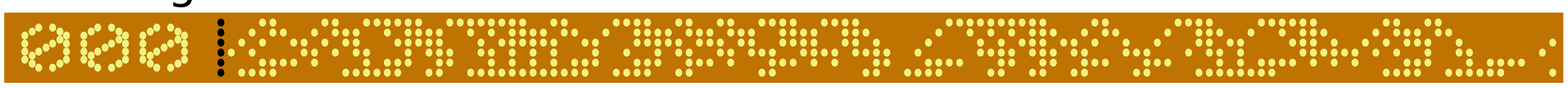
Can simulate any Turing machine

But

Bits are expensive

II. Two Counting Problems

Counting to 63:

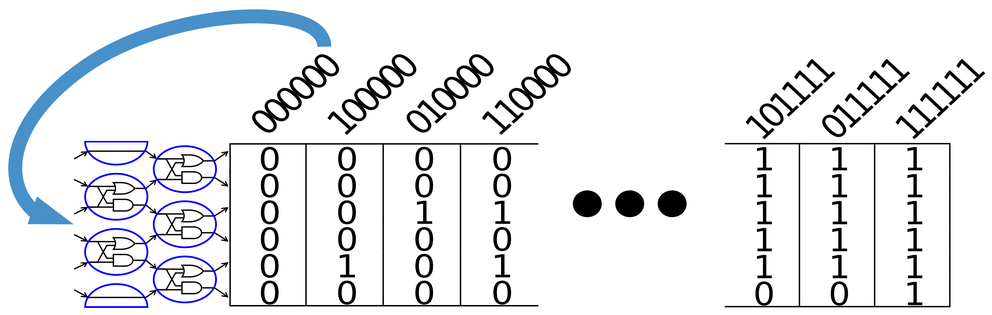


Is missing!

Is it possible to count to 64 in the 6-bit model? To 2^n in the n-bit?

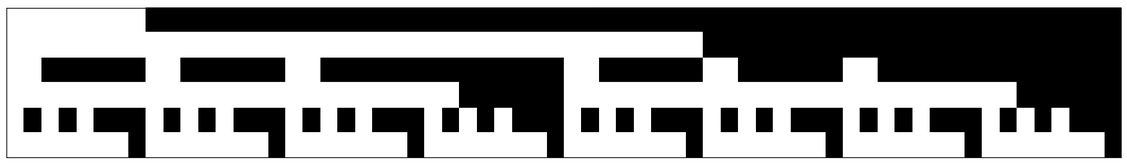
III. The Layer Function

1) Definition



The Layer Function is represented by a 6x64 binary image.

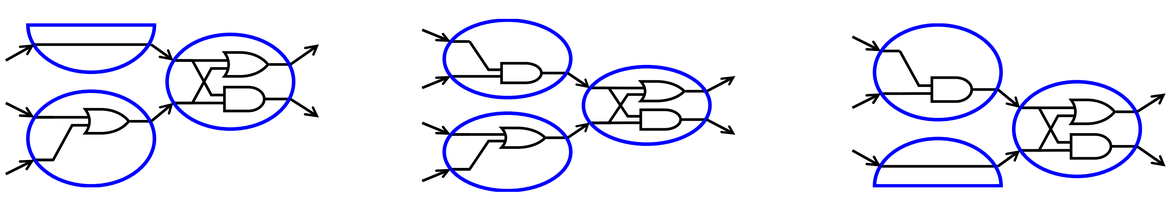
0 = □
1 = ■



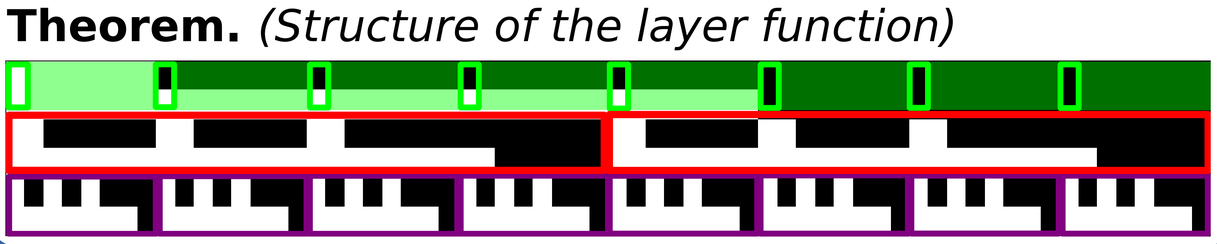
2) Why Focus on the Layer Function?

- Same layer function = same computation
- $2^{44} = 1.8*10^{14}$ 6-bit layers
- But merely $32*10^9$ layer functions
- More abstract and **structured** object

3) A Structural Result



Theorem. *(Structure of the layer function)*



IV. No 6-bit Counters

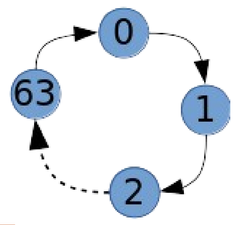
1) Candidate Layer Function

Bijection

i.e. Each string on 6 bits must appear once.

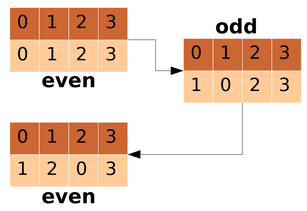
One 64-orbite

i.e.



0dd Bijection

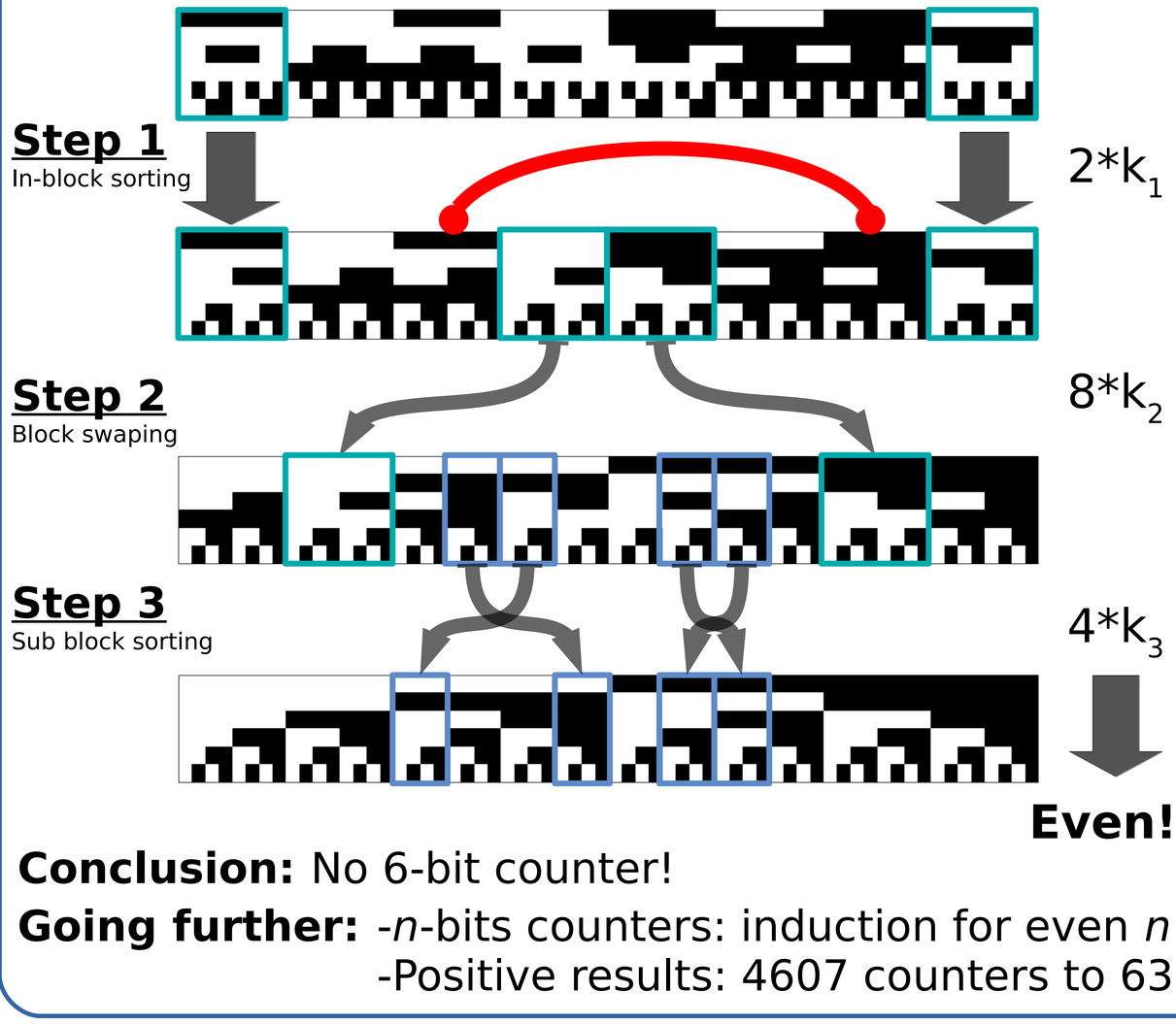
i.e.



Theorem. *(No odd bijections)*
The model produces only **even** bijections.

2) Proof

#Swaps



Even!!

Conclusion: No 6-bit counter!

Going further: -*n*-bits counters: induction for even *n*
-Positive results: 4607 counters to 63